Geometry, Gravity, Matter, Dark Energy

Utrecht UPAC Workshop
"Philosophy of Dark Energy"
13 Jan 2025

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(partially funded by an ERC Collaborator grant to Bonn
and a John Templeton Foundation grant to QISS)

Outline

[W]e must bear in mind that the scientific or science-producing value of the efforts made to answer these old standing questions is not to be measured by the prospect they afford us of ultimately obtaining a solution, but by their effect in stimulating men to a thorough investigation of nature. To propose a scientific question presupposes scientific knowledge, and the questions which exercise men's minds in the present state of science may very likely be such that a little more knowledge would shew us that no answer is possible. The scientific value of the question, How do bodies act on one another at a distance? is to be found in the stimulus it has given to investigations into the properties of the intervening medium.

– James Clerk Maxwell (1875)
"Attraction" (*Encyclopædia Britannica*, edition IX)

Mise en scène

Geometry versus Gravity versus Matter: First, Abortive Attempts

The Metaphysics of Matter and Spacetime Geometry: Substance versus Accident

The Metaphysics of Matter and Spacetime Geometry: Stress-Energy and a Deeper Analysis of Coupling

- Stress-Energy
- Comcomitants of the Metric
- Non-Existence of a Gravitational Stress-Energy Tensor and the Uniqueness of the Einstein Field Equation

The Ἀπορία of Dark Energy

in this talk, by 'dark energy' I mean the cosmological constant (Λ), not (*e.g.*) any commonly considered alternatives:

1. promote Newton's constant to dynamical degree of freedom by allowing it to depend on scalar field $\phi(t, r)$: *scalar-tensor theories*, most general known as *Horndeski theories* (Horndeski 1974; Kobayashi 2019)
2. promote Λ to dynamical degree of freedom, treating as scalar field with kinetic and potential terms: quintessence, K-essence, tachyon, phantom, dilatonic, ... (Caldwell 2002; Copeland et al. 2006; Scherrer 2022)
3. modify gravitational field equations, most often $f(R)$ theories, "add higher-order curvature terms to Einstein-Hilbert action" (de Felice and Tsujikawa 2010)
4. modifed dispersion relations in the trans-Planckian regime (Mersini et al. 2001)
5. call upon quantum gravity (QG) to save the day—I consider these too speculative, even for this most speculative branch of cosmology

main puzzle

how to determine whether the cosmological constant is "matter" or "geometry"?

$\Rightarrow$ *how do we distinguish "matter" from "geometry" at all in GR?*

from hereon, I'll use '*LHS*' and '*RHS*' to refer to sides of EFE, meaning respectively '*geometry terms*' and '*matter terms*'

why is it of interest to distinguish "geometry" from "matter"?

1. may want to treat the terms differently in attempts to modify/extend/quantize theory
2. may provide "deeper understanding" of conceptual structure of general relativity, and gravitational theories more generally
3. may provide "deeper understanding" of nature of the world
4. status of equivalence principle, *e.g.*, do test bodies follow geodesics? (if "new geometry terms" are stress-energy of exotic matter, yes; otherwise, in general no)

Mise en scène

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The Ἀπορία of Dark Energy

preliminary remark: standard versions of equivalence principle already suggest blurring of line between matter and geometry/gravity:

1. gravitational mass equals ("same as"!) inertial mass—property that sources geometry is property that is guided by geometry
2. but geometry "can source itself" (non-trivial vacuum spacetimes), and geometry can "guide itself" (characteristics of gravitational radiation)
3. $\Rightarrow$ no difference between geometry and matter here at all
4. presence of "effective gravitational field" even when no matter present and system "only accelerating" suggests, again, that the difference between matter and geometry/gravity is, perhaps not wholly "conventional", but, perhaps better, only effectively determined (in the sense of EFT, at least according to some QG programs)

in general relativity, "obvious" answer to "matter or geometry?" given by how matter and geometry couple, via EFE:

$$G_{ab} := R_{ab} - \frac{1}{2} g_{ab} = 8\pi T_{ab}$$

$$\Downarrow$$

matter is Ricci curvature and
gravity is Weyl curvature

unsatisfactory because in fact both matter and geometry (naively speaking) can constrain geometry:

- "geometry can source itself": non-trivial vacuum solutions
- causal structure and topology can, *e.g.*, by themselves place constraints on affine structure (singularity and black hole theorems in vacuum spacetimes)
- causal structure and topology can, *e.g.*, in conjunction with matter place constraints on affine structure (singularity and black hole theorems in non-vacuum spacetimes)
- indeed, there are highly non-trivial constraints that *every* type of spacetime geometry (causal, topological, projective, conformal, affine, curvature, metric) can place on each other (Curiel 2021), for the most part independent of the EFE and matter

interesting project: which of these inter-relations and contraints depend on EFE, which don't?

