On the Existence of Spacetime Structure: Technical Appendices[†]

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A Technical Appendix: Limits of Spacetimes

I sketch here the construction of Geroch (1969) (whose exposition I closely follow), which grounds the arguments of section 3 in Curiel (2016). (I simplify his construction in non-essential ways for our purposes, and gloss over unnecessary technicalities.) Consider a 1-parameter family of relativistic spacetimes, by which I mean a family $\{(\mathcal{M}_\lambda,\ g^{ab}(\lambda))\}_{\lambda \in (0,1]}$, where each $(\mathcal{M}_\lambda,\ g^{ab}(\lambda))$ is a relativistic spacetime with signature $(+,\ -,\ -,\ -)$ for $g^{ab}(\lambda)$. (It will be clear in a moment why I work with the contravariant form of the metric tensor.) In particular, I do not assume that $\mathcal{M}_\lambda$ is diffeomorphic to $\mathcal{M}_{\lambda'}$ for $\lambda \neq \lambda'$. The problem is to find a limit of this family, in some suitable sense, as $\lambda \to 0$. To solve the problem in full generality, we will use a geometrical construction, gluing the manifolds $\mathcal{M}_\lambda$ of the family together to form a 5-dimensional manifold $\mathfrak{M}$, so that each $\mathcal{M}_\lambda$ is itself a 4-dimensional submanifold of $\mathfrak{M}$ in such a way that the collection of all of them foliate $\mathfrak{M}$.[1] λ becomes a scalar

[†]These are technical appendices to the paper "On the Existence of Spacetime Structure" (forthcoming 2016 in *British Journal for Philosophy of Science*, early online publication, free access: doi:10.1093/bjps/axw014), working out details of some of that paper's constructions and arguments.

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[1]In general what will result is not a foliation in the strict sense of differential topology, but will rather be a stratified space (Thom 1969). It is close enough to a foliation, however, to warrant using the more familiar term for simplicity of exposition, as nothing hinges on the technical differences between the two.

field on $\mathfrak{M}$, and the metrics $g^{ab}(\lambda)$ on each submanifold fit together to form a tensor field g^{AB} on $\mathfrak{M}$, of signature $(0, +, -, -, -)$. (I use majuscule indices for objects on $\mathfrak{M}$.) The gradient of λ on $\mathfrak{M}$ determines the singular part of g^{AB}: $g^{AN}\nabla_N \lambda = 0$. (This is why I work with the contravariant form of the metric; otherwise, we could not contravect its five-dimensional parent in any natural way with the gradient of λ.) Note that g^{AB} by itself already determines the submanifolds $\mathcal{M}_\lambda$ (*viz.*, as the surfaces defined by $g^{AN}\nabla_N \lambda = 0$), and that it does so in a way that does not fix any identification of points among them. In other words, the structure I posit does not allow one to say that a point in $\mathcal{M}_\lambda$ is "the same point in spacetime" as a point in a different $\mathcal{M}_{\lambda'}$ (as I shall discuss at some length below).

To define a limit of the family now reduces to the problem of the attachment of a suitable boundary to $\mathfrak{M}$ "at $\lambda = 0$". A *limiting envelopment* for $\mathfrak{M}$, then, is an ordered quadruplet $(\hat{\mathfrak{M}}, \hat{g}^{AB}, \hat{\lambda}, \Psi)$, where $\hat{\mathfrak{M}}$ is a 5-dimensional manifold with paracompact, Hausdorff, connected and non-trivial boundary $\partial\hat{\mathfrak{M}}$, $\hat{g}^{AB}$ a tensor field on $\hat{\mathfrak{M}}$, $\hat{\lambda}$ a scalar field on $\hat{\mathfrak{M}}$ taking values in $[0, 1]$, and Ψ a diffeomorphism of $\mathfrak{M}$ to the interior of $\hat{\mathfrak{M}}$, all such that

1. Ψ takes g^{AB} to $\hat{g}^{AB}$ (*i.e.*, Ψ is an isometry) and takes λ to $\hat{\lambda}$
2. $\partial\hat{\mathfrak{M}}$ is the region defined by $\hat{\lambda} = 0$
3. $\hat{g}^{AB}$ has signature $(0, +, -, -, -)$ on $\partial\hat{\mathfrak{M}}$

