

On the Existence of Spacetime Structure

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ABSTRACT

I examine the debate between substantivalists and relationalists about the ontological character of spacetime and conclude it is not well posed. I argue that the hole argument does not bear on the debate, because it provides no clear criterion to distinguish the positions. I propose two such precise criteria and construct separate arguments based on each to yield contrary conclusions, one supportive of something like relationalism and the other of something like substantivalism. The lesson is that one must fix an investigative context in order to make such criteria precise, but different investigative contexts yield inconsistent results. I examine questions of existence about spacetime structures other than the spacetime manifold itself to argue that it is more fruitful to focus on pragmatic issues of physicality, a notion that lends itself to several different explications, all of philosophical interest, none privileged *a priori* over any of the others. I conclude by suggesting an extension of the lessons of my arguments to the broader debate between realists and instrumentalists.

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[...] we must bear in mind that the scientific or science-producing value of the efforts made to answer these old standing questions is not to be measured by the prospect they afford us of ultimately obtaining a solution, but by their effect in stimulating men to a thorough investigation of nature. To propose a scientific question presupposes scientific knowledge, and the questions which exercise men's minds in the present state of science may very likely be such that a little more knowledge

would shew us that no answer is possible. The scientific value of the question, How do bodies act on one another at a distance? is to be found in the stimulus it has given to investigations into the properties of the intervening medium. (Maxwell [1965b])

[...] between a cogent and enlightened 'realism' and a sophisticated 'instrumentalism' there is no significant difference—no difference that *makes* a difference. (Stein [1989])

1 Introduction

The revival of the debate in the philosophical community over the ontic status of spacetime can trace its roots, in part, to its revival in the community of physicists. Belot ([1996]) and Belot and Earman ([2001]), for instance, claim that philosophers ought to take the debate seriously because many physicists do. I do not think that fact suffices as reason for philosophers to take the debate as interesting, much less even well posed. The active work of physicists on our best physical theories should provide the fodder for the work of the philosopher of physics most of the time. Sometimes, however, the physicists are confused or just mistaken, and it is then our job to try to help set matters straight. I believe that is the case here.¹

A virtue of the work of many contemporary philosophers on the issue is the foundation of their metaphysical conclusions on arguments based on the structures of our best physical theories. I think the method falls short, however, in so far as it treats those structures in abstraction from their uses in actual scientific enterprises, both theoretical and experimental. This lacuna leaves the debate merely formulaic, without real content, at the mercy of clever sophistications without basis in scientific knowledge in the fullest sense.

Stein ([1994], pp. 1–2) admirably sums up the situation as I see it. I quote him at length, as he says it better than I could:

[...] let me [...] hazard a rough diagnosis of the reason why some things that are (in my view) true, important, and obvious tend to get lost sight of in our discussions [...] Philosophy] has (I believe) in our own time been affected by an excess of what might be called the *esprit de technique*[...]: a tendency both to concentrate on such matters of detail as allow of highly formal systematic treatment (which can lead to the neglect of important matters on which sensible even if vague things can be said), and (on the other hand), in treating matters of the latter sort, to subject them to quasi-technical elaboration beyond what, in the present state of knowledge, they can profitably bear [...] what I have described can be characterized rather precisely as a species of scholasticism [...] In so far as the word 'scholasticism', in its application to medieval thought, has a

¹ See (Curiel [2001], [2009]) for arguments to this effect on closely related matters, and for a defence of this claim as a fruitful philosophical attitude.

pejorative connotation, it refers to a tendency to develop sterile technicalities—characterized by ingenuity out of relation to fruitfulness; and to a tradition burdened by a large set of standard counterposed doctrines, with stores of arguments and counterarguments. In such a tradition, philosophical discussion becomes something like a series of games of chess, in which moves are largely drawn from a familiar repertoire, with occasional strokes of originality—whose effect is to increase the repertoire of known plays.

In the spirit of Stein's diagnosis, rather than something formally sophisticated, I'm going to propose something crude and simple: to avoid the sterility that formal technical elaboration can lead to, we should look at the way that spacetime structures are used in practice to model real systems in order to make progress on issues pertaining to the standard debate. For I do think there are important, deep questions we can make progress on in the vicinity of that debate, questions of the sort Maxwell alludes to in this article's epigraph. As Maxwell intimates, however, for such questions to be investigated profitably, they must be such as to support and stimulate 'the investigation of nature'. And that, I submit, can be accomplished only when the questions bear on scientific knowledge in all its guises, as theoretical comprehension and understanding, as evidential warrant and interpretative tool in the attempt to assimilate novel experimental results, as technical and practical expertise in the design and performance of experiments, and as facility in the bringing together of theory and experiment in such a way that each may fruitfully inform the other.

I will argue that the way to find the philosophically and scientifically fruitful gold in the metaphysical dross is to formulate and address the questions in a way that makes explicit contact with our best current knowledge, in its fullest form, about the kinds of physical system at issue. One way to do that is to pose and investigate the questions explicitly in the context of what I will call an investigative framework—roughly speaking, a set of more-or-less exactly articulated theoretical structures for the modelling of physical systems, along with a family of experimental practices and techniques suited to their investigation. Different investigative frameworks, as I show by constructive example, provide different natural criteria for rendering determinate the question of the ontic status of spacetime, with none privileged *sub specie aeternitatis* over any other. Those different criteria yield different answers to the question, suitably formulated in the given frameworks. This should not be surprising. After all, different sorts of scientific investigations naturally assume and rely on different relations between individual spacetime points and metrical (and other forms of spatiotemporal) structure, and it is those relations that are supposed to provide the criteria for the existence of spacetime points. The

mathematical formalism of the theory does not by itself fix a unique such relation with clear ‘physical’ significance.

I begin in Section 2 with an examination of the hole argument. I do this for two reasons. First, because invocation of the argument has become a mannerism in the debate, it must be confronted; I conclude that it has no bearing on the issue. Second, I discuss it because it yields a useful schema for the production of concrete criteria that one can use to explicate the differences between substantivalists and relationalists. I use that schema to frame the arguments of the subsequent two sections, of the article. In each of those two sections, I make the schematic criterion concrete in the context of a particular form of investigative framework, constructing two arguments with contrary conclusions, one for something like relationalism and the other something like substantivalism, to show that one can make the debate concrete in any of a number of precise, physically significant ways, none *a priori* privileged over the others, and that those ways will not in general agree in their consequences.

In Section 5, I urge that the contrary conclusions of Sections 3–4 strongly suggest that issues of ontology are best addressed in the context of a particular form of investigation. For a given spacetime theory—and even a given model within the theory—depending on one’s purposes and the tools one allows oneself—either one can treat spacetime points as entities and individuate and identify them *a priori*, or one can in any of a number of ways construct spacetime points as factitious, convenient pseudo-entities. Nothing of intrinsic physical significance hangs on the choice, and so *a fortiori* science cannot guide us if we attempt to choose *sub specie aeternitatis* between the alternatives—such a choice must become, if anything, an exercise in scholastic metaphysics only.

In Section 6, I extend the discussion to a host of other types of spacetime structure, such as Killing fields and topological invariants. The attempt to formulate criteria for the physicality of such other structures adds weight to the conclusion that such questions require concrete realization in the context of something akin to real science in order to acquire substantive content. I conclude in Section 7 with a brief attempt to show that my arguments ramify into the debate between realists and instrumentalists more generally, by dint in part of the picture of science the arguments implicitly rely on.

The overarching lesson I draw is that metaphysical argumentation abstracted from the pragmatics of the scientific enterprise as we know it—science as an actually achieved state of knowledge and as an ongoing enterprise of inquiry—is vain. Very little of real substance can be learned about the nature of the physical world by studying only theoretical structures in isolation from how they hook up to experimental knowledge in real scientific practice, a

practice endemic not only to the current debate, but to the entirety of philosophy of physics as a discipline.

The constructions I found the arguments on require the use of advanced mathematical machinery from the theory of general relativity. (For the interested reader, (Wald [1984]; Malament [2012]), for example, contain comprehensive coverage of all material required.) Limitations of space have required me to elide many of the technical details of the constructions the arguments are based on. The interested reader can find them in a separate manuscript (Curiel [unpublished]) containing technical appendices to this article, in which the details are worked out.

2 The Hole Argument

In recent times, several physicists and philosophers have treated Einstein's infamous hole argument as being at the heart of questions about the ontology of spacetime (Earman and Norton [1987]; Belot [1996]; Gaul and Rovelli [2000]). The lesson most often claimed is that one cannot identify spacetime points without reliance on metrical structure, that there is no bare manifold of points under the metric field.

The debate is often posed thus: should the manifold \mathcal{M} by itself or the ordered pair (\mathcal{M}, g_{ab}) be properly construed as the representation of physical spacetime? This, in brief, is the argument: Fix a spacetime model (\mathcal{M}, g_{ab}) .² For ease of exposition, we stipulate that it possess a global Cauchy surface, Σ . (We could do without this condition at the cost of unnecessary technical details.) Say that we know the metric tensor on Σ and on the entire region of spacetime to its causal past, $J^-[\Sigma]$. (Note that $J^-[\Sigma]$ contains Σ .) This forms a well set Cauchy problem, and so there is a solution to the Einstein field equation (EFE) that extends g_{ab} on $J^-[\Sigma]$ to a metric on all of \mathcal{M} , yielding the original spacetime.³ Now, let ϕ be a diffeomorphism that is the identity on $J^-[\Sigma]$ and smoothly becomes non-trivial on $J^+[\Sigma] - \Sigma$. No matter what else one takes the diffeomorphism invariance of general relativity to mean, at a minimum it must be that a diffeomorphism applied to a solution of the EFE yields another, possibly distinct solution. Apply ϕ to g_{ab} (but not to M itself); this yields a seemingly different metric—a different physical state of the

² I am not biasing the argument by demanding a model of spacetime consist of both a manifold and a metric. By 'model of spacetime' here, I mean just 'manifold cum metrical structure as purely computational tool', irrespective of how the debate resolves itself.

