A Glossary for Riemann's "On the Hypotheses, Which Lie at the Basis of Geometry"

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Riemann (1854) writes in a somewhat archaic style, and uses terms whose sense may not be immediately clear. He also uses some mathematical terminology, which may not be familiar to the reader. I give a glossary, which is intended only as a rough guide to his usage; it does not pretend to give precise or thorough explications of the terms. The reader should work those out for him- or herself.

magnitude-concept a measure of "size" used to quantify the magnitude of any instance of a given concept; "length", *e.g.*, is a magnitude-concept used to quantify spatial measurements; "number" is a magnitude-concept used to quantify the counting of discrete objects such as apples

mode of specification a unit or standard of magnitude, used to fix the "amount" of an instance of a given concept, as measured by the associated magnitude-concept; "meter", *e.g.*, is a mode of specification of spatial length, "5 meters" a fixed value of that mode; "dozen" is a mode of specification of the magnitude of a collection of apples; modes can vary either continuously (as for spatial lengths) or discretely (as for apples)

multiply extended magnitude a concept with an associated fixed number of magnitude-concepts, each of which must be specified according to its mode in order to individuate and identify an instance of that concept; ordinary physical space, *e.g.*, is a triply extended magnitude, because it needs three spatial lengths (coordinates, say, in a fixed coordinate system) to fix one of its points; the space of visual colors is also a triply extended magnitude, with three different types of magnitude-concepts, say, hue, saturation and brightness, that one must fix values for in order to fix an individual point in the space

manifold a collection of points or elements (objects, entities) that has the structure of a multiply extended magnitude, *i.e.*, has a fixed number of associated modes of specification of magnitude-concepts; the collection of all points of physical space is a 3-tuply extended, continuously varying manifold; the collection of all possible physical colors is as well, since an individual physical color can be uniquely identified by the values (modes of specification) of its hue, saturation and brightness, all of which vary continuously

measure relation on a manifold, a relation between pairs of points or elements that quantifies a notion of "distance", or "separation" more generally, between the pair (for *cognoscenti*: a Riemannian metric); correlatively or derivatively (depending on one's method of presentation), these relations also include other quantitative relations among geometrical objects living in the manifold, such as the angle between two intersecting curves (conformal structure), the volume of a solid figure (volume element), the intrinsic curvature of a curve, *etc.*

extension (or domain) relation a relation among points of a manifold that depends only on the modes of specification used to identify a point of the manifold, as opposed to a measure relation which imposes additional structure; unboundedness is an extension relation, because it is qualitative and not quantitative, as opposed to infinitude, which is a measure relation because it is quantitative; (for *cognoscenti*: the extension relations are the differential structure and topology of a manifold)

References

Riemann, B. (1854). Über die Hypothesen, welche der Geometrie zu Grunde liegen. Habilitationsschrift, Königlichen Gesellschaft der Wissenschaften zu Göttingen. English translation by Howard Stein (unpublished), "On the Hypotheses, Which Lie at the Basis of Geometry", available at http://strangebeautiful.com/other-minds.html#riemann.