Singularities in Classical and Semi-Classical Gravity—Theory Failure or Groovy New Physics?

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Outline

Propædeutic to Singularities in General Relativity

Cosmic Censorship and Determinism

Singularities and Cosmic Censorship in Semi-Classical Gravity

Breakdown of Theory or Groovy New Physics?

Propædeutic to Singularities in General Relativity

Cosmic Censorship and Determinism

Singularities and Cosmic Censorship in Semi-Classical Gravity

Breakdown of Theory or Groovy New Physics?

incomplete worldlines

- sometimes worldlines "just end" after a finite period of time
- cannot extend them in the spacetime, cannot extend spacetime itself
- no event "at which they terminate": breakdown of spacetime geometry?
- observer would experience only finite length of time of existence without ever dying or ceasing to exist in any way

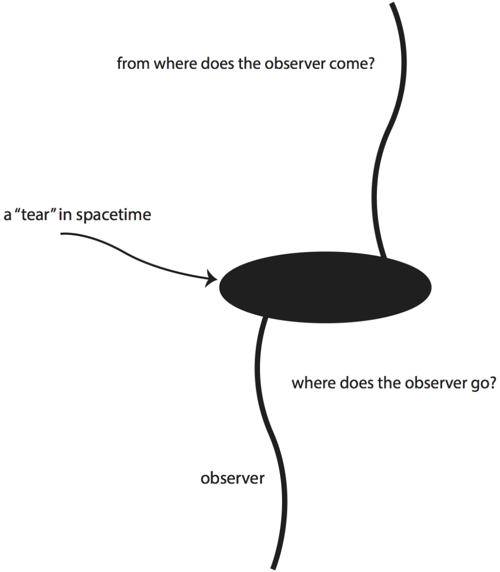

Figure: incomplete curves due to "holes" in spacetime (taken from Curiel 2019)

singular structure: canonical definition schema

Definition (schema)

in a spacetime satisfying a maximality condition: an inextendible causal curve incomplete with respect to some measure of "length"

places almost no substantive constraint on nature or physical properties of spacetime

("schema" because "maximality condition" and "some measure of length" need to be specified)

Examples

- delete from spacetime set in shape of Ingrid Bergman—but seems like cheating
- blow-up of spacetime curvature in gravitational collapse: metric "loses definition at $r = 0$"
- some incomplete, inextendible curves in spacetimes with no surgery and no pathology in curvature (Taub-NUT, colliding plane gravitational waves, . . .)
- generic features of otherwise "well behaved" spacetimes (Penrose, Geroch, and Hawking singularity theorems)

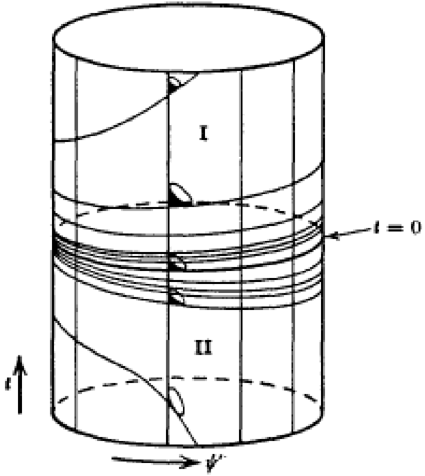

Figure: Misner's 2-d Taub-NUT (taken from Hawking and Ellis 1973)

incomplete, inextendible null geodesics "bunch up" at $t = 0$, but timelike curves pass through them without problem

Physical Problems

- GR is time-reversible: worldline ("observer") could just "pop into existence" right in middle of spacetime ("Principle of Sufficient Reason"!)
- any physical mechanism that could cause such a thing, going forward or backward in time?
- how to formulate laws when spacetime geometry loses definition (curvature pathology)?
- physical significance of curvature pathology? (often ambiguous, depends on motion of observer probing spacetime)
- spacetimes can be timelike-complete but null-incomplete, and vice-versa: how to understand this physically?
- spacetimes can be geodesically complete, but have incomplete timelike curves of bounded total acceleration

(Earman 1995; Curiel 1999)

Philosophical Problems

- plethora of definitions (not only ones based on canonical schema): such a thing as "the single, correct definition"?
- if not, what is relation among all the viable definitions? are they different explications of the same concept?
- ambiguity, meaning of different notions of "length" used to determine incompleteness
- choice of reference class of spacetimes to define "maximality" (Manchak 2016, 2017; Doboszewski 2017, 2019)
- "to exist is to exist in space and time": incompleteness of curves don't—what sort of existence to attribute to them?
- breakdown of determinism?
- breakdown of GR as physical theory?
- what to go on besides intuitions of eminent physicists?