This makes precise the sense in which $\hat{\mathfrak{M}}$ represents $\mathfrak{M}$ with a boundary attached in such a way that the metric on the boundary ($\hat{g}^{AB}$ restricted to $\partial\hat{\mathfrak{M}}$) can be naturally identified as a limit of the metrics on the $\mathcal{M}_\lambda$ (g^{AB} on $\mathfrak{M}$). I call $\{(\mathcal{M}_\lambda, g^{ab}(\lambda))\}_{\lambda\in(0,1]}$ an *ancestral family* of the spacetime represented by $\partial\hat{\mathfrak{M}}$, and I call $\partial\hat{\mathfrak{M}}$ the *limit space* of the family with respect to the given envelopment. In general, a given spacetime will have many ancestral families, and an ancestral family will have many different limit spaces. For the sake of convenience I will often not distinguish between $\mathfrak{M}$ and the interior of $\hat{\mathfrak{M}}$. (Although it is tempting also to abbreviate '$\partial\hat{\mathfrak{M}}$' by '$\mathcal{M}_0$', I will not do so, because part of the point of the construction is that different spacetimes can have the same ancestral family.)

To characterize the metrical structure of the limit space using structure of members of the ancestral family, I introduce one more construction. An orthonormal tetrad $\xi(\lambda)$ at a point $p_\lambda \in \mathcal{M}_\lambda$ is a collection of 4 tangent vectors at the point mutually orthogonal with respect to $g_{ab}(\lambda)$. Let γ be a smooth curve on $\mathfrak{M}$ nowhere tangent to any $\mathcal{M}_\lambda$ such that it intersects each exactly once. γ then is composed of a set of points $p_\lambda \in \mathcal{M}_\lambda$, one for each λ. A *family of frames* along γ is a family of orthonormal tetrads, one at each point of the curve such that each vector in the tetrad is tangent to its associated $\mathcal{M}_\lambda$, whose members vary smoothly along it. In general, a family of frames will have no well defined limit in $\hat{\mathfrak{M}}$ as $\lambda \to 0$, *i.e.*, there will be no tetrad $\xi(0)$ at a point of $\partial\hat{\mathfrak{M}}$ that the family $\xi(\lambda)$ converges to; in this case, I say the family is *degenerate*. It is always possible, however, given a tetrad $\xi(0)$ at a point on the boundary to find some family of frames that does converge to it.

Now, fix $\xi(0)$ at $p_0 \in \partial\hat{\mathfrak{M}}$ and a family of frames $\xi(\lambda)$ that converges to it. We can represent the metric tensor $g_{ab}(\lambda)$ in a normal neighborhood of p_λ in $\mathcal{M}_\lambda$ using the normal coordinate system that $\xi(\lambda)$ defines in the neighborhood. In a normal neighborhood of p_0, the components of the metric with respect to these coordinates converge as $\lambda \to 0$, and the limiting numbers are just the components of $g_{ab}(0)$ at p_0 with respect to the normal coordinates that $\xi(0)$ defines. In this way, we can characterize all structure on the limit space based on the behavior of the corresponding structures along the family of frames in the ancestral family.

I turn now to an example immediately relevant to my arguments. Consider a family $\{(\mathcal{M}_\lambda, g^{ab}(\lambda))\}$ of Reissner-Nordström spacetimes each element of the family having the same fixed value M for its mass and all parametrized by electric charge λ, which converge smoothly to 0.[2] Construct their envelopment. One can now impose a natural collection of families of frames on the family, with Schwarzschild spacetime as the limit.[3] Now, comparison of figures 3.1 and 3.2 in Curiel (2016) suggests that something drastic happens in the limit. All the points in the throat of the Reissner-Nordström spacetimes (the shaded region in the diagram) seem to get swallowed by the central singularity in Schwarzschild spacetime—in some way or other, they vanish. Using our machinery we can make precise the question of their behavior in the limit $\lambda \to 0$ in the envelopment.