³ This is not, strictly speaking, accurate. If no restrictions are placed on the matter fields, then in general the initial-value problem is not well set. Indeed, even a few known physical solutions to the EFE possess no well set initial-value formulation, for example, those representing homogeneous dust and some types of perfect fluid (Geroch [1996]). We can ignore these technicalities, though it may raise a serious problem about indeterminism in the theory, one which has not been addressed in the literature.

gravitational field. This is the crux of the issue: that the points of $J^+[\Sigma] - \Sigma$ carry a different metric tensor than before.

We now face a dilemma, the argument continues (Earman and Norton, [1987]): we can either hold that fixing the metric on $J^-[\Sigma]$ does not determine the metric on $J^+[\Sigma] - \Sigma$, a radical indeterminism, or else we can conclude that spacetime points in some sense have no identifiability or existence independent of the prior fixing of the metric, with most researchers opting for the dilemma's second horn.⁴

I want to make a crude and simple proposal, for I think the debate has lost sight of a crude and simple, and yet fundamentally important, fact: just because the mathematical apparatus of a theory appears to admit particular mathematical manipulations does not *eo ipso* mean that those manipulations admit of physically significant interpretation. One has the mathematical structure of the theory; one is not free to do whatever it is one wants with that formalism and then claim, with no foundation in practice, that what one has done has physical import. The mathematical formalism by itself cannot tell us what manipulations it admits have physical significance; one must determine what one is allowed to do with it—'allowed' in the sense that what one does respects the way that the formalism actually represents physical systems. A simple example illustrates the point: adding 3-vectors representing spatial points in Newtonian mechanics. As a physical operation, adding spatial points is meaningless—the idea of linearly superposing spatial points in Newtonian theory as a representation of a physical state of affairs makes no sense. For computing factitious quantities such as the centre of mass, however, it does make sense. Just because one can add two vectors in the mathematical formalism of a theory does not by itself make the operation physically significant.

General relativity is (usually) formulated with the use of differential Lorentz manifolds. Not every well-formed mathematical operation on a Lorentz

⁴ There are actually two different versions of the argument in the literature, though this goes unremarked. The one I rehearse here can be thought of as a generalization of the other. The more specialized form, which Einstein himself formulated and used, assumes that spacetime has a region of compact closure, the hole, in which the stress-energy tensor vanishes, though it itself is surrounded by non-zero stress-energy. The diffeomorphism is then stipulated to vanish everywhere except in the hole, and the argument goes more or less as in the general case, with the emendation that now it is the distribution of ponderable matter that does not suffice to fix the physical state of the gravitational field. (Earman [1989], for example, uses the more general argument, whereas Stachel [1993] uses the more specialized form.) I think the specialized form of the argument introduces a red herring, namely, physical differences between regions of spacetime with stress-energy and those without. There is no principled way within the theory itself to distinguish between such regions in a way that bears on ontological issues. One of the regions has non-trivial Ricci curvature; the other does not, though it may have non-trivial Weyl curvature. That difference, the only one formulable in the terms of the theory, can tell us nothing about the ontic status of the spacetime manifold. The introduction of the difference seems rather to bespeak an old prejudice that material sources should suffice to determine the physical state of associated fields, but this is not true even in classical Maxwell theory.

manifold has physical significance. It arguably makes mathematical sense to apply a diffeomorphism of the manifold to the metric only, and not to the underlying manifold at the same time.⁵ That fact by itself does not imbue the operation with physical significance. Considerations such as the hole argument show how diffeomorphisms ought to be applied to spacetime models so as to have physical significance.

What is of intrinsic physical significance in the possible states and interactions of physical systems does not depend on the diffeomorphic presentation of the manifold cum metric. (Are those two bodies in physical contact? Is stress-energy being transferred from this one to that or vice versa? Can a light-signal be sent from this to that? Is gravitational radiation present? And so on.) To ensure this equivalence of physical significance across diffeomorphic presentations, however, one must stipulate that, in the context of general relativity, the application of a diffeomorphism to the metric is a ‘physically’ well defined procedure only when one also applies it to the (given presentation of the) manifold itself. Thus the hole argument is obviated by the fact that the application of ϕ to the manifold cum metric results only in a different presentation of the same intrinsic physical structure, and so the worry about determinism evaporates, doing away with the dilemma. How one tries to characterize the ontology of the spacetime manifold, if that is the sort of thing one is into, may be influenced by this restriction on the applicability of diffeomorphisms to spacetime models, or it may not. The important point is that this restriction results from conditions imposed by the way one may employ the formal apparatus of the theory so as to respect how in scientific practice spacetime models represent physically possible spacetimes—how it is that the formal structures of the theory acquire real physical meaning.

In sum, the hole argument has no bearing on whether existence should be attributed to spacetime points independent of metrical structure. The diffeomorphic freedom in the presentation of relativistic spacetimes does not *ipso facto* require philosophical elucidation, for it in no way prevents us from investigating what is of true physical significance in systems that general relativity models (Curiel [2009]). It is neither formal relations nor substantive entities that remain invariant when one applies a diffeomorphism to a

⁵ If one adopts a certain definition of a differential manifold, namely, that it is an equivalence class of diffeomorphic presentations, then the operation underlying the hole argument does not make even mathematical sense. (Weatherall [forthcoming] concludes this, based on related considerations; I am sympathetic with his arguments.) \mathbb{S}^2 , for example, can be presented as a submanifold of a 179-dimensional hyperboloid, or as \mathbb{R}^2 with a point added, or as a manifold in its own right; $\mathbb{S}^2 \times \mathbb{R}^2$ can be presented as a direct product of manifolds (as here), or as \mathbb{R}^4 with a line removed; and so on. In this case, pushing tensors around on the manifold by a diffeomorphism without also pushing the points around, as required by the hole argument, is not an unambiguous notion. I do in fact accept the definition of a differential manifold as an equivalence class, but I am trying to be as charitable as possible to the proponents of the debate, so I am willing to grant for the sake of argument that the required manipulations make mathematical sense.

relativistic spacetime; it is the family of physical facts the spacetime represents. One may represent those facts in a language some of whose primitive terms designate ‘spacetime points’ or not. It is irrelevant to our capacity to use them in profitable ways in science and, more important, to our understanding of those facts in our broader attempts to comprehend the physical world. This line of thought already suggests that the debate between substantialists and relationalists is not well posed.

In the event, my rejection of the hole argument rests on a deeper point. I think the most unproblematic and uncontroversial fact about diffeomorphic freedom is that it embodies an inevitable arbitrariness in the mathematical apparatus the theory uses to model physical systems: the choice of the presentation of the spacetime manifold and metric one uses to model a physical system is fixed only up to diffeomorphism. A comparison will help illuminate the character of this arbitrariness.

Hamiltonian mechanics has a similar arbitrariness: one is free to choose any symplectomorphism between the space of states and the cotangent bundle of configuration space, that is, one may choose, up to symplectomorphism, any presentation of phase space (or, in more traditional terms, any complete set of canonical coordinates), without changing the family of solutions the possible Hamiltonians determine (Curiel, [2014]). One is not driven to investigate the ontic status of points in phase space merely because one is free to choose any symplectomorphism in its presentation. Indeed, one can run an argument analogous to the hole argument here, substituting ‘phase space’ for ‘spacetime manifold’, ‘symplectomorphism’ for ‘diffeomorphism’, and ‘symplectic structure’ for ‘metric’. Does that show anything of intrinsic physical or metaphysical significance? No serious person would argue so. And in this case, it would be manifestly absurd to ‘apply a symplectomorphism only to the symplectic structure and not the underlying manifold’: in general the underlying manifold is a cotangent bundle and the symplectic structure is the canonical one on it; pushing the symplectic structure around on its own will yield a new symplectic structure that is not the canonical one, and so is manifestly unphysical for the purpose of formulating Hamilton’s equation.

It is clear that the existence of inevitable, more or less arbitrary, non-physical elements in the presentation of the models of a theory by itself does not require that one decide on the ontic status of any entities putatively designated by its mathematical structures. More to the point, it is clear in such cases that the physical significance of the theory’s models is not masked or polluted by the unavoidable arbitrariness in the details of their presentations.⁶

⁶ It is a deep puzzle that every known physical theory has such arbitrariness in its formal representations of physical systems. Does this imply that our mathematics is not so well suited to modelling the physical world as we tend to assume?

In the end, however, the most serious problem I have with the hole argument, and all other arguments analogous to it, comes to this: nothing I can see militates in favour of taking the hole argument as bearing on the ontic status of spacetime points, just because the hole argument by itself provides no independent, clear, and precise criterion for what ‘existence independent of metrical structure’ comes to. That idea has no substantive content on its own. In the next two sections, I will show this by exhibiting two plausible, precise criteria for what the idea may mean in the contexts of two different types of investigation, which in the event lead to opposing conclusions. The criteria are based on the criterial schema I have implicitly relied on so far: whether the identification of spacetime points must depend on the prior stipulation of metrical structure.⁷

3 Limits of Spacetimes

In this section, I propose an argument in favour of the view that one cannot attribute to the spacetime manifold any existence independent of metric structure; the provision of a precise criterion for the existence of spacetime structure, grounded in both the structure and the application of physical theory, drives the argument. Two criteria natural to the investigative context will suggest themselves, a weaker one based on the idea of the identifiability of spacetime points and a stronger one based on their existence (in a precise sense).