(Earman 1995; Curiel 1999)

Psychological Problems: The Finitude of Existence

The mind of man, by nature a monist, cannot accept two nothings; he knows there has been one nothing, his biological inexistence in the infinite past, for his memory is utterly blank, and that nothingness, being, as it were, past, is not too hard to endure. But a second nothingness—which perhaps might not be so hard to bear either—is logically unacceptable.

Ada
V. Nabokov

the immodesty of naked singularities

Penrose (1969):

> *Even if [the No-Hair Theorem], or something like it, is true, have we any right to suggest that the only type of collapse which can occur is one in which the space-time singularities lie hidden, deep inside the protective shielding of an absolute event horizon? .... [If not, it would be] possible for information to escape from the singularity to the outside world.... In short, the singularity is visible, in all its nakedness, to the outside world! ....*
>
> *We are thus presented with what is perhaps the most fundamental unanswered question of general-relativistic collapse theory, namely: does there exist a "cosmic censor" who forbids the appearance of naked singularities, clothing each one in an absolute event horizon?*

cosmic censorship can't just be "no visible (naked) singularities"

there's one in our universe!

the big bang

two responses

1. refine what one means by “naked singularity”
2. provide necessary and sufficient conditions for “cosmic censorship” in general, independent of the idea of a naked singularity

modern impetus

ensure determinism/predictability, "physics as we know it"

a breakdown in the fundamental structure of spacetime so severe that it could wreak havoc anywhere it were visible

- the structures that break down are required for the formulation of known physical laws
- and in particular for initial-value problems

$\Rightarrow$ determinism and predictability, perhaps the possibility of physics as we know it, would collapse wherever the pathology were causally visible

but what determinism?

there are many forms of "determinism", and they can all fail in different ways

(Earman 1986, 1995; Doboszewski 2017, 2019)

well-posed Cauchy problem

example: one standard “strong” formulation

The maximal globally hyperbolic future developments of generic asymptotically flat initial data are future inextendible to regular Lorentzian manifolds.

very popular!

BUT:

- anything remotely resembling a proof depends on assuming auxiliary conditions (*e.g.*, energy or causality conditions) that themselves are as weakly supported or otherwise problematic as cosmic censorship itself
- problems with ambiguity of "inextendibility of spacetime" (Manchak 2016, 2017; Doboszewski 2017, 2019)
- problems with making "generic" well defined and physically significant (Curiel 2017)
- counter-examples have been found to essentially every precisely formulated proposal along those lines (and all others), leading to *weak* "strong formulations" (*e.g.*, ruling out only *discontinuous* extensions!)

several open and serious conceptual problems

1. there are *many* different formulations like this, none equivalent, all capturing subtlely different conceptions, all with their own conceptual virtues and demerits—which to choose? (Earman 2020, personal communication)
2. $3{+}1 \neq 4$ (canonical formulation – evolved globally hyperbolic – is not GR, neither mathematically, nor physically, nor conceptually)
3. why focus on indeterminism rather than other forms of "pathology"?
4. why expect that one principle will cover all important cases?
5. what justifies the urge to find such a thing in the first place?

why one principle?

Why try to find *The* Principle of Cosmic Censorship, rather than several different propositions that make precise and capture different aspects of what we roughly and crudely have an idea we want, each relevant in its own context but none applicable universally?

An unrepentant and degenerate old Aristotelian essentialism, I think, that *demands* a single definition for what we use a single name for.

why look for it in the first place?