Consider the points in the shaded region in figure 3.2 in Curiel (2016), between the lines $r = 0$ and $r = r^-$. (r is the radial coordinate in a system that respects the spacetime's spherical symmetry; the coordinate values r^- and r^+ define boundaries of physical significance in the spacetime, which in large part serve to characterize the central region of the spacetime as a black hole.) Fix a natural family of frames along a curve in $\mathfrak{M}$ composed of points q_λ each of which lies in the shaded region in its respective spacetime. It is straightforward to verify that the family of frames along the curve does not have a well defined limit: roughly speaking, the curve runs into the Schwarzschild singularity at $r = 0$. In this sense, no point in Reissner-Nordström spacetime to the future of the horizon $r = r^-$ has a corresponding point in the limit space. To sum up: one begins with a family of Reissner-Nordström spacetimes continuously parametrized by electric charge, which converges to 0, and constructs the envelopment of the family; one constructs the limit space by a choice of families of frames; the collection of families of frames enforces an identification of points among different members of the family of spacetimes, including a division of those points that have a limit from those that do not; and that identification, in turn, dictates the identification of spacetime points in the limit space (which points in the ancestral family lie within the Schwarzschild radius, *e.g.*, and which do not). Thus one can identify points within the limit Schwarzschild spacetime, one's idealized model, only by reference to the metrical structure of members of the ancestral family; one can, moreover, identify points in the limit space with points in the more complex, initial models one is idealizing only by reference to the metrical structure of the members of the ancestral family as well. It is only by the latter identification, however, that one can construe the limit space as an idealized model of one's initial models, for the whole point is to simplify the reckoning of the

[2] I ignore the fact that electric charge is a discrete quantity in the real world, an appropriate idealization in this context.

[3] The frames are natural in the sense that they conform to and respect the spherical and the timelike symmetries in all the spacetimes. One could use this fact to explicate the claim that Schwarzschild spacetime is the canonical limit of Reissner-Nordström spacetime, in the sense that it is what one expects on physical grounds, whatever exactly that may come to, in the limit of vanishing charge while leaving all else about the spacetime fixed.

physical behavior of systems at particular points of spatiotemporal regions of one's initial models.

One can, moreover, use different families of natural frames to construct Schwarzschild spacetime from the same ancestral family, with the result that in each case the same point of Schwarzschild spacetime is identified with a different family of points in the ancestral family. More generally, different families of frames will yield limit spaces different from Schwarzschild spacetime, with no canonical way to identify a point in one limit space (one idealized model the theoretician constructs) with one in another. In other words, the identification of points in the limit space depends sensitively on the way the limit is taken, *i.e.*, on the way the model is constructed. In consequence, in so far as one conceives of Schwarzschild spacetime as an idealized model of a richer, more complete representation, one can identify points in it only by reference to the metrical structure of one of its ancestral families, and one can do that in a variety of ways.

Every spacetime has at least one ancestral family, the trivial one consisting of the continuous sequence of itself, so to speak. Construct an envelopment $\mathfrak{M}$ for it, with it itself as the limit space, and apply a slight twist, so to speak, to every metric in every model in the family so as to render each model non-isometric to any other, *i.e.*, so as to render the family non-trivial. (One can make this idea precise in any of a number of simple ways, such as using a smoothly varying 1-parameter family of homotopies or linear perturbations.) On a curve in $\mathfrak{M}$, fix a family of frames that has a well defined limit on $\partial\hat{\mathfrak{M}}$. Now, define a family of Lorentz transformations along that curve, one transformation at each point, such that the family varies smoothly along the curve, and such that when one applies each transformation to the tetrad at its point, the result is a family of frames that has no well defined limit. (One can always do this; for example, the Lorentz transformations can cause the tetrads to oscillate wildly as $\lambda \to 0$.) The points of the ancestral family along that curve have no corresponding point in the limit space defined by the resulting family of frames. This proves:

Proposition A.1 *Every spacetime has a non-trivial ancestral family with vanishing points. Every non-trivial ancestral family has a limit space with respect to which some of its points vanish.*

