Classical Black Holes Are Hot*†

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ABSTRACT

In the early 1970s it is was realized that there is a striking formal analogy between the so-called laws of black-hole mechanics and the laws of classical thermodynamics. Before the discovery of Hawking radiation, however, it was generally thought that the analogy was only formal, and did not reflect a deep connection between gravitational and thermodynamical phenomena. It is still commonly held that the surface gravity of a stationary black hole can be construed as a true physical temperature and its area as a true entropy only when quantum effects are taken into account; in the context of classical general relativity alone, one cannot cogently construe them so. Does the use of quantum field theory in curved spacetime offer the only hope for taking the analogy seriously? I think the answer is 'no'. To attempt to justify that answer, I shall begin by arguing that the orthodxy is not physically well founded, and in any event begs the question. Looking at the various ways that the ideas of "temperature" and "entropy" enter classical thermodynamics then will suggest arguments that, I claim, show the analogy between classical black-hole mechanics and classical thermodynamics should be taken more seriously, without the need to rely on or invoke quantum mechanics. In particular, I construct an analogue of a Carnot cycle in which a black hole "couples" with an ordinary thermodynamical system in such a way that its surface gravity plays the role of temperature and its area that of entropy. This strongly suggests that the connection between classical general relativity and classical thermodynamics on their own is already deep and physically significant, independent of quantum mechanics.

1 Introduction

Based on the striking formal similarities between, on the one hand, the so-called Zeroth, First, Second and Third Laws of classical thermodynamics, and, on the other, the mechanics of black holes in stationary, axisymmetric, asymptotically flat spacetimes (Bardeen, Carter, and Hawking 1973), it is tempting to try to conceive of classical black holes as thermodynamical objects. When it is also noted that classical black holes, like ordinary thermodynamical systems, are characterized by a small number of gross parameters independent of any details about underlying microstructure, and that each version of the First Law states a conservation principle for essentially the same quantity as the other, *viz.*, mass-energy, it becomes tempting to surmise that some deep or fundamental connection between classical black holes and thermodynamics is being uncovered. But is it of real physical significance in some sense?

The conventional answer to this question is 'no'. Because classical black holes are perfect absorbers, they would seem to have a temperature of absolute zero, even when they have non-zero surface gravity. It is only with the introduction of quantum considerations, orthodoxy runs, in particular the derivation of Hawking radiation (Hawking 1974; Hawking 1975), that one finds grounds for taking the analogy seriously.[1] I believe, however, that the orthodoxy field is mistaken. The use of quantum field theory in curved spacetime is not the only hope for taking the analogy between black holes and thermodynamics seriously. There are two steps to my argument. First, I show that the standard argument to the contrary is not physically well founded. Then, looking at the various ways that the ideas of "temperature" and "entropy" enter classical thermodynamics will suggest positive arguments that show the analogy between classical black-hole mechanics and classical thermodynamics should be taken seriously indeed, in particular, the construction of the analogue of a Carnot cycle with the heat sink provided by a stationary black hole, without the need to rely on or invoke quantum mechanics. In the cycle, the black hole's surface gravity and area play, respectively, the physical roles of temperature and entropy of an ordinary heat sink in an ordinary Carnot cycle. There also follows from the construction the existence of a universal constant with the physical dimensions needed to give surface gravity the physical dimension of temperature and area the physical dimension of entropy. If surface gravity and entropy couple to ordinary thermodynamical systems in the same way as temperature and entropy, respectively, do, then there can be no grounds for denying that they physically *are* a real temperature and entropy.