What justifies the urge to find such a thing in the first place? What reasons may we have beyond the brute deliverance of entrenched physical theory—which we don't have—to expect such a proposition may hold?

three kinds of physical principles

1. those required by the physics (for any of several reasons)
2. those required on pragmatic grounds (any of several)
3. those desired for psychological comfort

required by the physics

1. conflicts with another *prima facie* fundamental, entrenched principle (conservation of energy, *e.g.*)
2. blocks theoretical development (QG, *e.g.*)
3. ...

required on pragmatic grounds

1. conflicts with how we like to "do" physics (predictability based on IVPs, *e.g.*)
2. makes modeling systems difficult
3. ...

desired for psychological comfort

1. conflicts with how we like to "do" physics (predictability based on IVPs, *e.g.*)
2. aesthetic grounds
3. fear of unpredictability
4. neurotic Cartesian phobia of uncertainty
5. ...

the demand for cosmic censorship, as the ethically evaluative flavor of its very name suggests, and as the connection with determinism shows, arises from both pragmatic and psychological grounds

not physical grounds

in sum, we have. . .

an example of a sizable subculture in physics working on matters that:

- have no clearly or even unambiguously defined physical parameters to inform the investigations
- have no empirical evidence to guide or even just constrain them
- have the parameters of the debate imposed by and large by the intuitions of a handful of leading researchers

From sociological, physical, and philosophical vantage points, one may well wonder why so many physicists continue to work on it, and what sort of investigation they are engaged in.

52 years of Cosmic Censorship...

and still we do not know what it is or why we should care this is not to say that mathematical work on the initial-value formulation is not interesting and important in and of itself—only that it is not clear to me that the attempt to "resolve the problem of cosmic censorship" ought to frame and motivate that work

cf. the, to my mind, unhealthy obsession with differentiability class of extensions—can it possibly make a *physical* difference whether a metric is $C^{1,1}$ but not C^2 extendible, to take what is these days a tame example, especially given that no one expects GR to be a "final" theory anyway? do we seriously expect such a difference can register in the kinds of "emergence of classical spacetime geometry" that an adequate theory of QG will admit (except perhaps in absurdly peculiar and isolated cases)?

(to be fair: some conceptually oriented mathematical physicists have given this thought and have interesting things to say in response; *cf*. recent talk given by Martin Lesourd at BHI)

semi-classical Einstein field equation (SCEFE)

$$G_{ab} = 8\pi\langle\hat{T}_{ab}\rangle$$

(classical Einstein tensor =
expectation value of stress-energy tensor as quantum operator)

many fascinating mathematical, physical, conceptual, philosophical problems...

which we, sadly but expeditiously, here ignore semi-classical gravity (SCG) promiscuously, with joyous and libertine, even Corybantic, abandon, violates every energy condition under the sun—whither the singularity theorems? (and indeed essentially every important theorem about black holes as well, but we pretend not to care about that)

do we not need to await QG to efface singularities? does SCG do it for us already?

not so fast!

Theorem (Wall 2013)

Suppose there exists a globally hyperbolic spacetime with a quantum trapped surface T [i.e., a certain kind of null surface with semi-classical entropy decreasing to the future, as one expects to occur generically in collapse black holes]. Let the semi-classical approximation be valid near T (but not necessarily elsewhere). Then the fine-grained Generalized Second Law requires that spacetime is not null-geodesically complete.

fascinating! singularity formation is thermodynamically preferred—if anything like the GSL holds in QG, there seems no escape. . .

and there's more:

1. Fewster and Kontou (2020): like classic singularity theorems, but with weakened energy conditions ("quantum energy inequalities") that we expect to hold for evaporating black holes
2. Freivogel et al. (2020): more singularity theorems with even weaker energy conditions, also applicable to evaporating black hole spacetimes
3. Minguzzi (2020): a singularity theorem with causality conditions relaxed enough to accommodate expected pathologies in evaporating black hole spacetimes

BUT: are complete causal curves compatible with QG in the first place?

any curve heading into a region where (our descriptive resources for) classical spacetime geometry breaks down will *ipso facto* be incomplete and inextendible, *according to the classical theory* straightforward violation of weak cosmic censorship?