To treat a spacetime as the limit, in some sense, of an ancestral family of continuously changing spacetimes is one of the ways of embodying in the framework of general relativity two of the most fundamental and indispensable tools in the physicist’s workshop: the idealization of a system by means of the suppression of complexity, so as to render the system more tractable to investigation; and the enrichment of a system’s representation in a theory by the addition (or reimposition) of complexity previously ignored (or ellided). As a general rule, the fewer degrees of freedom a system has, the easier it is to study. Schwarzschild spacetime (Figure 1) is far easier to work with than Reissner–Nordström (Figure 2) in large part because one ignores electric charge, and there is a natural sense in which one can think of Schwarzschild spacetime as the limit of Reissner–Nordström as the electric charge of the

⁷ I know of no one who adopts exactly this schematic criterion. (Perhaps Hofer ([1996], [1998]) comes the closest.) I use it because it captures the essence of the criteria that are often stipulated in the debate, that the question of the existence of spacetime points devolves upon the relation of those points to some geometrical structure, such as the metric. See, e.g., Butterfield ([1989]); Earman ([1989]); Maudlin ([1990], [1993]); Pooley ([2006], [2013]); Rynasiewicz ([1994]); Belot ([1999]), ([2011]); Dorato ([2000]); Huggett ([2006]); DiSalle ([1994], [2006]); is a notable example of a contemporary philosopher who takes an approach sympathetic to my own; Robert Geroch, in private conversation, is a notable example of a contemporary physicist who does so.

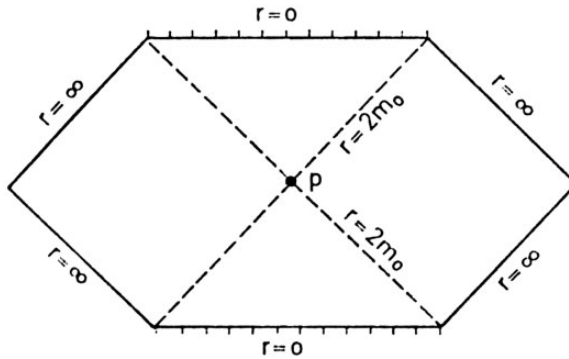


Figure 1. Carter–Penrose diagram of Schwarzschild spacetime. Each point in the diagram represents a 2-sphere in the spacetime manifold. (This diagram is taken from (Geroch [1969]), with the author’s permission.)

central black hole shrinks to zero.⁸ Contrarily, by reversing the sense of that limiting procedure, one can think of Reissner–Nordström spacetime as the complexification of Schwarzschild spacetime induced by the introduction of a smoothly increasing central electric charge.⁹ A generic representation of such a limiting process can provide schemas of both of these theoretical tools, depending on whether one enlarges or shrinks the number of degrees of freedom in the limiting process. As we will see, what in the idealized model one may reasonably identify and attribute existence to may depend in sensitive ways on the character of the more complex or simpler models one starts with and the nature of the limiting process itself. This fact drives the argument I propose. I will discuss two examples of such a limiting process in order to motivate the two precise criteria I propose for the existence of spacetime points independent of metrical structure.

A complete treatment of the limiting process grounding the examples would require the use of heavy machinery from differential geometry, based on a construction of Geroch ([1969]). Limitations of space prevent me from working it out in detail here.¹⁰ I will instead sketch the features relevant to our problem and describe salient examples.

⁸ Schwarzschild spacetime is the unique spherically symmetric vacuum solution to the EFE (other than Minkowski spacetime); it represents a spacetime that is empty except for an electrically neutral, spherically symmetric, static central body or black hole of a fixed mass. Reissner–Nordström is the generalization of Schwarzschild spacetime that allows the central structure to have an electric charge; see, for example, (Hawking and Ellis ([1973]), Chapter 5, Section 5) for an exposition.

⁹ The idea of complexification I employ here has nothing to do with the idea bandied about in other contexts in mathematical physics, also called ‘complexification’, in which one extends a mathematical structure based on the real numbers to one based on the complex numbers.

¹⁰ To see the constructions and examples worked out in detail, see (Curiel [unpublished]).

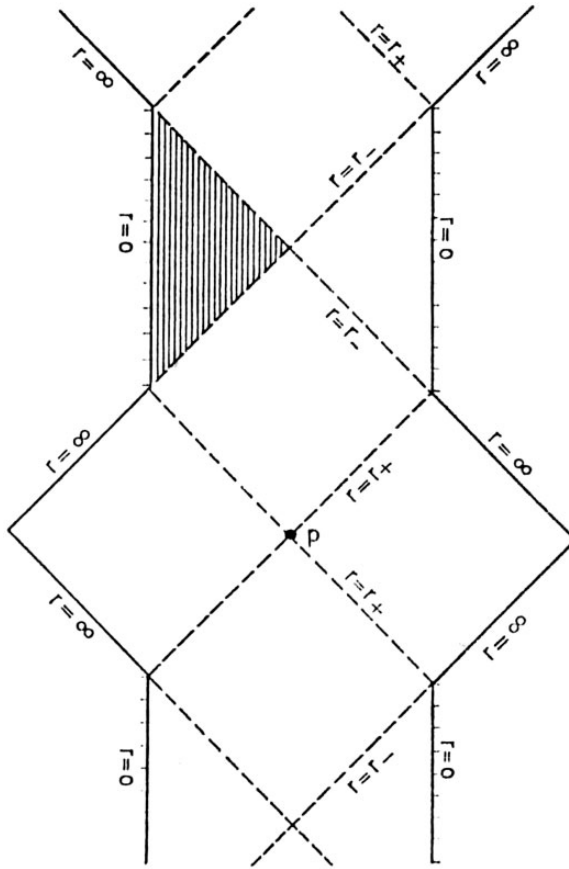


Figure 2. Carter–Penrose diagram of Reissner–Nordström spacetime. Each point in the diagram represents a 2-sphere in the spacetime manifold. (This diagram is taken from (Geroch [1969]), with the author’s permission.)

Before giving an example of the construction directly relevant to my argument and putting it to work, however, I discuss one of its most important and powerful features, that the constructed limiting family does not parametrize metrics on a fixed manifold, but rather parametrizes the spacetime manifolds themselves. Geroch ([1969], p. 181) himself states in illuminating terms the reason behind this:

It might be asked at this point why we do not simply [use a] 1-parameter family of metrics on a given fixed manifold [...]. Such a formulation would certainly simplify the problem: it amounts to a specification of when two points [in different members of the limiting family] are to be considered as representing ‘the same point’ of [the limit spacetime]. It is not appropriate to provide this additional information, for it always

involves singling out a particular limit, while we are interested in the general problem of finding all limits and studying their properties.

To make the force of these remarks clear, consider the attempt to take the limit of Schwarzschild spacetime as the central mass goes to 0. In Schwarzschild coordinates, using the parameter $\lambda \equiv M^{-1/3}$ (the inverse-third root of the Schwarzschild mass), the metric takes the form

$$\left(1 - \frac{2}{\lambda^3 r}\right) dt^2 - \left(1 - \frac{2}{\lambda^3 r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

This clearly has no well-defined limit as $\lambda \rightarrow 0$. Now, apply the coordinate transformation

$$\tilde{r} \equiv \lambda r, \tilde{t} \equiv \lambda^{-1} t, \tilde{\rho} \equiv \lambda^{-1} \theta.$$

In these coordinates, the metric takes the form

$$\left(\lambda^2 - \frac{2}{\tilde{r}}\right) d\tilde{t}^2 - \left(\lambda^2 - \frac{2}{\tilde{r}}\right)^{-1} d\tilde{r}^2 - \tilde{r}^2(d\tilde{\rho}^2 + \lambda^{-2} \sin^2(\lambda\tilde{\rho}) d\phi^2).$$

The limit $\lambda \rightarrow 0$ (now representing the limit of the central mass going to zero) exists and yields

$$-\frac{2}{\tilde{r}} d\tilde{t}^2 + \frac{\tilde{r}}{2} d\tilde{r}^2 - \tilde{r}^2(d\tilde{\rho}^2 + \tilde{\rho}^2 d\phi^2),$$

a flat solution discovered by Kasner ([1921]). If instead of that coordinate transformation we apply the following to the original Schwarzschild form In Equation (1),

$$x \equiv r + \lambda^{-4}, \rho \equiv \lambda^{-4} \theta,$$

then the resulting form also has a well-defined limit, which is the Minkowski metric. The two limiting processes yield different spacetimes because behind the scenes the same points of the underlying manifold get pushed around relative to each other in different ways. Because the coordinate relations of initially nearby points differ in different coordinate systems, those differences get magnified in the limit, so that their final metrical relations differ. Thus, the limits in the different coordinates yield different metrics.