To put it more provocatively, if my claim is correct, then gravity on its own, independent of its relation to the other three known fundamental forces so successfully treated by quantum field theory, and independent of the postulation of any underlying statistical kinematics and dynamics, is already a fundamentally thermodynamical phenomenon. I want to stress, nonetheless, that I do not consider quantum effects to be irrelevant when considering possible relations between gravitational physics and thermodynamics. I want only to argue for the idea that the analogy between the laws of classical thermodynamics and those of black hole mechanics in classical general relativity is robust, physically significant and deep in its own right. If this is correct, then this could have fruitful consequences for work on general relativity as an effective field theory (Jacobson 1995), proposals for general measures of gravitational entropy (Clifton, Ellis, and Tavakol 2013), Penrose's Conformal Curvature Hypothesis (Penrose 1979), and several programs in quantum gravity (Rovelli 2008; Amelino-Camelia 2013; Ashtekar, Reuter, and Rovelli 2015; Elvang and Horowitz 2015).

2 The Laws of Black Hole Mechanics and Thermodynamics

Within the context of general relativity, one can derive laws describing the behavior of black holes in stationary, asymptotically flat spacetimes bearing a remarkable resemblance to the classical laws of equilibrium thermodynamics. I restrict attention to the asymptotically flat case, because those are the simplest natural analogue of an isolated system for black holes. I restrict attention to stationary black holes because those are the simplest natural analogue of a thermally equilibrated system for black holes.

[1] See, for example, the remarks of Hawking (1976) and Wald (1984, p. 337). Indeed, many researchers make even stronger claims. Unruh and Wald (1982, p. 944), for example, say that "the existence of acceleration radiation [outside the event horizon, a fundamentally quantum phenomenon,] is vital for the self-consistency of black-hole thermodynamics."

Now, for the laws themselves:[2]

Zeroth Law

[**Thermodynamics**] The temperature T is constant throughout a body in thermal equilibrium.

[**Black Holes**] The surface gravity κ is constant over the event horizon of a stationary black hole.

First Law

[**Thermodynamics**]

$$\mathrm{d}E = T\mathrm{d}S + p\mathrm{d}V + \Omega\mathrm{d}J$$

where E is the total energy of the system, S the entropy, p the pressure, V the volume, Ω the rotational velocity and J the angular momentum.

[**Black Holes**]

$$\delta M = \frac{1}{8\pi}\kappa\delta A + \Omega_{\mathrm{BH}}\delta J_{\mathrm{BH}}$$

where M is the total black hole mass, A the surface area of its horizon, Ω_{BH} the "rotational velocity" of its horizon, J_{BH} its total angular momentum, and 'δ' denotes the result of a first-order, linear perturbation of the spacetime.

Second Law

[**Thermodynamics**] $\delta S \geq 0$ in any process.

[**Black Holes**] $\delta A \geq 0$ in any process.

Third Law

[**Thermodynamics**] $T = 0$ is not achievable by any process.

[**Black Holes**] $\kappa = 0$ is not achievable by any process.

Perhaps the most striking architectonic similarity between the two sets of laws is that in each case the behavior of the system, irrespective of any idiosyncracies in the system's constitution or dynamical history, is entirely captured by the values of a small number of physical quantities, 6 for ordinary thermodynamical systems, 4 for black holes: in the former case, they are temperature, entropy, pressure, volume, angular velocity and angular momentum; in the latter, they are surface gravity, area, angular velocity and angular momentum. The Zeroth and Third Laws suggest that we take the surface gravity of a black hole as the analogue of temperature. The Second Laws suggest that we take area as the analogue of entropy. This is consistent with the First Law, if we treat $\frac{1}{8\pi}\kappa\delta A$ as the Gibbsian "heat" term for a system in thermal equilibrium. Indeed, if we do so then the analogy for the First Law becomes exact: relativistically, energy just is mass, so the lefthand

[2] For proofs of the laws, see Wald and Gao (2001), Israel (1986) and Wald (1994).

side terms of the First Law for ordinary systems and for black holes are not just analogous, they are physically identical; likewise, $\Omega_{\text{BH}}\delta J_{\text{BH}}$ as a work term in the law for black holes is physically identical to the corresponding term in the law for ordinary systems.

Now the force of the question motivating this paper should be clear: the mathematical analogy is perfect, and there are already some indications that the analogy may reach down to the level of physics, not just mathematics. But how far should we take the analogy? What can it mean to take seriously the idea that the surface gravity of a black hole is a physical temperature, and its area a physical entropy?