- Hod (2018, 2019): Hawking radiation can turn near-extremal black-hole spacetimes into horizonless naked singularities
- BUT: unclear whether effect is stable under asymmetric perturbations (depends on suppression of large angular momentum modes of Hawking radiation, seemingly based on spherical symmetry)

other worries related to cosmic censorship in SCG

trapped surfaces outside event horizons:

1. happens in black hole evaporation (Bousso and Engelhardt 2016): should we worry?
2. many important BH theorems depend on negation
3. seems to suggest singularities can appear outside event horizon (but problem with singularity theorems: they don't allow us to "locate" singularity, except Lesourd 2018a)

but:

1. Engelhardt and Folkestad (2020): in holographic spacetimes (*i.e.*, with well behaved dual CFTs), trapped surfaces lie inside horizons—but their assumptions are suspect, seem question-begging (even though their results are formulated for *classical* spacetimes in the bulk, this still has direct relevance to SCG in so far as they do not assume an energy condition; their definition of "evaporating singularity" in classical spacetime is problematic)
2. still, if right: evaporating black holes are not holographic?! whither all the sanctimonious holographic "proofs" of unitarity?

other worries about evaporating black holes, besides the standard "loss of unitarity and Page-time" problems:

1. Lesourd (2018b): evaporating black hole spacetimes are causally discontinuous—a violation of something morally equivalent to some form of cosmic censorship
2. this should be *seriously* disturbing: no way unitarity could survive this
3. ...

what hangs on cosmic censorship in SCG?

1. many theorems in BHT require, *inter alia*, trapped surfaces behind event horizon, and spacetime to be strongly asymptotically predictable
2. Hawking radiation itself?
3. definitions of temperature/entropy of BH?

energy conditions in SCG:

1. seems unlikely that violation in evaporating black hole spacetimes will suppress singularities, as no significant violations appear until well after collapse—need calculations to show that, *e.g.*, immediate influx of negative-energy modes of Hawking radiation would suppress formation of trapped surfaces to argue otherwise, I think
2. violations affect other laws of BHT?
3. in particular, GSL requires energy conditions/quantum energy inequalities for proof?

where does GR breakdown? what is "high-energy regime"?

1. "large stress-energy tensor"—no physically significant norms known
2. "large curvature"—ditto
3. "large curvature scalars"—no relation to singularities
4. "high gravitational energy"—no known invariant formulations

$\Rightarrow$ no unambiguous demarcation of regime

Role of Einstein Field Equation?

for all classical singularity theorems, I claim: *none* depends on it!

in very strong sense. . .

assume logical negation of the EFE and theorems still hold

Wall's theorem may depend on the EFE or SCEFE—it's a subtle issue, depending on how one interprets Bekenstein entropy and the GSL; none of the other semi-classical singularity theorems depend on the SCEFE

Actual Role of Einstein Field Equation:

provides physical interpretation

gives physical interpretation to conditions on “purely geometrical” objects

logical form of arguments:

1. assume condition on some geometrical object (Einstein, Ricci, . . .)
2. derive mathematical theorem
3. invoke EFE to give physical interpretation of condition assumed in step 1 and result derived in step 2

if singularity theorems do not depend on EFE, in what sense can they mark a breakdown of *general relativity* itself?

- what actually "breaks down", if anything, is the "idea" of Lorentzian geometry, not GR itself
- but it is difficult to make this precise: a singular spacetime, after all, is *everywhere* a well defined Lorentzian manifold
- illuminating comparison: divergence of kinetic energy terms in finite time in solutions of Navier-Stokes equations—that *really is* a breakdown of the terms of the theory, for the KE terms become ill-defined at a determinate point of space at a determinate instant of time

Of course, GR *depends on* Lorentzian geometry, but the “breakdown” here, if any, does not come from the EFE itself, but rather generic features of Lorentzian geometry, such as the Raychaudhuri equation.

So the breakdown, if there is one, seems to be:

- conditions for the definition of structures required for the formulation of the theory no longer apply
- or, perhaps: since the EFE only gives physical interpretation to conditions on Ricci tensor, breakdown is in possibility of giving physical interpretation to geometrical structures in the conceptual terms of the theory
- when *Ricci* curvature pathology is involved, that may mark a breakdown of the theory (or theories) of *matter* at issue, but, again, seemingly not GR

But keep in mind: singularities do not even herald the breakdown of the theory, in any straightforward sense, because the Lorentzian geometry is always, almost by definition, *well defined at every point of the spacetime manifold*.