B Technical Appendix: Pointless Constructions

I give here the constructions relevant to the arguments of section 4 in Curiel (2016). The basic idea of the construction of a pointless manifold is simple. I posit a class of sets of rational numbers to represent the possible values of physical fields, with a bit of additional structure in the form of primitive relations among them just strong enough to ground the definition of a derived relation whose natural interpretation is "lives at the same point of spacetime as". A point of spacetime, then, consists of an equivalence class of the derived relation. The derived relation, moreover, provides just enough rope to allow for the definition of a topology and a differential structure on the family of all equivalence classes, and from this the definition of all tensor bundles over the resultant manifold, completing the construction. The posited primitive and derived relations have a straightforward physical interpretation, as the designators of instances of a schematic representation of a fundamental type of procedure the experimental physicist performs on physical fields when he attempts to ascertain relations of physical proximity and superposition among their observed values. An important example of such an experimental procedure is his use of the observed values of physical quantities associated with experimental apparatus to determine the values of quantities associated with other systems, those he investigates by use of the apparatus. This interpretation of the relations motivates the claim that the constructed structure suffices, for our purposes, as a representation of spacetime in the context of a particular type of experimental investigation as modeled by mathematical physics, and is not (only) an abstract mathematical toy. Because of limitations of space, I give only a bare sketch of the construction. SeeCuriel (2015)

I begin the construction by laying down some definitions. Let $\mathbb{Q}$ be the set of rational numbers. A *simple pointless field* ϕ (or just *simple field*) is a disjoint union $\biguplus_{p \in \mathbb{Q}^4} f_p$, indexed by the set $\mathbb{Q}^4$, such that

1. every $f_p \in \mathbb{Q}$
2. there is exactly one $f_p \in \phi$ for each $p \in \mathbb{Q}^4$
3. there are two strictly positive numbers B_{l} and B_{u} such that $B_{\mathrm{l}} < |f_p| < B_{\mathrm{u}}$ for all $p \in \mathbb{Q}^4$
4. the function $\bar{\phi} : \mathbb{Q}^4 \to \mathbb{Q}$ defined by $\bar{\phi}(p) = f_p$ is continuous in the natural topologies on those spaces, except perhaps across a finite number of compact three-dimensional boundaries in $\mathbb{Q}^4$

Our eventual interpretation of such a thing as a candidate result for an experimentalist's determination of the values for a physical field motivates the set of conditions. That we index ϕ over $\mathbb{Q}^4$ means we assume that the experimentalist by the use of actual measurements and observations alone can impose on spacetime at most the structure of a countable lattice indexed by quadruplets of rational numbers (and even this only in a highly idealized sense); in other words, the spatiotemporal precision of measurements is limited. Condition 1 says that all measurements have only a finite precision in the determination of the field's value. Condition 2 says that the field the experimentalist measures has a definite value at every point of spacetime. Condition 3 says that there is an upper and a lower limit to the magnitude of values the experimentalist can attribute to the field using the proposed experimental apparatus and technique; for instance, any device for the measurement of the energy of a system has only a finite precision, and thus can attribute only absolute values greater than a certain magnitude, and the device will be unable to cope with energies above a given magnitude. Condition 4 tries to capture the ideas that (local) experiments involve only a finite number of bounded physical systems (apparatuses and objects of study), and that classical physical systems bear physical quantities the magnitudes of which vary continously (if not more smoothly), except perhaps across the boundaries of the systems.

Fix a family Φ of simple pointless fields. The *link at* p, λ_p, is the set containing the elements from all simple fields in Φ indexed by $p \in \mathbb{Q}^4$. A *linked family of simple pointless fields* $\mathfrak{F}$ is an ordered pair (Φ, Λ) where Φ is a collection of distinct simple fields, and Λ is the family of links on Φ, a *linkage*, complete in the sense that it contains exactly one link for each $p \in \mathbb{Q}^4$. The idea is that the values of the simple fields in the same link all live "at the same point of spacetime", namely that designated by p. One can think of the linkage as a coordinate system on an underlying, abstract point set.