3 The Standard Argument Does Not Work

There are well-known difficulties with taking the surface gravity of a classical black hole to represent a physical temperature. One important method for defining the thermodynamic temperature of an object derives from the theory of thermal radiation from black bodies. If a normal black body immersed in a bath of thermal radiation settles down to thermal equilibrium, it will itself emit thermal radiation with a power spectrum characteristic of its equilibrium temperature as measured using a gas thermometer. This power spectrum can then be used to define a temperature scale. It is this definition of thermodynamic temperature that is almost always (at times implicitly) invoked when the claim is made that if one considers classical general relativity alone then black holes, being perfect absorbers and perfect non-emitters, have an effective temperature of absolute zero.[3] A Kerr black hole in a perfectly reflective box filled with thermal radiation will absorb all the radiation and emit none. The orthodoxy thus has it that it is only with the derivation of Hawking radiation when quantum particle-creation effects near the black hole horizon are taken into account (Hawking 1974; Hawking 1975), which shows that stationary, axisymmetric black holes appear to radiate as though they were perfect black-body emitters in thermal equilibrium with temperature $\frac{\hbar}{2\pi}\kappa$, that black holes can be considered truly thermodynamical objects, and in particular that κ *does* in fact represent the physical temperature of a black hole, and therefore A its entropy.

The physical content of the standard argument, however, does not stand up to scrutiny. While it is true that the Kerr black hole in the box, according to classical general relativity, will emit no blackbody radiation while it absorbs any incident on it, that is not the end of the story. Classical general relativity does tell us that the Kerr black hole will emit some radiation, *viz.*, *gravitational* radiation, while it is perturbed by the infalling thermal radiation, and that gravitational radiation will in fact couple with the thermal radiation still outside the black hole. If we are trying to figure out whether purely gravitational objects, such as black holes, have thermodynamical properties, then we must allow for the possibility that gravitational radiation, or, indeed, the exchange of "gravitational energy" in any form, will count as a medium for thermodynamical coupling capable of supporting a definition of temperature.[4] Indeed, just as electromagnetic radiation turned out to be a medium

[3] See for example the remarks in Carter (1973) and and Wald (1999).

[4] I use scare-quotes for 'gravitational energy' because that is an infamously vexed notion in classical general relativity, with no cogent way known to localize it, and indeed strong reasons to think there can be no localization of it

capable of supporting a physically significant coupling of electromagnetic systems with classical thermodynamical systems, and so allowing for a definition of temperature for electromagnetic fields consistent with the classical scale, it seems *prima facie* plausible that gravitational radiation may play the same role for gravitational systems. Just as "heat" for an electromagnetic system may be measured by intensity of electromagnetic radiation, at least when transfer processes are at issue, so it may be that "heat" for a gravitational system may be measured by intensity of gravitational radiation, or any form of exchange of gravitational energy, again at least when transfer processes are at issue. Electromagnetic energy is just not the relevant quantity to track when analyzing the thermodynamic character of purely gravitational systems.

There is yet another *prima facie* problem, however, with trying to interpret surface gravity as a true temperature and area as a true entropy: neither has the proper physical dimension. In geometrized units, the physical dimension of temperature is mass (energy), and entropy is a pure scalar. The physical dimension of surface gravity, however, is the inverse of mass, and that of area is mass squared. There are no purely classical universal constants, moreover, available to fix the dimensions by multiplication or division. The only available universal constant to do the job seems to be $\hbar$, which has the dimension mass squared. Remarkably, however, a natural sequela to the construction of the appropriate analogue of a Carnot cycle for black holes will be the existence of a universal constant in the classical regime with the proper dimension.