so, think about differences in respective symmetries of local degrees of freedom:

- for leptons generally $SU(2)$ or $U(1)$ (or some combination thereof), for hadrons $SU(3)$, $SU(2)$ or $U(1)$ (or some combination thereof), and same for exchange bosons
- one wants to say: in GR, for local gravitational degrees of freedom, the symmetry group is $O(1, 3)$ (or, if one restricts to the proper Lorentz group, $SL(2, \mathbb{C})$—
- but that works for curvature only when there is no matter present; when there is matter present coupling to the curvature. . .
- the "gravity" and "matter" groups mix, so one already needs to know what is gravity and what is matter to apply this criterion

we use matter to operationalize geometry, not conversely:

- rods and clocks (Einstein 1984; Brown 2005)! or paths of light rays and massive particles (Weyl 1918; Malament 2012)!
- BUT: restriction is not in principle, only from constraints imposed by our current epistemic control over GR
- *no principled reason* we can't use, *e.g.*, geons and gravitational radiation to operationalize rods and clocks

a few difficulties and problems for the whole idea, based on fungibility:

1. quantum effects in gravitational fields, *e.g.*, Hawking effect: "geometry transforms into matter"
2. gravitational collapse: "matter transforms into geometry (singularity)"
3. globally hyperbolic spacetimes vacuum everywhere except in bounded region [draw pretty picture]: "matter and geometry mutually transform into each other"
4. certain FLRWs smoothly extendible through $\mathscr{I}^-$ into Minkowski spacetime (*e.g.*) [draw pretty picture]: "flat vacuum spacetime erupting into both matter and gravity/non-trivial geometry"

   (and time-reverse for "both matter and gravity/non-trivial geometry condense into flat vacuum spacetime")

(can be formally represented by writing the Bianchi identity using the Lanczos tensor[1])

1. $\nabla_n C^n{}_{abc} = J_{abc} := \frac{1}{2}\nabla_{[b}R_{a]c} + \frac{1}{6}g_{c[a}\nabla_{b]}R$

moreover, in same vein:

- if Unruh radiation is related to excitation of "spatiotemporal" or "geometrical" micro-degrees of freedom—
- as one may expect from the equivalence principle or certain programs of QG—
- and it occurs in flat spacetime with a quantum field in its own vacuum state...
- then this suggests that distinction between "matter" and "geometry/gravity" breaks down severely already in Minkowski spacetime

$\Rightarrow$ may all suggest that "matter" and "geometry/gravity" just different manifestations of underlying "unified" entity...

like $\mathbf{E}$ and $\mathbf{B}$ in relation to F_{ab}

profound difference between the two in GR?

- makes sense to talk about changing matter against a fixed metric ("test matter")
- *BUT* makes *NO* sense to talk about unchanging, fixed matter on a changing metric ("test metric")...
- *because the metric is required to formulate the field equations and the stress-energy tensor of matter*
- but not vice-versa

I don't know what to think about this—suggestions?[2]

2. This may suggest, to the more metaphysically minded, that the proper way to understand the EFE is by taking the lefthand side to be "more fundamental" than the right (which can be spelled out in a number of ways, *e.g.*, as a causal relation), but I think that would be a mistake.

to sum up what hasn't worked so far:

1. difficulties already suggested by equivalence principle
2. how matter and gravity/geometry couple to and constrain each other (and themselves!)
3. symmetries of local degrees of freedom
4. possible operationalization procedures for each
5. AND difficulties suggested by seeming fungibility of matter and geometry in gravitational collapse, spacetimes with exotic conformal and affine structure, and Unruh and Hawking effects
6. and ambiguous observation about propriety of approximations and idealizations of metric versus matter

let's try some metaphysics!

but let's do it right

in particular, let's try different approaches, based on two criteria:

1. substance versus accident (body/physical system versus property/quantity)
2. deeper analysis of how matter and geometry couple in GR, focus on stress-energy

accidents ("properties", "quantities") in classical (Newtonian) physics:

- consider inertial mass
- to know only the inertial mass of a system does not determine the kind of system...
- for every system in classical mechanics has inertial mass
- to know that a classical system has shear-stress, and even to know its value, again, does not determine the type of system
- for, although not every classical system has a shear-stress tensor, many do, (*e.g.*, viscous fluids, elastic solids, ...)
- there never seems to be a single "aspect" of a classical system, for lack of a better word, such that to know the details of that aspect determines the kind of system it is

staying in classical Newtonian physics:

- for every kind of classical physical system, generally to know the value of every of its physical quantities at an instant except for one. . .[3]
- does not determine the value of that last quantity
- to know the position and velocity of a Newtonian particle at an instant does not tell you its acceleration
- to know the values of a Navier-Stokes fluid's shear and bulk viscosities, mass-density, flow velocity, flow acceleration and isotropic pressure at an instant does not tell you the value of its shear-stress tensor
- . . .
- there never seems to be a single aspect of it such that to know the details of the aspect tells you the values of all (or all the rest) of the system's quantities

3. That is to say, to know the values of the quantities relevant to physics of the system as treated by the theory one uses in the regime one considers.

– that is why, for systems treated by classical Newtonian physics, such things ought to be thought of as "properties" of physical systems, or "quantities" associated with physical systems, not physical systems in and of themselves

> *classical physics does not appear to admit a representation of "substance", of "the thing itself and the entire thing"*

– no explicit representation of "point particle", "rigid body", "fluid", "pendulum", "spring", . . . , in classical physics![4]

4. Stuff that in your pipe and smoke it, Quine.

field theories, however, even classical ones, are different

– consider a classical Maxwell field *in vacuo*

the Faraday tensor F_{ab} determines every other quantity associated with the Maxwell field itself:

1. field intensity and flux
2. stress-energy
3. energy density, Poynting vector, shear-stress in 3+1 decompositions
4. amplitude and frequency of $\mathbf{B}$ and $\mathbf{E}$ in 3+1
5. frequency (for a radiative Maxwell field), and wavelength and wave number as well) in 3+1
6. polarization in 3+1
7. *etc.*
8. and when coupled to external sources, the 4-d charge distribution

if I know properties of matter that Maxwell field interacts with (spatial configuration, permittivity, conductivity, susceptibility, permeability, ...), then can calculate further behavior and properties:

1. wave, phase and group velocities
2. coefficients of reflection and absorption
3. refrangibility
4. diffraction
5. displacement current
6. ...

- in short, the Faraday tensor determines the *entire state* of the field, under *all conditions*
- in the general case (no special symmetries, *etc.*), no subset of those quantities determines the Faraday tensor[5]
- *same for quantum fields*

– speed of light is interesting: it is determined by the permittivity and permeability of the media (including the vacuum); no worries, however, as it's a kinematical constant, so no reason to expect theory to determine it

5. Interesting problem: to determine the circumstances under which some subset does determine the state entirely.

F_{ab} seems to represent a *substance*—the Maxwell field—in the sense that it encodes, but is not encoded in, all accidents (quantities—amplitude, stress-energy, . . .) accruing to the substance[6]

6. Quine strikes back? NEVER!!!

criterion for being "substance-like" (body, physical system)

the one entity in the formalism of the theory that encodes—allows one to recover—all features of the thing relevant in the regime at issue

GR:

- G_{ab} is "substance-like" and T_{ab} is "accident-like"—
- to know the value of the Einstein tensor determines (in general) the metric, and so essentially everything about the spacetime[7]
- to know the values of the stress-energy tensor, in general, does not even determine the type of matter field...
- perfect fluid, Maxwell field, Dirac field ... (just as with charge-current in classical Maxwell theory)

7. Still open problem: necessary and sufficient conditions for a G_{ab} not to uniquely determine a metric. Solve it!

If we are to surround ourselves with a perceptual world at all, we must recognize as substance that which has some element of permanence. We may not be able to explain how the mind recognizes as substantial the world-tensor [i.e., the Einstein tensor], but we can see that it could not well recognize anything simpler. There are no doubt minds which have not this predisposition to regard as substantial the things which are permanent; but we shut them up in lunatic asylums.

– Arthur Eddington (1923)
Mathematical Theory of Relativity (pp. 120–121)

- a ***genus*** (or ***kind***) of physical field is characterized in part by the fact that it interacts with other types by couplings of different forms and strengths, fixed unambiguously for each other type of system

- but what counts as a coupling is characterized in part by the physical dimensions of the coupling-coefficient terms appearing in the constitutive relations

consider 2 of the Maxwell equations:

$$\dot{\mathbf{B}} = -\frac{1}{c}\boldsymbol{\nabla} \times \mathbf{E}$$

versus

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{1}{\varepsilon_0}\rho$$

1. c is dimensionless (to be explained shortly): $\mathbf{E}$ and $\mathbf{B}$ don't couple—they are different manifestations of underlying unified entity F_{ab}
2. ε_0 is dimensionful: $\mathbf{E}$ and ρ couple—they are different types of matter field (or properties of different matter fields)

necessary condition for "geometry"

all the terms to count as "geometry" must have same physical dimension without modification by multiplication by various dimensionful constants term by term, because geometry should couple *en bloc* with all other fields in the same way, ***on pain of there being "different" genera of geometry in a radical sense***