We are almost ready to define the point-structure of the spacetime manifold. We require only a few more definitions and two more constructions, which I give in an abbreviated fashion so as to convey the main points without getting bogged down in unnecessary technical detail. First, given an open set $O \in \mathbb{Q}^4$ and a simple pointless field ϕ, let ϕ_O be the field restricted to f_p for $p \in O$. Given a family of simple pointless fields Φ, denote by 'Φ_O' the family of simple pointless fields in Φ restricted to O, and similarly for 'Λ_O', defined in the obvious way. Now, let $\mathfrak{F} = (\Phi,\, \Lambda)$ be a linked family containing a countable number of simple fields; we call it a *simple fundamental family.* Let $\hat{\mathfrak{F}} = (\hat{\Phi},\, \hat{\Lambda})$ be another. Let $\mathfrak{F}_O$ be the family of simple pointless fields and the linkage in $\mathfrak{F}$ restricted to the open set O. We want a way to relate the linkages of $\mathfrak{F}_O$ and $\hat{\mathfrak{F}}_{\hat{O}}$, for open sets O and $\hat{O}$, so as to be able to represent the relation between the coordinate systems of two different charts on the same neighborhood of the spacetime manifold, or on the intersection of two neighborhoods. A *cross-linkage* between two simple fundamental families $\mathfrak{F}$ and $\hat{\mathfrak{F}}$ is an ordered triplet $(O,\, \hat{O},\, \chi)$ where $O, \hat{O} \subseteq \mathbb{Q}^4$ are open sets, such that either both are the null set or else both are homeomorphic to $\mathbb{Q}^4$, and χ is a homeomorphism of O to $\hat{O}$. The link $\lambda_p \in \Lambda$ for $p \in O$, then, will designate the same point in the underlying manifold as $\hat{\lambda}_{\chi(p)} \in \hat{\Lambda}$ for $\chi(p) \in \hat{O}$; in this case, we say the links *touch.* If O and $\hat{O}$ are the null set, then the represented neighborhoods do not intersect. (We do not require that the values of the scalar fields in the two different simple fundamental families be numerically equal at any given point, as the two scalar fields may represent different physical quantities, *e.g.*, a component of the fluid velocity and a component of the shear-stress tensor of a viscous fluid.) $\mathfrak{F}$ and $\hat{\mathfrak{F}}$ are to represent coordinate charts on open sets of the underlying spacetime manifold, and the cross-linkage the relation between the ways that $\mathfrak{F}_O$ and $\hat{\mathfrak{F}}_{\hat{O}}$ respectively "assign coordinates" to the same spacetime region, *viz.*, the one defined by the intersection of the "domains" of $\mathfrak{F}$ and $\hat{\mathfrak{F}}$. (The idea of a cross-linkage can be extended to cover more than two simple fundamental families in the obvious way.)

To finish the preparatory work, we must move from rationals to reals. Fix a simple fundamental family $\mathfrak{F}$ containing all simple pointless fields, a *complete simple fundamental family.* First, we attribute to $\mathfrak{F}$ the algebraic structure of a module over $\mathbb{Q}$. For example, the sum of two simple pointless fields ϕ and ψ in Φ is a simple pointless field ξ such that $x_p \equiv f_p + g_p$ is the value in ξ labeled by the index p, where $f_p \in \phi$ and $g_p \in \psi$. ξ is clearly itself a simple pointless field with a natural embedding in the linkage on $\mathfrak{F}$, and so belongs to Φ. Now, roughly speaking, we take a double Cauchy-like completion of Φ over both the points $p \in \mathbb{Q}^4$ and the values $f_{\hat{p}} \in \mathbb{Q}$, yielding the family $\bar{\Phi}$ of all disjoint unions of real numbers continuously indexed by quadruplets of real numbers.[4] This procedure makes sense, because every continuous real scalar field on $\mathbb{R}^4$ is, again roughly speaking, the double limit of some sequence of bounded, continuous rational fields defined on $\mathbb{Q}^4$. We thus

[4]In order to get the completion we require, standard Cauchy convergence does not in fact suffice. We must rather use a more general method, such as Moore-Smith convergence based on topological nets. The technical details are not important. See, *e.g.*, Kelley (1955, ch. 2) for details.