This example suggests why, in working with limits of spacetimes, it is inappropriate to work with a fixed manifold from the start. To do so determines a unique limit, but we want to allow ourselves different ways to take the limit, so that our ideal scientist can ignore different facets of the complex system under study, and so produce different idealized models of it.¹¹ For example,

¹¹ Of course, sometimes is appropriate for the scientist to take the limit of a family of metrics on a fixed background manifold. An excellent example is in the statement and proof of the geodesic

she may want to take the limit of Reissner–Nordström spacetime as the mass goes to zero while leaving the electric charge fixed, rather than taking the limit as the electric charge vanishes, or she may want to take the limit in a way that does not respect the spherical symmetry of the initial system in order, for example, to study small perturbations of the original system.¹²

I turn now to an example immediately relevant to my arguments. Consider a family $\{(\mathcal{M}_\lambda, g^{ab}(\lambda))\}$ of Reissner–Nordström spacetimes, each element of the family having the same fixed value, M , for its mass and all parametrized by electric charge λ , which converges smoothly to zero.¹³ Geroch's construction shows that there are innumerable ways of fixing a limit, each leading to a different topology and metric. Fix one that has Schwarzschild spacetime as the limit, 'natural' in the sense that it respects the spherical and the timelike symmetries in all the spacetimes in the limiting family. (There is not even a unique limiting family in this special case.) Now, comparison of Figures 1 and 2 suggests that something drastic happens in the limit. All the points in the throat of the Reissner–Nordström spacetimes (the shaded region in the diagram) seem to get swallowed by the central singularity in Schwarzschild spacetime—in some way or other, they vanish. Using Geroch's machinery we can make precise the question of their behaviour in the limit $\lambda \rightarrow 0$.

Consider the points in the shaded region in Figure 2, between the lines $r = 0$ and $r = r^-$ (r is the radial coordinate in a system that respects the spacetime's spherical symmetry; the coordinate values r^- and r^+ define boundaries of physical significance in the spacetime, which in large part serve to characterize the central region of the spacetime as a black hole). The machinery allows one to trace individual points through the given limiting process, in effect identifying the same point in the different member spacetimes of the limiting family, in a way peculiar to that limiting process. (Of course, part of the point of the construction is that there is no single, *a priori* privileged way of doing this.) One can use this inter-family identification of points to make precise the sense in which something drastic does indeed occur in such a limit that takes Reissner–Nordström to Schwarzschild spacetime. In each Reissner–Nordström spacetime in the limiting family, any point lying in the shaded region does not have a well defined limit: no point in the resulting Schwarzschild spacetime limit can be identified with it. (Roughly speaking, the

theorem of Ehlers and Geroch ([2004], p. 233). In fact, they give an illuminating discussion of this very issue on p. 233.

¹² Paiva *et al.* ([1993]) discuss in some detail an interesting class of different limiting spacetimes one can induce from Schwarzschild spacetime by taking the limit as the mass goes to zero and to infinity, respectively, in different ways. See (Bengtsson *et al.* [2014]) for a similar discussion for Reissner–Nordström spacetime, as the electric charge and the mass are each taken to zero.

¹³ I ignore the fact that electric charge is a discrete quantity in the real world, an appropriate idealization in this context.

points, in the limit, run into the Schwarzschild singularity at $r=0$.) In this precise sense, no point in Reissner–Nordström spacetime to the future of the horizon $r=r^-$ has a corresponding point in the limit space.

To sum up: one begins with a family of Reissner–Nordström spacetimes continuously parametrized by electric charge, which converges to zero; one uses Geroch’s machinery to construct a limit space by a choice of how to track the identification of individual points across members of the limiting family; this choice enforces a division of points that have a limit from those that do not; and that identification, in turn, dictates the identification of spacetime points in the limit space (which points in the ancestral family lie within the Schwarzschild radius, for example, and which do not). Thus one can identify points within the limit Schwarzschild spacetime—one’s idealized model—only by reference to the metrical structure of members of the ancestral family; one can, moreover, identify points in the limit space with points in the more complex, initial models one is idealizing only by reference to the metrical structure of the members of the ancestral family as well. It is only by the latter identification, however, that one can construe the limit space as an idealized model of one’s initial models, for the whole point is to simplify the reckoning of the physical behaviour of systems at particular points of spatiotemporal regions of one’s initial models.

One can, moreover, use different choices of the inter-family identification of points to construct Schwarzschild spacetime from the same ancestral family, with the result that in each case the same point of Schwarzschild spacetime is identified with a different family of points in the ancestral family. More generally, different choices will yield limit spaces that differ from Schwarzschild spacetime, with no canonical way to identify a point in one limit space (one idealized model the theoretician constructs) with one in another. In other words, the identification of points in the limit space depends sensitively on the way the limit is taken, that is, on the way the model is constructed. In consequence, in so far as one conceives of Schwarzschild spacetime as an idealized model of a richer, more complete representation, one can identify points in it only by reference to the metrical structure of one of its ancestral families, and one can do that in a variety of ways.

Now, say one wants to treat slightly aspherical, almost-Schwarzschild spacetimes as a complexification of Minkowski spacetime, in order to study how asphericities affect metrical behaviour. Because the limit spacetime will be almost-Schwarzschild, its appropriate manifold is still $\mathbb{R}^2 \times \mathbb{S}^2$, the natural topology of Schwarzschild spacetime. In this case, in an intuitive sense, points will appear, because the topology of Minkowski spacetime is \mathbb{R}^4 , so in some sense one must compactify two topological dimensions to derive a Schwarzschildian spacetime as a more complex limit. There are many ways to effect such a compactification; all the simplest, such as Alexandrov compactification, work by the addition of an extra point or set of points to the

topological manifold to represent, intuitively speaking, the bringing in of points at infinity to a manageable distance from everything else.¹⁴ The difficulty of these issues, however, is underscored by the fact that one can also think of this as a case in which points rather disappear; $\mathbb{R}^2 \times \mathbb{S}^2$, after all, is homeomorphic to \mathbb{R}^4 with a line removed! Thus one could use an ancestral family every member of which is \mathbb{R}^4 but that has as limit space the manifold of Schwarzschild spacetime presented as the manifold \mathbb{R}^4 with a line removed.¹⁵

In this example, we will consider the attempt to introduce a central, slightly aspherical body by physical construction in a Minkowskian laboratory, as an experimentalist might do it. The physical construction will proceed in infinitesimal stages, with a tiny portion of matter introduced at each step distributed in a slightly aspherical way (keeping, in an intuitive sense, the aspherical shape of the body the same), and an allowance of a finite time so that the ambient metrical structure can settle down to an almost-Schwarzschild character before the next step is initiated, until the central body's mass reaches the desired amount. (Intuitively, the finite time period allows the metrical perturbations introduced by the movement of the matter in, and its distribution around, the central body to radiate off to infinity.) One can represent this process with a limiting ancestral family of Geroch's type in, a more or less obvious way, starting with Minkowski spacetime—namely, the empty, flat laboratory, with each member of the ancestral family representing the laboratory at each stage of the construction as a bit more matter has been introduced and the perturbations have settled down.

Now, consider at the beginning of the process a small patch of space in the laboratory not too far from the position where the central body will be constructed. We want to try to track, as it were, the spacetime points in that patch during the enlargement of the central body because we plan to investigate, say, how the metrical structure in regions at that spatiotemporal remove from a central aspherical body differ from each other for different masses of the central body. (Because the EFE is non-linear, and there is no exact symmetry, one cannot just assume that slightly aspherical spacetimes will scale in any straightforward way with increases in the central mass.) There are several ways one might go about trying to track the region as the construction progresses. One obvious, simple way is by the triangulation of distances from some fixed markers in the laboratory. Because the metrical structure within the lab is constantly changing, however, and doing so in very complex ways during the periods when new matter is being introduced and distributed, and

¹⁴ See, for example, (Kelley [1955]) for an account of methods of compactification, including the Alexandrov type.

¹⁵ This is a concrete instance where thinking of two different diffeomorphic presentations of the same manifold—in this case, $\mathbb{R}^2 \times \mathbb{S}^2$ and \mathbb{R}^4 with a line removed—as different manifolds leads to obvious difficulties, if not downright confusions.

the concomitant metrical perturbations are radiating away, there is no canonical way of implementing the triangulation procedures. In fact, the different ways of doing so are exactly captured by the different choices of how to identify points among the members of the ancestral family of spacetimes (which in this case, recall, now represent the spacetime region enclosed by the laboratory at different stages of the construction of the central body). According to some of the concrete implementations of the triangulation procedure, that is, according to different choices of how to identify points among the several members of the ancestral family, the patch one tries to track will end up inside the central body; according to other procedures, it will end up outside the central body. In consequence, what one means by ‘the set of spacetime points composing a small region at a fixed spatiotemporal position relative to the central body’ will depend sensitively on how one fixes and tracks relative spatiotemporal positions, which is to say, depends sensitively on one’s knowledge of the spacetime’s metrical structure.¹⁶

We are finally in a position to offer a precise criterion for the existence of spacetime points independent of metrical structure that is to the investigative contexts we have considered. There are, in fact, two natural criteria that suggest themselves, one weaker than the other. The first is suggested by the example of complexification and stated somewhat loosely:

Definition 1: Points in a spacetime manifold have existence independent of metrical structure if there is a canonical method to identify spacetime points during gradual modifications to the local spacetime structure.

My discussion of the example of complexification shows that, in this context and using this criterion, spacetime points do not have existence independent of metrical structure.

Now, based on the discussion of simplification, I propose a second criterion, stronger than the first and formulated more precisely and rigorously. Fix a limiting family with a choice of definite limit space. I say that a point in the initial member of the limiting family vanishes (or that the point itself is a vanishing point) with respect to the given family of frames if it has no point

¹⁶ One might object that, in this example, the experimentalist is really trying to track the same points through space over time, not the same spatiotemporal points in different spacetimes. In fact, though, since the goal of the investigation is to determine how global metrical structure in slightly aspherical spacetimes differ for different values of the central mass, it is natural for the experimentalist to consider each static phase of the laboratory—the period after the last bit of mass has been added and the perturbations have settled down, but before the next bit of mass is added—as a separate spacetime in its own right, for the purposes of comparison. An appropriate analogue is the so-called physical process version of the first law of black-hole mechanics (Wald [1994]; Wald and Gao [2001]), where one must identify two separate spacetimes (in the sense of two different solutions to the EFE) that differ in that one conceives of the spacetime as the result of a dynamical evolution of the other, even though there is no concrete representation of that evolution as occurring in a single spacetime.

identifiable with it in the limit determined by the fixed choice of how to identify that point across all the members of the family. I say that a point in the limit space appears if there is no limiting sequence of points that converges to it.