4 Temperature and Entropy in Classical Thermodynamics

Imagine that we are physicists who know only classical general relativity and classical thermodynamics, but have no knowledge of quantum theory. How could we determine whether or not to take black holes as thermodynamical objects in a substantive, physical sense, given that we know the deep formal analogy between the two sets of laws? In such a case, we would look to the way that temperature and entropy are introduced in classical thermodynamics and the various physical roles they play there. If the surface gravity and area of black holes play the analogous physical roles, and obey the conditions needed for the introduction of the classical quantities, then the physical analogy is already on strong ground. If, futhermore, it can be shown that surface gravity couples to ordinary classical thermodynamical systems in the same formal way as ordinary temperature does, then there are no grounds for denying that it is a true physical temperature.[5] And if area for black holes is related to surface gravity and to the proper analogue of heat in the same way as entropy is to ordinary temperature and heat, and if it is required for formulating an appropriately generalized Second Law, then there are no grounds for denying that it is a true physical entropy. Indeed, it was exactly on these grounds that physicists in the 19th century concluded that the power spectrum of blackbody radiation itself encoded a *physical* temperature and entropy, not merely that there was an

in general.

[5] Since entropy directly mediates no coupling between thermodynamical systems, the same argument is not available for it. This is one of the properties of entropy that makes it a truly puzzling physical quantity: there is no such thing, not even in principle, as an entropometer.

analogy between thermodynamics and the theory of blackbody radiation. Planck (1926) himself had doubts about the thermodynamical character of blackbody radiation until he had satisfied himself on these points.

There are two fundamental, related ways that temperature is introduced in classical thermodynamics, which themselves ground the various physical roles temperature can play in the theory (how it serves as the mediator of particular forms of coupling between different types of physical system, *e.g.*). The first arises from the fact that increase in temperature is positively correlated with increases in the capacity of a system to do work.[6] This fact allows one to define an empirical scale of temperature through, *e.g.*, the use of a gas thermometer: the temperature reading of the thermometer is made directly proportional to the volume of the thermometric gas used, which is itself directly proportional to the work the gas does on its surrounding container as it expands or contracts in response to its coupling with the temperature of the system being measured. The utility of such a scale is underwritten by the empirical verification that such empirical scales defined using a multitude of different gases under a multitude of different conditions are consistent among themselves.[7] The second arises from the investigation of the efficiency of reversible, cyclic engines, *viz.*, Carnot engines, which yields a definition of the absolute temperature scale associated with the name of Kelvin. It is the possibility of physically identifying the formally derived absolute scale with the empirically derived scale based on capacity to do work (increase in volumes, *e.g.*) that warrants the assertion that they both measure the same physical quantity.[8]

Likewise, there are (at least) three ways that entropy enters classical thermodynamics. The first historically, and perhaps the most physically basic and intuitive, is as a measure of how much energy it takes to transform the heat of a thermal system into work: generally speaking, the free energy of a thermodynamical system is inversely proportional to its entropy.[9] The second is as that perfect differential $\mathrm{d}S$ into which temperature, as integrating factor, transforms exchanges of heat $\mathrm{d}Q$ over the course of quasi-stationary processes: the integral of $\mathrm{d}Q$ along a quasi-stationary path between two equilibrium states in the space of states of a thermodynamical system is not independent of the path chosen, whereas the integral of $\frac{\mathrm{d}Q}{T}$ is. The third also arises from the analysis of the efficiency of Carnot engines.

There are two classical postulates, which can be argued on physical grounds both to be equivalent to each other and to directly imply the Second Law of thermodynamics, which underlie all the ways temperature and entropy are introduced:

[6] See, *e.g.*, the exemplary remarks of Sommerfeld (1964, p. 36): "Thermodynamics investigates the conditions that govern the transformation of heat into work. It teaches us to recognize temperature as the measure of the work-value of heat. Heat of higher temperature is richer, is capable of doing more work."

[7] Planck (1926, §1, p. 1) remarks that quantitative exactness is introduced into thermodynamics through this observation, for changes of volume admit of exact measurements, whereas sensations of heat and cold do not, nor even comparative judgments of hotter and cooler on their own.