obtain what is in effect the family $\bar{\Phi}$ of all continuous real scalar fields on $\mathbb{R}^4$, though I refer to them as *pointless fields*, in so far as, at this point, they are still only indexed disjoint unions. The limiting procedure, moreover, induces on $\bar{\Phi}$ the structure of a module over $\mathbb{R}$ from that on Φ. Finally, in the obvious way, we take the completion, as it were, of Λ using the same limiting procedure to obtain a linkage $\bar{\Lambda}$ on $\bar{\Phi}$. I call $\bar{\mathfrak{F}} = (\bar{\Phi}, \bar{\Lambda})$ a *complete fundamental family.* A cross-linkage on a pair of fundamental families is the same as for simple fundamental families, except only that one uses homeomorphisms on subsets of $\mathbb{R}$ rather than $\mathbb{Q}$. If we have two simple fundamental families with a cross-linkage on them and take limits to yield two fundamental families, then the nature of the limiting process guarantees a unique cross-linkage on the two fundamental families consistent with the original.

We can at last construct a real topological manifold from a collection a cross-linkage on a family of simple fundamental fields. The basic idea is that a complete fundamental family represents the family of continuous real functions on the interior of a bounded, normal neighborhood of what will be the spacetime manifold. Because a spacetime manifold must be paracompact (otherwise it could not bear a Lorentz metric), there is always a countable collection of such bounded, normal neighborhoods that cover it. This suggests

Definition B.1 *A* pointless topological manifold *is an ordered pair* $(\{\mathfrak{F}_i\}_{i\in\mathbb{N}}, \chi)$ *consisting of a countable set of complete simple fundamental families and a cross-linkage on them.*

To justify the definition, I sketch the construction of the full point-manifold and its topology. First, we take the joint limit of all simple fundamental families to yield a countable collection of fundamental families with the induced cross-linkage. A point in the manifold, then, is an equivalence class of links, at most one link from each family, under the equivalence relation "touches". The set of links associated with one of the families, then, becomes a representation, with respect to the equivalence relation, of the interior of a compact, normal neighborhood in the manifold, and the fields in that family represent the collection of continous real functions on that neighborhood. The cross-linkage defines the intersections among all these neighborhoods, yielding the entire point-set of the manifold. By assumption, the collection of all such neighborhoods forms a sub-base for the topology of the manifold, and so, by constructing the unique topological base from the given sub-base, the point-set becomes a true topological manifold. It is straightforward to verify, for example, that a real scalar field on the constructed manifold is continuous if and only if its restriction to any of the basic neighborhoods defines a field in the fundamental family associated with that neighborhood.

Now, to complete the construction, we can define the manifold's differential structure in a straightforward way using similar techniques. First, demarcate the family of smooth scalar fields as a sub-set of the continuous ones, which one can do in any of a number straightforward ways with clear physical content based on the idea of directional derivatives. (The algebraic modular structure of the fields comes into play in the definition of the directional derivative.) The family of all smooth scalar fields on a topological manifold, however, fixes its differential structure (Chevalley 1947). The directional derivatives themselves suffice for the definition of the tangent bundle over the manifold, and from that one obtains all tensor bundles.

C Appendix: Observability

This discussion supplements that of section 6 in Curiel (2016).

One does not have to be an instrumentalist or an empiricist to accept that the possible observability of physical phenomena is one of the most fundamental reasons we have to think such things are physical in the first place. The question of the observability of various kinds of global structure in general relativity, therefore, poses particularly interesting problems for arguments about physicality. Manchak (2009, 2011) shows that, in a precise sense, local observations can never suffice to determine the complete global structure of spacetime in general, and in particular cannot determine whether a spacetime is inextendible or stably causal (Manchak 2011, p. 418, proposition 3). Nonetheless, there remain several things to say and ask about the matter of physicality here.