Definition 2: Points in a spacetime manifold have an existence independent of metrical structure if there is no way to identify points across members of any ancestral family of the spacetime so that points vanish or appear.

I do not demand that one be able to identify in a preferred way a spacetime point in the limit with any point of any member of one of its ancestral families, much less for all its ancestral families. This allows us to hold on to diffeomorphic freedom in the presentation of the limit space. I do not even demand that the criterion hold for every possible spacetime model—perhaps in some spacetimes it makes sense to attribute existence to spacetime points independent of metrical structure, whereas in others (say, completely homogeneous spacetimes) it does not. I demand only that, for a given spacetime, one not be able to make points in any of the spacetime’s ancestral families vanish and not be able to make points in the spacetime, as the limit space, appear—a weak demand. This should capture the idea that when we construct a spacetime model and treat it as an idealized representation of a more complex system—as it always is—then we can reliably identify spacetime points in our model with points in the more complex system, albeit up to diffeomorphic presentation. If we cannot do this irrespective of the more complex model we start from, then we cannot—without arbitrariness and artifice—regard results of an investigation in the context of the idealized model as relevant to the physics of the more complex system, for we will be unable to identify the regions in the more complex system to which the results of the idealizing investigation pertain. The example of Schwarzschild spacetime as a limit of a family of Reissner–Nordström spacetimes clearly does not satisfy the criterion, for there are points that vanish in the limiting procedure (for example, those in the shaded region of [Figure 2](#)). One may suspect that the existence of singular structure in the two spacetimes fouls things up. The following result, however, establishes that no spacetime satisfies the criterion, namely, that its failure is universal and depends on no special properties of any spacetime model.¹⁷

Proposition 1. Every spacetime has a non-trivial ancestral family with vanishing points. Every non-trivial ancestral family has a limit space with respect to which some of its points vanish.

(The analogous proposition holds for points that appear.) In consequence, in every relativistic spacetime we treat as an idealized model in the context of this

¹⁷ See (Curiel [unpublished](#)) for the proof.

sort of scientific investigation, we can attribute existence to individual spacetime points (or not) only by reference to the metrical structure of the ancestral family we use to construct the model, and the limiting process we choose for the construction.

An obvious objection to the relevance of these arguments to the ontic status of spacetime points is that I deal here only with idealizations and approximations, not with a real model of real spacetime. But we never work with anything that is not an idealization—it's idealizations all the way down, young man, as part of the human condition. If you can't show me how to argue for the existence of spacetime points independently of metrical structure using our best scientific theories as they are actually used in successful practice—a large and essential component of scientific knowledge—then you are not relying on real science to ground your arguments. You are paying only lip-service to the idea that science should ground these sorts of metaphysical issues.

4 Pointless Constructions

The argument of Section 3 yields a conclusion that holds only in a limited sphere, namely, those investigations based on the idealization of models of spacetime by means of limits. One may wonder whether it could be parlayed into a more general argument. I do not think so. Indeed, I think there is no sound argument to the effect that, regardless of the context of the investigation, one can identify spacetime points or attribute existence to them only by reference to prior metrical structure. Sometimes, in some contexts, one can identify and attribute existence to spacetime points without any such reference. To show this, I will present an argument that all the structure accruing to a spacetime, considered simply as a differential manifold that represents the collection of all possible (or, depending on one's modal predilections, actual) physical events, can be given definition with clear physical content in the absence of metrical structure. The argument takes the form of the construction of the point-manifold of a spacetime, its topology, its differential structure, and all tensor bundles over it from a collection of primitive objects that, when the construction is complete, acquires a natural interpretation as a family of covering charts from the manifold's atlas, along with the families of bounded, continuous scalar fields on the domain of each chart. That idea yields the following precise criterion the argument will rely on.

Definition 3. Points in a spacetime manifold have existence independent of metrical structure if the manifold can be constructed from a family of scalar fields, the values of which can be empirically determined without knowledge of metrical structure.

The basic idea of the construction is simple. I posit a class of sets of rational numbers to represent the possible values of physical fields, with a bit of additional structure in the form of primitive relations among them just strong enough to ground the definition of a derived relation whose natural interpretation is ‘lives at the same point of spacetime as’. A point of spacetime, then, consists of an equivalence class of the derived relation. The derived relation, moreover, provides just enough rope to allow for the definition of a topology and a differential structure on the family of all equivalence classes, and from this the definition of all tensor bundles over the resultant manifold, completing the construction. The posited primitive and derived relations have a straightforward physical interpretation, as the designators of instances of a schematic representation of a fundamental type of procedure the experimental physicist performs on physical fields when attempting to ascertain relations of physical proximity and superposition among their observed values. An important example of such an experimental procedure is the use of the observed values of physical quantities associated with experimental apparatus to determine the values of quantities associated with other systems, those investigated with the use of the apparatus. This interpretation of the relations motivates the claim that the constructed structure suffices, for our purposes, as a representation of spacetime in the context of a particular type of experimental investigation as modelled by mathematical physics, and is not (only) an abstract mathematical toy. Because of limitations of space, I give only a bare sketch of the construction; see (Curiel [unpublished]) for an exposition of the complete construction.

A ‘simple pointless field’ (or just ‘simple field’), ϕ , is a disjoint union $\bigsqcup_{p \in \mathbb{Q}^4} f_p$, indexed by \mathbb{Q}^4 (the set of quadruples of rational numbers), such that

- (1) every $f_p \in \mathbb{Q}$;
- (2) there is exactly one $f_p \in \phi$ for each $p \in \mathbb{Q}^4$;
- (3) there are two strictly positive numbers, B_l and B_u , such that $B_l < |f_p| < B_u$ for all $p \in \mathbb{Q}^4$;
- (4) the function $\bar{\phi} : \mathbb{Q}^4 \rightarrow \mathbb{Q}$ defined by $\bar{\phi}(p) = f_p$ is continuous in the natural topologies on those spaces, except perhaps across a finite number of compact three-dimensional boundaries in \mathbb{Q}^4 .

Our eventual interpretation of such a thing as a candidate result for an experimentalist’s determination of the values for a physical field motivates the set of conditions. That we index ϕ over \mathbb{Q}^4 means we assume that the experimentalist by the use of actual measurements and observations alone can impose on spacetime at most the structure of a countable lattice indexed by quadruplets of rational numbers (and even this only in a highly idealized sense); in other words, the spatiotemporal precision of measurements is limited. Condition 1 says that all measurements have only a finite precision in the

determination of the field's value. Condition 2 says that the field the experimentalist measures has a definite value at every point of spacetime. Condition 3 says that there is an upper and a lower limit to the magnitude of values the experimentalist can attribute to the field using the proposed experimental apparatus and technique. For instance, any device for the measurement of the energy of a system has only a finite precision, and thus can attribute only absolute values greater than a certain magnitude, and the device will be unable to cope with energies above a given magnitude. Condition 4 tries to capture the ideas that (local) experiments involve only a finite number of bounded physical systems (apparatuses and objects of study), and that classical physical systems bear physical quantities the magnitudes of which vary continuously (if not more smoothly), except perhaps across the boundaries of the systems.

A linkage is a relation imposed on a family of simple pointless fields, capturing the idea that values of the various fields all live at the same point of spacetime. One can think of the linkage as a coordinate system on an underlying, abstract point set, homeomorphic to an open set of \mathbb{Q}^4 . (For simplicity, we restrict attention to linkages that define convex normal neighbourhoods; this entails no real loss of generality.) To capture the idea of transformations between coordinate systems, one defines a relation between linkages, a 'cross-linkage', inducing a homeomorphism between two open sets of \mathbb{Q}^4 , naturally construed as the intersection of the two coordinate systems. Now, to complete the construction, we need to move from the rationals to the reals, to define the manifold structure of the abstract point-set represented by a maximal set of families of simple pointless fields. Roughly speaking, we take a double Cauchy-like completion over elements of \mathbb{Q}^4 linked with rational numbers (values of the fields with their associated points in the underlying space).¹⁸ We thus obtain what is in effect the family of all continuous real scalar fields on \mathbb{Q}^4 , though I refer to them as 'pointless fields' in so far as, at this point, they are still only indexed disjoint unions. The limiting procedure, moreover, induces on the family of pointless fields the structure of a module over \mathbb{Q} , from the modular structure over \mathbb{Q} that accrued to the maximal family of simple pointless fields. Finally, in the obvious way, we take the completion, as it were, of a family of maximal cross-linkages on the original family of simple pointless fields, resulting in a maximal family of homeomorphisms between open sets of \mathbb{Q}^4 , the allowed transformations among all the induced coordinate systems on our abstract point set, a complete fundamental family.

To complete the construction, we need only to define a topology and then a compatible differential structure on the point-set, turning it into a true

¹⁸ In order to get the completion we require, standard Cauchy convergence does not in fact suffice. We must instead use a more general method, such as Moore–Smith convergence based on topological nets; see (Curiel [unpublished]) for details.

differential manifold. The basic idea is that a complete fundamental family represents the family of continuous real functions on a bounded, normal neighbourhood of what will be the spacetime manifold. Because a spacetime manifold must be paracompact (otherwise it could not bear a Lorentz metric), there is always a countable collection of such bounded, normal neighbourhoods that cover it. This suggests:

Definition 4. A pointless topological manifold is an ordered pair consisting of a countable set of maximal simple pointless families and a cross-linkage on them.