[8] Maxwell (1888, chs. VIII, XIII) gives a wonderfully illuminating discussion of the physical basis of the equivalence of the absolute temperature scale with the one based on gas thermometry.

[9] Again, the discussion of Maxwell (1888, ch. XII) about this idea is a masterpiece of physical clarification and insight.

Postulate 4.1 (Lord Kelvin) *A transformation whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.*

Postulate 4.2 (Clausius) *A transformation whose only final result is to transfer heat from a body at a given temperature to a body of a higher temperature is impossible.*

(Indeed, in purely classical treatments, these postulates, and not the principle of entropy increase, are taken to embody the Second Law; see, *e.g.*, Fermi (1956).) The Clausius Postulate captures the idea that when two bodies are brought into thermal contact, heat flows from the body of higher temperature to the other unless work is done. The Kelvin Postulate captures the idea that the capacity of a body to do work on its environment tends to increase as its temperature increases. If one could show that appropriately formulated analogues to these two propositions hold in general relativity about black holes, with surface gravity playing the role of temperature and area that of entropy, one would have gone a long way towards showing that surface gravity *is* a true thermodynamical temperature and area a true entropy. If one could moreover construct an absolute temperature scale for black holes that was essentially equivalent to surface gravity, and then show that surface gravity so characterized plays the role of integrating factor for the gravitational analogue of heat, turning it into the perfect differential of area, the conclusion would be even more secure. If, finally, such an absolute scale for surface gravity would be shown to be empirically equivalent to that constructed for ordinary thermodynamical systems, then the analogy would have been shown to be far more than analogy: it would be physical equivalence in the strongest sense.

5 Taking Black Holes Seriously as Thermodynamical Objects

Just as the concept of "thermal coupling" had to be emended in the extension of classical thermodynamics to include phenomena associated with radiating black bodies, so we should expect it to be in this case. In classical thermodynamics before the inclusion of black-body phenomena, thermal coupling meant immediate spatial contiguity: heat was known to flow among solids, liquids and gases only when they had surfaces touching each other. In order to extend classical thermodynamics to include black-body phenomena, the idea of thermal coupling had to be extended as well: two black bodies thermally couple when and only when the ambient electromagnetic field each is immersed in includes direct contributions from the electromagnetic radiation emitted by the other. They do not need to have surfaces touching each other.

What is needed, then, is a way to characterize "thermal coupling" between black holes and ordinary thermodynamical systems: granted that "heat" in the gravitational context is gravitational energy of a particular form, such as that carried in the form of gravitational radiation or that responsible for red-shift effects in monopole solutions, then it follows that classical black holes *are not perfect absorbers*. When there is an ambient electromagnetic field, the black hole will radiate gravitationally as it absorbs energy and grows from the infalling electromagnetic radiation. To conclude that surface gravity is a physical temperature, therefore, one need show only that the gravitational energy exchanged between a black hole and other thermodynamical systems in transfer processes depends in the appropriate way on the surface gravity of the event horizon.

In order to characterize the correct notion of thermal coupling among systems including black holes, we first need to characterize an appropriate notion of "heat" for black holes, and the concomitant notion of free energy. That will put us in a position to formulate the appropriate generalizations of the Clausius and Kelvin Postulates for such systems, and to construct the appropriate generalization of Carnot cycles for them.

5.1 Irreducible Mass, Free Energy and "Heat" of Black Holes

In analyzing the ideas of reversibility and irreversibility for processes involving black holes, Christodoulou (1970) introduced the *irreducible mass* M_{irr} of a black hole of mass M and angular momentum J:

$$M_{\text{irr}}^2 := \frac{1}{2}[M^2 + (M^4 - J^2)^{\frac{1}{2}}]$$

Inverting the definition yields

$$M^2 = M_{\text{irr}}^2 + \frac{1}{4}\frac{J^2}{M_{\text{irr}}^2}$$

and so, for a Kerr black hole,

$$M > M_{\text{irr}}$$

(Clearly, $M_{\text{irr}} = M$ for a Schwarzschild black hole.) Thus, the initial total mass of a black hole cannot be reduced below the initial value of M_{irr} by any physical process. A simple calculation for a Kerr black hole, moreover, shows that,