1. "the equivalence principle"
2. "the LHS entirely arises from the same type of physical entity"

One fundamental and characteristic property of "matter":

it possesses stress-energy as represented by a T_{ab}

some entities may possess quantities similar to stress-energy only not representable by a tensor, and so not "localizable", such as "gravity" (gravitational radiation, *e.g.*) in GR (Curiel 2019)

– all "stress-energy" ultimately will be included in $8\pi\gamma T_{ab}$ (the RHS), and so ought to have the "physical dimension" of T_{ab}[8]

– ***but*** this can come about by multiplying T_{ab} by a dimensionful coupling constant, possibly different for each type of matter

8. I now explicitly write Newton's constant γ, for reasons soon to be clear.

Properties of T_{ab}

10 components 6 for the classical stress tensor; 3 for linear momentum; 1 for scalar energy density

two covariant indices linear mapping from tangent vector of worldline to 4-momentum covector of field; bi-linear mapping from tangent vectors to energy-density—momental phenomena linear in velocity, energetic phenomena quadratic in velocity

symmetric infinitesimally, yields "conservation of angular momentum"; part of relativistic equivalence of momentum flux and energy density

Properties of T_{ab}

covariantly divergence-free infinitesimally, yields "classical energy and linear momentum conservation"; part of relativistic equivalence of momentum flux and energy density

a tensor the localization of ponderable stress-energy and its invariance as a physical quantity (multi-linear map acting only on the tangent plane)

all stress-energy tensors have same physical dimension thermodynamic fungibility of energetic phenomena

The Thermodynamical Fungibility of Stress-Energy

The First Law of Thermodynamics in Curved Spacetime:[9]

> all forms of stress-energy are in principle ultimately fungible: any form of stress-energy can in principle be transformed into any other form.

$\Rightarrow$ the family of "all possible" T_{ab} has a natural linear structure, and thus all stress-energy tensors must have the "physical dimension of stress-energy" in order to make physical sense of adding them together

9. This is a necessary precondition not only for the standard First Law in ordinary thermodynamics, but for any general principle of energy conservation outside context of GR.

Physical Dimension

- "geometrized units": all fundamental physical units in GR expressible as a constant multiple of a single fundamental unit...
- say time (*e.g.*, traversal time of a proton's diameter by light)
- to multiply the metric by a constant rescales the measure of time in a fixed way, and so fixes rescalings of all other physical units
- in particular, *e.g.*, length and mass definable by time in such a way that they rescale the same as time
- and so all other units derivable therefrom with fixed scalings depending on the combination of `t`, `l`, `m`)

- thus, necessary condition for a quantity to have a given physical dimension is that it transform under constant re-scaling of metric in a particular way

- in particular, stress-energy remains fixed under constant re-scaling

- from EFE: G_{ab} remains fixed (simple calculation), as does γ $(= \frac{(\texttt{length})^3}{(\texttt{mass})\,(\texttt{time})^2})$

this may still come about because the “bare” stress-energy needs to be multiplied by a dimensionful coupling constant in order for the total to re-scale properly

Concomitant: tensor (or more general geometric entity) that is "invariant function" of other tensors (or more general geometric entities): mapping from fiber bundle to fiber bundle

a little more precisely, A is a concomitant of B if the action of the mapping $\chi : A \to B$ commutes with the natural action of diffeomorphisms on each:

$$\phi_B(\xi(A)) = \xi(\phi_A(A)(A)$$

where 'ϕ_A' is the natural lift of the spacetime manifold diffeomorphism ϕ to the fiber bundle A lives on, *etc*.

if there is a gravitational stress-energy tensor, it ought to be a concomitant of the metric or Riemann tensor

Jet Bundles

A bundle over a primary fiber bundle capturing the idea of a generalized derivative of a geometrical object—"equivalence of Taylor-series expansions to n^{th} order" gives $J^n\mathcal{B}$, n^{th}-order jet bundle over $\mathcal{B}$

(one should generally use the invariant formulation—obvs; see Curiel 2019)

n^{th}-order Concomitant: tensor (or more general geometric entity) that is "invariant function" from n^{th} jet bundle of other tensors (or more general geometric entities

Concomitants of the Metric

Theorem (Curiel, a long time ago, published 2019)

1. *First jet bundle of bundle of metrics, $J^1\mathcal{B}_g$, is naturally diffeomorphic to the fiber bundle with fibers consisting of all (g_{ab}, ∇_a);*
2. *the second jet bundle, $J^2\mathcal{B}_g$, is naturally diffeomorphic to the fiber bundle with fibers consisting of all $(g_{ab}, \nabla_a, R^a{}_{bcd})$.*

Corollary

All concomitants of Riemann tensor are second-order concomitants of metric.