It is straightforward to verify, when one works all the details out, for example, that a real scalar field on the constructed manifold is continuous if and only if its restriction to any of the basic neighbourhoods defines a field in the family associated with that neighbourhood. Now we can define the manifold's differential structure in a straightforward way using similar techniques. First, demarcate the family of smooth scalar fields as a subset of the continuous fields. One can do this in any of a number straightforward ways with clear physical content based on the idea of directional derivatives, such as measuring the rate of change of a physical scalar field in a given spatiotemporal direction. (The algebraic modular structure of the fields comes into play in the definition of the directional derivative.) The family of all smooth scalar fields on a topological manifold, however, fixes its differential structure (Chevalley [1947]). The directional derivatives themselves suffice for the definition of the tangent bundle over the manifold, and from that one obtains all tensor bundles, completing the construction.

After so much abstruse and, worse, tedious technical material, we can now judge whether the construction supports the argument I want to found on it. The use of \mathbb{Q}^4 to index a simple pointless field represents the fact that all points in a laboratory have been uniquely labelled by four rational numbers, say, by the use of rulers and stop-watches. Such an operation neither measures nor relies on knowledge of metrical structure, for it yields in effect only a chart on that spacetime region. (No assumption need be made about the 'metrical goodness' of the rulers and clocks.) Neither does any other operation used in the construction rely on or even pertain to metrical structure. One determines the values of the simple fields, for example, by use of physical observations, none of which necessarily depend on knowledge of the ambient metrical structure. To illustrate the idea, consider the use of a gravity gradiometer to measure the components of the Riemann tensor in a region of spacetime, which exemplifies many of the ideas in the construction. The gradiometer is essentially a sophisticated torsion balance for measuring the quadrupole (and higher) moments of an acceleration field.¹⁹ Its fixed centre and the ends of its

¹⁹ See, for example, (Misner *et al.* [1973], Section 16.5, pp. 401–2) for a description of the device and its use.

two rotatable axes continuously occupy at any given moment five proximate points, and the values of linear and angular acceleration of each point yield direct measures of the Riemann tensor's components in a Fermi frame adapted to the position and motion of the instrument. One then identifies the spacetime points occupied by the parts of the instrument, by the Riemann tensor's components and their derivatives, by the values of its scalar invariants, and so on.²⁰ One does not have to postulate a prior metric structure in order to perform the measurements and label the points, nor need one have already determined the metrical structure by experiment. Indeed, in the performance of the gradiometer measurements one determines much of spacetime's metrical structure. Because, moreover, the facts of intrinsic physical significance that the values of the fields and the relations among them embody (Is this body in contact with another? Does heat flow from that body to this or vice versa?) remain invariant under the action of a diffeomorphism, it follows that the equivalence classes we used to construct points does so as well. Thus, we can fix all the manifold structure, including metrical, only up to diffeomorphism, as we expect. This shows that the construction delivers everything we need and nothing more.

There is an obvious response to the argument based on this construction. One may object that far from the argument's having shown that the construction pushes us to attribute independent existence to spacetime points, it instead suggests that points are defined only by reference to prior physical systems, and hence exist in only a Pickwickian sense, dependent on the identifiability of those physical systems. This objection can be answered by, as it were, throwing away the ladder. Once one has the identification of spacetime points with equivalence classes of values of scalar fields, one can as easily say that the points are the objects with primitive ontological significance, and the physical systems are defined by the values of fields at those points, those values being attributes of their associated points only *per accidens*. I do not pretend to endorse such a move, but I do not have to. My constructive argument is *ad hominem*.

5 The Debate between Substantivalists and Relationalists

I do not consider the idea of pointless manifolds deep or of great interest in its own right.²¹ There are, I am sure, many other constructions in the same spirit. If one were so inclined, I suppose one could try to take something like it to give

²⁰ See, for example, (Bergmann and Komar [1960], [1962]) for a concrete, albeit purely formal, example of a procedure for implementing this idea.

²¹ There are a few questions of potential interest that accrue to it. Is it possible to determine the topology of a non-compact manifold by the postulation of a finite number of simple fields? If so, does the minimum number depend on a topological invariant? Is it greater than the number of fields we currently believe to have physical import in any case?

a precise way for a relationalist to characterize the spacetime manifold.²² I am not so inclined, because I do not think the contemporary debate between the relationalist and the substantivalist has been well posed, and I am inclined to think it never will be in any interesting sense. That is what I take to be the force of the opposed constructions of Sections 3 and 4, taken in tandem. They show that ‘dependence on prior metrical structure’ is formal, that is, without substantive content until given explication in the framework of an investigative enterprise, even if that framework is given only in schematic form. Once one grants this, however, the game is up. Different investigative frameworks can and do yield natural criteria that lead to contrary conclusions.²³

An amusingly poignant feature of the constructions shows this clearly: each yields a conclusion contrary to what the traditional debates would have led one to have expected based on the tools and techniques it employs. In the second, one uses independent values of physical quantities (a stock in trade of the relationalist) in order to identify and attribute existence to spacetime points without a prior assumption of metric structure; and in the first, one uses structures in mathematical physics that seem to presuppose the independent identifiability of spacetime points (a stock in trade of the substantivalist) in order to argue that in fact they are not identifiable without a prior postulation of metric structure. One may think that these features of the arguments make them, in the end, self-defeating, but I do not think so. In the first, one implicitly assumes that complex models are themselves only idealizations of yet more complex models. In the second, one implicitly assumes that, say, the gradiometer is small enough and the temporal interval of the measurement short enough in the experiment to justify the use of the Minkowski metric in making the initial attributions of the magnitudes of spatiotemporal intervals and relations of orthogonality among vectors; one then uses this to bootstrap one’s way to a more accurate representation of the metrical structure of spacetime, which is what is done in practice. I think that these facets of the arguments, perhaps more than anything else, illustrates the vanity of the traditional debate: one can use the characteristic resources and moves of each side to construct arguments contrary to it, once one takes the trouble to make the question precise.

Most damning in my eyes, the constructions show the futility of the debate, for they make explicit how very little one gains in comprehension or understanding by having taken the considerable trouble to have made the questions precise. Indeed, one may feel with justice that nothing has been gained, but

²² See (Butterfield [1984]) for a survey of some ways one might attempt such a project.

²³ This line of argument bears fruitful comparison to the ideas of Ruetsche ([2011]) in the context of interpretations of quantum field theory.

rather something has been lost in a pettifoggery of irrelevant technical detail.²⁴

Although I conclude the traditional debate is without real content, I think there is a related, interesting question one can give clear sense to: what in one's investigative framework is naturally taken to—or must one take to—have intrinsic physical significance? Even putting aside existence and ontology as emotive distractions, however, I do not think one can give even this question substantive sense in the abstract: the question is a formal template that one must give substance to by fixing the significance of its terms in presumably different (but, also presumably, related) ways in different particular contexts.

Consider one way to rephrase the question that may seem, on the face of things, to give it concrete content in abstraction from any schematic framework: what propositions would all observers agree on? One cannot answer this question in the abstract, or even give it definite sense, because one has not yet fixed the way that one will schematically represent the observer (or experimental apparatus) and the process of observation. In order to do so, one must settle many questions of a more concrete nature. Will one use the same theory to model the observation as one uses to model the system? Will one take the observer to be a test system, in the sense that the values of its associated physical quantities do not contribute to the initial-value formulation of the equations of motion of one's models? And so on. Until one settles such issues, one cannot even say with precision what any single observer can or will observe, much less what all will agree on. In this sense, even claims such as 'in general relativity, only what is invariant under diffeomorphisms has intrinsic physical significance' have only schematic content. One must give definite substance to the 'what' in 'what is invariant'—substance that involves the forms of the physical systems at issue and the methods available for their probing and representation—before one can make the claim play any definite role in our attempts to comprehend the world. I take this to be the lesson of Stein ([1977]), namely, that the way to proceed in these matters is the one Newton and Riemann relied on: we must infer what we can about the spatio-temporal structure of the world from the roles it plays in characterizing physical interactions as revealed by our best experimental techniques and modelled by our best theories; and on this basis, neither substantivalism nor relationalism can claim any great victory.

In the end, why should we ever have expected there to have been a single, canonical way to explicate the physical significance of the idea of a spacetime point, on the basis of which we might then attempt to determine whether such a thing exists or not in some lofty or mundane sense? What, after all, is lost to our comprehension of the physical world without such a unique, canonical

²⁴ Jeremy Butterfield has tried to convince me that I dismiss too readily the possible philosophical value of the technical constructions and arguments of Sections 3 and 4. I would like to think he is right.

explication? After all, in these debates we purport to better comprehend the ‘physical’ world. Hadn’t we better ensure, then, that the terms of our arguments have the capacity to come into contact in some important way with the physical world by way of experiment and theory? Once we take that demand seriously, we find an orgiastic throng of possible candidates to serve as concrete realizations of the question, some of which will be fruitful in some kinds of enterprises, others in others, and, most likely, several in none at all. I think a necessary (though not sufficient) condition for the scientific cogency and relevance of the question of the existence of spacetime points is a demonstration that an answer to it would contribute fruitfully to the proper comprehension of the performance of an experiment or the proper construction of a model of a physical system in the context of general relativity. (Recall this article’s epigraph by Maxwell.)