Take, for example, the Euler number of the spacetime manifold, a global topological structure.[5] It is a topological invariant that, in part, constrains the possible existence of everywhere non-zero vector fields on a manifold. That an even-dimensional sphere, for example, possesses no everywhere non-zero vector field (and indeed no Lorentzian metric) follows directly from the computation of its Euler number. If we were to live in a world whose underlying manifold possessed a non-trivial Euler number, and so could support no physical process that would manifest itself as an everywhere non-zero vector field, this would constitute a physical fact about the world in an indubitable sense. It is not clear to me, however, whether in some precise sense the Euler number of the spacetime manifold could ever be determined by direct observation.

The orientability of spacetime is an example of a global topological structure that seems to be strictly inobservable in an intuitive sense. This follows from the fact that one can construct an orientable manifold from any non-orientable one by lifting the structures on it to a suitable covering space, which is automatically orientable. The lift of the spacetime metric to a covering manifold, however, would yield a representation of exactly the same physical spacetime as the original: every physical phenomena in the one has an isometric analogue, as it were, in the other, and vice-versa. Whether or not a spacetime manifold is simply connected, moreover, seems to be in the same boat, for the universal covering manifold of a manifold is guaranteed to be simply connected.[6]

Nonetheless, I think those answers about the possible observability of a manifold's orientability and simple connectedness may be too pat. If one were to observe that any member of a certain

[5]See, *e.g.*, Alexandrov (1957, ch. VIII).

[6]In order for a manifold to possess a universal covering manifold, it must be semi-locally simply connected. Intuitively, this means that it cannot contain "arbitrarily small holes". More precisely, it means that every point in the space has a neighborhood such that every loop in the neighborhood can be continuously contracted to a point. (The contraction need not occur entirely with the given neighborhood.) The so-called Hawaiian Ear-Ring is an example of a topological space that is not semi-locally simply connected (Biss 2000). Whether or not a spacetime manifold is semi-locally simply connected presents us with yet another type of question related to physicality: strictly speaking, there is no physical need for a manifold to possess a universal cover, and it is difficult, to say the least, to see what other physical relevance being semi-locally simply connected could have; and yet the construction of the universal cover is such an extraordinarily useful theoretical device (Geroch 1967) that one wants to demand that a candidate spacetime manifold be semi-locally simply connected. What status does such a demand have? A purely pragmatic one?

family of closed, physically distinguished spatiotemporal loops could not be continuously deformed into any member of another family of closed, physically distinguished spatiotemporal loops, one would have shown that the spacetime manifold is not simply connected. Similarly, if one could show that to parallely propagate a fixed orthonormal tetrad around a given closed spatiotemporal loop would result in its inversion, one would have demonstrated that spacetime is not orientable. I personally have no idea what sorts of experiment could show either of those things. The history of physics, however, if it shows us nothing else, does show us never to underestimate the ingenuity of experimentalists, no matter what the theoretician may tell them is impossible to observe or measure.

The first Betti number of the spacetime manifold offers another interesting example of this sort. The first Betti number of a topological space is the number of distinct connected components it has; any manifold with a first Betti number greater than one is *ipso facto* not connected. Say that we posited a non-connected spacetime manifold. According to the principles of general relativity, any phenomena in one component would be strictly inobservable in any other. By this criterion, it makes no sense to attribute physicality to regions of spacetime disconnected from our own.

So, are these possibly inobservable global structures physical? Well, it seems to me that in some senses they are, and in others they are not. The only lesson I want to draw here is that questions of this sort require in-depth investigation sensitive both to the technical details of the mathematics and to the physical details of how such structures may and may not bear on other phenomena we think of as manifestly physical, even if they turn out to be indubitably inobservable.[7]

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Curiel, E. (2015). On the existence of spacetime structure: Technical appendices. This paper consists of technical appendices to the paper "On the Existence of Spacetime Structure" (forthcoming 2016 in *British Journal for Philosophy of Science*), working out details of some of

[7] The family of phenomena in relativistic spacetimes grouped under the rubric "singular stucture" (or "singularities") provides on its own a rich and diverse selection of examples, which I do not have room even to sketch here. See Curiel (1999) for an extended discussion.

that paper's constructions and arguments. URL: http://strangebeautiful.com/papers/curiel-exist-st-struct-tech-apdx.pdf.

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