Homogeneous Concomitants

A concomitant is ***homogeneous of weight*** $\mathbf{w}$ if for any constant scalar field σ

$$\chi(\phi(\sigma u_1)) = \sigma^w \phi(\chi(u_1))$$

Homogeneous Concomitants of the Metric

Lemma

If, for $n \geq 2$, S_{ab} is an n^{th}-order homogeneous concomitant of g_{ab}, then to rescale the metric by the constant real number λ multiplies S_{ab} by λ^{n-2}.

1. S_{ab} has physical dimension of stress-energy only if it is zero-weight homogeneous concomitant of metric
2. in particular, S_{ab} does not rescale when $g_{ab} \to \lambda g_{ab}$ if and only if it is a second-order concomitant of g_{ab}, and so zeroth-order concomitant of $R^{a}{}_{bcd}$

Demand Vanishing of Covariant Derivative?

Can the "gravitational field" interact with ponderable matter fields so as to exchange stress-energy? If so, then, presumably, there could be interaction states characterized (in part) by:

1. $\nabla^n(T_{na} + S_{na}) = 0$
2. $\nabla^n T_{na} \neq 0$
3. $\nabla^n S_{na} \neq 0$

to avoid deep and speculative waters about the way S_{ab} might enter the righthand side of the Einstein Field Equation, safest to make partial demand: vanishing covariant divergence when there is no ponderable matter ($T_{ab} = 0$)

to sum up, we have the following necessary condition:

the only viable candidates for a gravitational stress-energy tensor are:

1. two covariant-index, symmetric, second-order, zero-weight homogeneous concomitants of the metric
2. that are not zero when the Riemann tensor is not zero
3. and that have vanishing covariant divergence when the stress-energy tensor of ponderable matter vanishes

There Is No Gravitational Stress-Energy Tensor

Theorem (Curiel, a long time ago, published 2019)

The only two covariant-index, divergence-free concomitants of the metric that are homogeneous of zero weight are constant multiples of the Einstein tensor.

Corollary (Curiel, a long time ago, published 2019)

There are no two covariant-index, divergence-free concomitants of the metric that are homogeneous of weight zero that do not identically vanish when the Riemann tensor is not zero.

a different interpretation, relevant to our purposes: *the uniqueness of the Einstein field equation in all dimensions*

stronger than classic Lovelock theorem:

1. does not assume that concomitant is at most second-order in partial-derivatives of the metric
2. holds in all dimensions, not just 4
3. physical perspecuity of assumption ("having physical dimension of stress-energy", based on re-scaling behavior)
4. simple, straightforward, geometrical proof

Upshot:

1. all other terms constructed from the Riemann tensor and other geometrical entities (*e.g.*, the metric or topological invariants) must be multiplied by dimensionful coupling constants for consistency
2. thus any such term ought to be considered a stress-energy tensor for an ("exotic") matter field, and so put on the RHS
3. in particular, Λg_{ab} ought to be put on the RHS

interesting aside: Jacobson's (1995) and Padmanabhan's (2005; 2009) derivations of EFE (and Lanczos-Lovelock theories) from thermodynamic considerations are special cases of the theorem

In general: any such term that requires dimensionful coupling constant, whether concomitant of Riemann or not, ought to be put on RHS of EFE and considered to be stress-energy of a form of matter, in the framework of GR

To the philosopher, the difference between "matter" and "empty space" is, I believe, merely a difference as to the causal laws governing succession of events, not a difference expressible as that between the presence or absence of substance, or as that between one kind of substance and another.

– Bertrand Russell (1927)
The Analysis of Matter
(part I, ch. XIII, pp. 121–122)

Question

- how does the criterion for "being substance-like" square (if at all) with criteria based on coupling constants?
- do we need them to deliver the same verdict?
- if they don't, is each capturing something orthogonal to the other, in so far (*e.g.*) as geometry may also have "substance-like" properties?

prima facie. . .

substance versus accident

- Λg_{ab} tells you the metric (up to a constant), so it seems like substance: *put it on LHS!*

stress-energy and coupling constants

- Λ is a dimensionful coupling constant, so Λg_{ab} is a stress-energy tensor: *put it on RHS!*

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