One tempting way to try to justify, on scientific grounds, the debate between the substantialist and the relationalist invokes the idea that ontological clarity by itself is a scientific virtue—it underpins real understanding of a theory; it facilitates novel investigations in, and applications of, a theory; it provides the resources for advancement of scientific knowledge in all its forms; and so on. Before getting carried away, however, it behooves us to look at the history of physics and ask when the settling of an intratheoretic ontological question ever led to a real scientific advance. I think, in fact, the opposite is the case: scientific advances often happen precisely when people stop worrying too much about ontology. It was, for example, Newton’s willingness to remain agnostic about the ontology of light that led him to develop his revolutionary mathematical theory of light and colour in the 1660s, just as it was a similar agnosticism with regard to the ontological basis of gravity that allowed him to take the steps necessary for deriving the law of universal gravitation in *Principia* (in particular, the application of the third law to the force the sun seems to exert on the planets).²⁵ In the development of electromagnetism, similarly, it was exactly when Maxwell stopped looking for an explicit model (ontology) of the electromagnetic field that he was able to construct the full, final theory as we know it today (Maxwell [1965a]). And I find it difficult to believe that quantum mechanics itself would ever have been discovered if Heisenberg, Schrödinger, Dirac, *et al.* had demanded resolution of all their many and deep ontological problems before they were willing to commit themselves and advance their new theory.

In the spirit, again, of the epigraph to this article by Maxwell, I think there is a better question at hand than that of the existence of spacetime points:

²⁵ See (Newton [1958a], [1958b], [1958c], [1958d]) for Newton’s exposition of his theory of light and colour, and for his own explicit explanation and defence of his ontological agnosticism. Stein ([unpublished (a)]) gives a detailed and insightful discussion of this point. Stein ([1990a]) also discusses the role of Newton’s agnosticism in his arguments for universal gravitation.

what mathematical structures best represent our scientific experience of spatio-temporal localization? Again, this question cannot be answered in the abstract, for it depends sensitively on the answers to other, more-or-less independent and yet inextricable questions, such as: What mathematical structures best represent our experience of other features of spatiotemporal phenomena, such as the lack of absolute simultaneity, the orientability of space, and so on? What structures representing various kinds of derivatives do we need to formulate equations of motion? What structures for representation of Maxwell fields? And so on. One has to attempt to address these questions in a dialectical fashion, answering part of one here, seeing what adjustments are then required in other parts of the manifold of possible structures, so to speak, and so on. The answer to one of these questions in one context may be individual points of a spacetime manifold; to another question in another context, it may be area and volume operators as in loop quantum gravity. Instead of asking whether the manifold itself or the manifold plus the metric is really spacetime, we should instead be asking what sorts of structure with real physical significance a manifold by itself and a manifold with a metric can each support—anything requiring only differential topology or geometry for the former, and anything requiring Lorentz geometry for the latter. It is to the investigation of such questions that I now turn.

6 Existence and Physicality: An Embarrassment of Spacetime Structures

The arguments of this article naturally extend themselves beyond the realm of the debate over the existence of spacetime points, and do so in a way that sheds further light on the futility of that debate. There are many different senses one can give to the question of whether some putative entity or structure of any type has real physical significance in the context of general relativity, each more-or-less natural in different contexts. For lack of a better term, I shall say that an entity (which, as we shall see, can encompass several different types of thing), purportedly represented by a theoretical structure, has physicality if one has a reason to take that structure seriously in a physical sense, namely, if one can show that it plays an ineliminable or at least fruitful and important role in the way that theory and experiment make contact with each other. Of course, as I stressed in Section 2 and elaborated on in Section 5, such an abstract, purely formal schema as ‘plays an ineliminable or at least fruitful and important role in the way that theory and experiment make contact with each other’ has no real content until one explicates it in the context of an investigative framework. It is, in fact, one of the ‘important matters on which sensible even if vague things can be said’, which Stein discussed at the beginning of this article. As such, it is the examples that give the idea life.

A Maxwell field, represented by the Faraday tensor, F_{ab} , is manifestly physical. One important sense in which this is true turns on the fact that it contributes to the stress-energy tensor on the righthand side of the EFE: the Maxwell field possesses stress-energy, and in general relativity nothing is physical if not that.

Consider now a Killing field on spacetime, a vector field ξ^a that satisfies Killing's equation

$$\nabla_{(a}\xi_{b)} = 0, \quad (2)$$

and so generates an isometry, in the sense that $\mathcal{L}_\xi g_{ab} = 0$. In this guise, it seems not to possess the characteristics of a physical field, in so far as it enters the equations of motion of no manifestly physical system, such as a Maxwell field. In other words, it does not couple with phenomena we consider physical, and so *a fortiori* does not contribute to the stress-energy tensor. Now, define the 2-index covariant tensor $P_{ab} \equiv \nabla_a \xi_b$. Equation (2) implies that it is anti-symmetric. Let us say that it happens to also have vanishing divergence and curl, $\nabla^n P_{na} = 0$ and $\nabla_{[a} P_{bc]} = 0$, and so satisfies the source-free Maxwell equations. Is it *eo ipso* a true Maxwell field, and so physical? Not necessarily. There are always an innumerable number of 2-forms on a spacetime that satisfy the source-free Maxwell equations. At most, one of them represents a physical Maxwell field. If, however, it just so happened that P_{ab} were to represent the physical Maxwell field on spacetime—one known as a Papapetrou field in this case—the fact that one natural way to represent the field happened to generate an isometry would appear to be an accident, in the sense that no property of the field accruing to it by dint of its physicality, which is to say, by dint of its satisfaction of the Maxwell equations and concomitant coupling with other manifestly physical phenomena (such as spacetime curvature, by way of the EFE), depends on the satisfaction of Equation (2) by ξ^a (except in the trivial sense that satisfaction of Equation (2) is necessary for ξ^a to be a 4-vector potential for a Maxwell field). Still, ξ^a as a Killing field is a naturally distinguished geometrical structure in the physical description of spacetime. It forms a part of the description of spacetime independent of the particulars of the physical constitution of any observed phenomena, particularly in so far as it places non-trivial constraints on a manifestly physical structure, the spacetime metric. In this sense, ξ^a is physical; for the Maxwell field, by contrast, is not naturally distinguished in this sense, but rather depends in an essential way on the peculiar, contingent physical constitution of a particular family of phenomena.

In what sense, though, is the metric manifestly physical? The metric does not itself contribute to the stress-energy content of spacetime, for one cannot attribute a localized gravitational stress-energy to it.²⁶ That is not to say that

²⁶ See, for example, (Curiel [forthcoming (a)]).

the metric does not appear in the stress-energy tensor of a given spacetime, for it is almost always required for the construction of the stress-energy tensor.²⁷ The stress-energy tensor of a Maxwell field, for example, is $F_{an}F^n_b + \frac{1}{4}g_{ab}F_{rs}F^{rs}$. (The metric appears not only explicitly in the second term, but also implicitly in both terms, raising the contracted indices.) The metric, however, is necessary both for posing the initial-value formulation of every possible kind of field that may appear in a relativistic spacetime—in particular all of those (such as the Maxwell field) that we regard as manifestly physical—and for formulating the equations of motion of the fields. In particular, the metric dynamically couples with other physical systems, namely, interacts with them in the strong sense that there always exist terms in the equations of motion for any given field in which the metric appears as one factor and the tensor representation of the field as another. For the Maxwell field, the metric appears contravected, with the Faraday tensor in the field equation representing the fact that its covariant divergence equals the charge-current density of matter.²⁸

The metric, of course, can play other roles as well, just as a Killing field can. A vacuum spacetime with non-zero cosmological constant is based on an extended form of the EFE, with an extra term equal to the metric times a constant. One plausible way of reading the extended EFE is to have the metric play two distinct roles simultaneously, one as the necessary ground of all spatiotemporal structure (embodied in the Einstein tensor) and the other as a component of the tensor representing the stress-energetic content of spacetime (that is, one interprets the extra term in the EFE as a stress-energy tensor), depending on contingent features of the ambient matter field—in this case, whatever field gives rise to the cosmological constant. Again, in the former sense, as the ground of spatiotemporal structure, the metric is a naturally distinguished structure in any physical description of spacetime; in the latter sense, it rather depends on the peculiar, contingent physical constitution of a particular family of phenomena.

Consider the Riemann tensor. Again, it manifests physicality in several different ways, in different contexts. Perhaps the most important is in the equation of geodesic deviation, where it directly measures the rate at which infinitesimally neighbouring geodesics tend to converge towards or diverge away from each other. In this case, the Riemann tensor's physicality consists in the fact that it encodes all information needed to model manifestly

²⁷ Indeed, the only example I know of a stress-energy tensor for which the metric is not needed for its definition is the case of a null gas, for which only the conformal structure of spacetime is required. See (Lehmkuhl [2011]) for discussion of these issues.

²⁸ That the other defining equation for a Maxwell field, representing the fact that the Faraday tensor is curl-free, does not require the metric at all for its formulation—the exterior derivative is determined by the differential structure of the underlying manifold—may push one to say that it is not a dynamical equation of motion, but rather a kinematical constraint.

observable phenomena, namely, the relative acceleration of nearby freely falling particles and the tidal force exerted between different parts of a freely falling extended body. Another important role it plays in general relativity is as the measure of the failure of the ambient covariant derivative operator associated with the spacetime metric to commute with itself when acting on vectors or tensors. The physical significance of this property is straightforward: it encodes the fact that in regions of non-trivial curvature, neighbouring, initially parallel geodesics do not remain parallel, but rather diverge away from or converge towards each other—the phenomenon of geodesic deviation.