That various measures of curvature (or derivatives of curvature) grow without bound along certain "truncated" curves may bother us psychologically, and may even be wrong as a prediction, but that does not necessarily mean the theory has "broken down" in any interesting sense. It may have just made a bad prediction.

Now, it may be that this particular bad prediction (if it be one) is fruitfully understood as indicating a regime where the effects of phenomena not treated by GR (*e.g.*, "quantum" or "thermodynamically non-equilibrium" behavior of some geometrical structure) become non-negligible compared to the scales at which GR is appropriately applied and predictively adequate. But it may not.

Compare Newtonian mechanics applied to bodies with arbitrarily high velocities. I think this is not fruitfully understood as a case in which the theory "breaks down", even though some quantities, *e.g.*, linear momentum and kinetic energy, grow without bound. I think it is rather that the theory makes bad predictions, and we now have one that does a better job (*viz.*, SR).

Briefly, then, what I mean by “breakdown” here is: the theory is no longer appropriately applied, *i.e.*, it fails to have the conceptual or mathematical resources needed to construct a model of the system at issue that can be checked for predictive accuracy.

If singularities herald a breakdown of GR, then it means that, as a theory, it does not have the conceptual or mathematical resources needed to model the relevantly “quantum” or “thermodynamically non-equilibrium” behavior that, *ex hypothesi*, manifests in regions where GR predicts strong curvature. But that has nothing to do with singularities.

N.b.: as remarked earlier, if classical "curvature blowup" heralds the need to use funky quantum structures to model a "region of spacetime", then *ipso facto* any classical curve that enters that region will be "incomplete and inextendible" according to GR...

so what exactly is this promise of "singularity resolution" by QG anyway?

Given the available data, as admittedly sparse as it is, I think that there are two possible retrodictions that best fit our best current theoretical understanding of early-state cosmology:

1. there is an initial singularity, an entirely generic prediction of classical cosmological models (*e.g.*, perturbed FLRW) in both spacetimes with a pre-inflationary phase and ones without inflation;
2. with either a pre-inflationary phase or one without inflation, eventually when one goes far enough back in dynamical evolution then QG phenomenology takes over and all bets are off about what "spacetime structure" is possible.

- even in the second possibility, it is still the case that, as modeled strictly by the classical theory, all past-directed causal curves still effectively "end" at finite proper time, in exactly the way that true singularities do in wholly classical spacetime models
- thus, from the point of view of analyzing the conceptual structure of the purely classical theory, this still in effect presents us with a situation in which there are "singularities visible to the past" of non-trivial regions of spacetime that don't cause any problems with determinism, in this particular way:
    1. there's no way to make sense, in the context of the classical theory, of doing anything like "using it as an initial-data surface"
    2. but, once stuff has randomly spewed out of it, as it were—in this case, the beginning of the universe as we know it (or at least something like its FLRW model)—then determinism takes over at any finite time thereafter, and we may cogently conceive of this simply as a (perhaps provocative, but still coherent) fixing of initial conditions

so maybe there really are "naked singularities", and they spew out old socks and TVs playing Nixon's "Checkers Speech" (in Earman's wonderfully evocative—and horrifying—conceit). . .

and if so, then we know that their behavior will not be governed by GR—but we could then begin to investigate their behavior as we would any other initially strange physical system, to see whether we can find patterns in it amenable to description by laws in the sense of "physics as know it"; maybe they do spew out old socks, but perhaps only when stimulated by the absorption of inter-stellar Hydrogen in particular ways after all, we already know of one case (the big bang) in which entrenched physical laws governing stuff "near but not at" a naked singularity still impose constraints on what "the singularity can do"

if there were a true big bang singularity, the fact that it spewed out stuff that immediately lent itself to accommodation by physics as we know it perhaps would give us some (minimal) confidence that other singularities would be so manageable as well

and so deliver us the joy of groovy new physics to investigate with a *frisson* of concupiscent pleasure

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