The Einstein tensor itself presents an interesting case. It has no straightforward geometrical interpretation.²⁹ It seems, moreover, to have no straightforward physical interpretation either—it enters into the equations of motion of no known fields; it measures no quantitative feature of any known physical phenomena; it does not represent a field possessing stress-energy; it constrains the behaviour of no other manifestly physical structure; and so on. And yet it is the structure that matter fields couple to (via the EFE) in their role as source for spatiotemporal curvature. In this role, it dynamically couples with no individual matter fields, but rather only to the aggregate physical quantity ‘stress-energy’ that they all possess and which, according to the fundamental principle of the fungibility of all forms of energy,³⁰ in no way differs qualitatively among all known fields. It seems, then, manifestly physical in some sense, but it is difficult to put one’s finger clearly on that sense. This is an example of a philosophically important problem whose resolution would provide real physical insight.

Global structures of various sorts (causal, topological, projective, conformal, affine, and so on) present interesting cases as well.³¹ Consider the conformal structure of a spacetime. It governs and is embodied in the relative behaviour of the null cones across all spacetime points. One natural interpretation of the null cones is as determining a finite, unachievable upper-limit for the velocities of material systems.³² The fact that the null cones determine a topological boundary for the chronological future and past of every spacetime point also has a natural interpretation in the same vein: if the chronological future or past were topologically closed, then there would be a limiting upper velocity for massive bodies that would be actually achievable by a massive

²⁹ See (Curiel [forthcoming (b)], Section 2.1) for a discussion.

³⁰ See (Maxwell [1952], Chapter 5, Section 97, [2001], Chapters 1,3,4,8,12) for illuminating discussion of this principle.

³¹ I take a structure to be global if it is not local in the sense explicated by Manchak ([2009], p. 55). I think Manchak’s definition of ‘local’ is superior, as judged by its physical significance in the context of general relativity, to the one I proposed in (Curiel [1999], Section 5), though the latter may still be of interest in purely mathematical contexts, or in contexts of physical investigation that transcend the scope of a single theory.

³² See, however, (Geroch [unpublished]; Earman [unpublished]) for dissenting views.

body using only a finite amount of energy. If one accepts these interpretative glosses, then the conformal structure has physicality in so far as it constrains the behaviour of manifestly physical systems.

So, to sum up, the notions of physicality mooted here are:

- contributes to T_{ab} (for example, Maxwell field);
- required for initial-value formulation of manifestly physical fields (for example, Maxwell field, g_{ab});
- dynamically couples to manifestly physical entities (for example, Maxwell field, g_{ab});
- dynamically couples to manifestly physical quantities that more than one type of physical system can bear (for example, Einstein tensor);
- acts as a measure of an observable aspect of manifestly physical entities (for example, Riemann tensor);
- enters the field equation of a manifestly physical structure (for example, Einstein tensor);
- constrains the behaviour of a manifestly physical entity (for example, Killing field, conformal structure);
- plays an ineliminable (albeit physically obscure) role in the mathematical structure required to formulate the theory (for example, Riemann tensor, Einstein tensor).

I am confident there are yet more senses of physicality I have not touched upon. One does not have to be an instrumentalist or an empiricist to accept that the possible observability of physical phenomena is one of the most fundamental reasons we have to think such things are physical in the first place; see (Curiel [unpublished]) for a discussion of the relation of this idea to that of physicality.

No matter how convincing or interesting or philosophically rich these examples and arguments may be, one might still want to respond that they show nothing about the possible existence of spatiotemporal entities, and so in the end they do not bear on the debate between substantivalism and relationalism. I do not think that is the correct lesson to leave with, though. I take physicality to be a necessary condition for the attribution of existence to a theoretical entity. Thus is there are many possible ways an entity can manifest physicality, and one can show that different entities manifest some but not others of them, then it follows that it is meaningless to attribute existence *simpliciter* to such theoretical entities. If there are two entities each manifesting a different type of physicality, then in so far as each is a necessary condition for existence, if one attributes existence to those entities, it must be of a different sort for each. Thus, in so far as one wants to make sense of the idea of existence in the

context of physical entities purportedly represented by theoretical structures (if that is the sort of thing one likes to do), it cannot be univocal. To paraphrase Aristotle, existence is said in physics, if at all, in many ways.

What light, if any, does all this shed on the cogency of the traditional debate about the ontic status of spacetime? I think quite a bit. A spacetime point is not physical in any of the ways I have explicated: they have no such things as an initial-value problem, they have no equation of motion, they have no property that dynamically couples to any physical field, and so on. How, then, is one supposed to try to answer the question of whether or not they exist in any way that purports to be grounded in physics?

7 Valedictory Remarks on Realism and Instrumentalism, and the Structure of Our Knowledge of Physics

I think my conclusions about the vanity of metaphysical argumentation abstracted from the pragmatics of the scientific enterprise carry over into the general debate over realism and instrumentalism. Indeed, I consider the argument about relationalism and substantivalism to be an instance of the more general form of argument one can give for existence claims about entities and structures in science. An example will make the point.

Consider the question, ‘Do electrons exist?’. On its face, it seems immune to the sorts of problems I raise about the ontic status of spatiotemporal structure. Surely one can attribute canonical significance to this question independent of investigative framework? In fact, one cannot. Think of the different contexts in which the concept of an electron may come into play, and the natural ways one may want to attribute physicality (or not) to electrons in those contexts. A small sample:

- as a component in a quantum, non-relativistic model of the Hydrogen atom;
- as an element in the relativistic computation of the Lamb shift;
- as a possible ‘constituent’ of Hawking radiation in an analysis of its spectrum;
- as a measuring device in the observation of quark structure from deep inelastic scattering of electrons off protons, as treated by the Standard Model.

In the first case, one may want to attribute physicality to the electron in so far as its associated quantities enter into the initial-value formulation of the system’s equations of motion. In the second, one may base the attribution on the fact that one identifies the electron as the bearer of definite values for the kinematic Casimir invariants of spin and mass. Generally, there is no good

definition of an electron in the third case, because there is no unambiguous, physically significant definition of particle in quantum field theory on a curved spacetime, and so *a fortiori* no way to attribute physicality to such a thing.³³ In the fourth and final case, one can attribute physicality to the electron because one can associate localized charge, spin, and lepton number with the mass-energy resonance that represents the electron. Now, one cannot even formulate in a rigorous, precise way (and, indeed, often not even in a loose and frowzy way) the criterion for physicality in any of these frameworks in the terms of at least some of the others.

It follows that even in this case, any formulation of the question in abstract terms—such as what all observers agree on, what has manifestly observable effects, what couples with other systems we already think of as physical, or what is essential to the formulation of the theory—remains empty until one gives content to it by the fixation of a framework, even if only schematic. To be clear, I do not claim that one must always make the investigative framework of one's work explicit, only that one ought to recognize it must be there in the background, specifiable when push comes to shove, as it will from time to time.

In the picture I have implicitly relied on in the construction of my arguments, the structure of physics may be thought of as something like a differential manifold itself, with different techniques and concepts that find appropriate application in different sorts of investigation, and even in similar sorts of investigation of different subject matters, all covering their own idiosyncratic patches of the global manifold, consonant with each other when they overlap but with none necessarily able to cover the entirety of the space. In that vein, I am confident there are many other interesting ways one can render the idea of the physicality of putative entities and structures represented by our best physical theories, variously useful or at least illuminating in investigations of different sorts. In some of those senses, one will rightly, or at least usefully or suggestively, say those things are physical. In others, one will not. The words we use to further all the sorts of scientific and philosophical investigations we pursue do not matter, only the concepts behind the words, some of which find natural application in some investigations and some of which do not.

This is not instrumentalism. Among other things, I neither make nor rely on any principled claim about how one ought to understand the structures of our best theories as formal systems, the terms and relations with which we formulate them, and their broader or deeper relation to the world itself, only about how we ought not understand them. The greatest physicists have always, it

³³ In essence, this is because one has no privileged group of timelike symmetries in a generic spacetime, as one has in Minkowski spacetime, on which to ground the notion of a particle; see (Wald [1994]) for a detailed explanation.

seems to me, had the capacity to think in both realist and instrumentalist ways about both the best contemporary theories and the most promising lines of theoretical attack as they were being developed. Often, they held both sorts of views in their minds at the same time, keeping many avenues open, sometimes moving forward along one, sometimes switching to another, sometimes straddling the line, as best befit the demands of the investigation, with a concomitant gain in richness of conception and depth of thought.³⁴ In some contexts and for some purposes, it is most useful to conceive, think, and speak in realist terms, and in others, to do so in instrumentalist terms. They are both good, in their place, and neither is correct *sub specie aeternitatis*. In any event, what I sketch here is certainly not anti-realism.

What I attempt in this article is a start to shifting the terms and viable fundamental positions of the debate. The traditional debate asks: what is a cogent ontological model of our best theory considered as a formal system? I have argued that we should instead be asking: what is essential for theory and experiment to make fruitful contact with each other? Only in that way does the complete depth, breadth, and scope of scientific knowledge in all its guises and aspects come to bear on the philosophical debate, as it should and must. And so in turn, only in that way can we reasonably hope that the philosophical debate will shed light on our scientific understanding of the world.

I am not against asking questions that, in traditional terms, seem to bear on issues of realism and instrumentalism. I am against the focus on the questions as meaningful and valuable in themselves, without regard to the roles they may or may not play in the ongoing scientific enterprise of attempting to comprehend the physical world. That focus, it seems to me, leads only to a sterile form of ideological back-and-forth that has all but crowded out the possibility of formulating and addressing questions of real scientific and philosophical clarity and value.

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³⁴ Stein ([1977], [1990b], [unpublished (b)]) forcefully argues this line of thought.

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