$$A = 16\pi M_{\text{irr}}^2 \tag{5.1.1}$$

Thus, it follows from the Second Law that M_{irr} itself cannot be reduced by any physical process, and so any process in which the irreducible mass increases is a physically irreversible process. In principle, therefore, the free energy of a black hole is just $M - M_{\text{irr}}$, in so far as its total mass M represents the sum total of all forms of its energies, and M_{irr} represents the minimum total energy the black hole can be reduced to.

In classical thermodynamics, it makes no sense to inquire after the absolute value of the quantity of heat a given system possesses. In general, that is not a well defined property of a system. One rather can ask only about the amount of heat tranferred between systems during a given process.[10] Consider, then, a classical thermodynamical system with total energy E and free energy E_{f}. $E - E_{\text{f}}$ is the amount of energy unavailable for extraction, what Kelvin called its dissipated energy, E_{d}. Say that, through some quasi-stationary process (we know not what), both E and E_{d} change so that it now has less free energy than it did before; therefore, the entropy of the system must have increased, which can happen only when it absorbs heat, which will in general be the difference between the total change in energy and the change in free energy. If they both change so that it has more free

[10] See Maxwell (1888, chs. I, III, IV, VIII, XII).

energy, the same reasoning applies, and it must have given up a quantity of heat equal to that difference.

These remarks suggest defining the "quantity of heat transferred" to or from a black hole during any quasi-stationary process to be the change in its free energy, which is to say the change in total black hole mass minus the change in its irreducible mass, $\Delta M - \Delta M_{\mathrm{irr}}$.[11] If, for instance, the irreducible mass of a black hole does not change, while the total mass decreases, then it would have given up a quantity of heat. As a consistency check, it is easy to see that, according to this definition, when an ordinary thermodynamical system in equilibrium is dumped into a Kerr black hole, the black hole absorbs the quantity of heat the ordinary matter contained as characterized by the Gibbs relation, *viz.*, its temperature times its entropy, as only that energy contributes to its total mass without directly changing its angular momentum. Based on this characterization of "quantity of heat transferred", I claim that the appropriate notion of thermal coupling for systems involving black holes is any interaction where there is a change in the black hole's free energy. For purely gravitational interactions, this includes emission and absorption of that part of the energy of gravitational radiation not due to angular momentum, energy exchange due to simple monopole- or multipole-moment couplings in the near-stationary case, and so on.

5.2 Carnot-Geroch Cycles for Schwarzschild Black Holes

The construction of the appropriate analogue for a Carnot cycle including black holes will kill three birds with one stone: not only will it show that κ can be characterized as the absolute temperature of the black hole using the same arguments as classical thermodynamics uses to introduce the absolute temperature scale; it will do so by showing that in the coupling of black holes with ordinary thermodynamical systems, κ does in fact play the physical role of temperature and area that of entropy; and it will have as a natural corollary the existence of a universal constant that renders the proper physical dimensions to surface gravity as a measure of temperature and area as a measure of entropy. I call the constructed process a "Carnot-Geroch cycle" both to mark its difference from standard Carnot cycles, and because it relies essentially on the mechanism at the heart of the most infamous example in this entire field of study, Geroch's putative counter-example against Bekenstein's original claim that one should think of the area of a black hole as its physical entropy.[12] I will first sketch the steps of the proposed cycle informally, then work through the calculations.

Reversible Carnot-Geroch Cycle Using a Schwarzschild Black Hole as a Heat Sink

1. start with a small, empty, essentially massless, perfectly insulating box "at infinity", one side of which is the outer face of a piston; the box is "small" in that it will experience

[11] Carathéodory (1909), in his ground-breaking axiomatization of classical thermodynamics, introduced the notion of heat in a way similar to this, not as a primitive quantity as is usually done, but as the difference between the internal and the free energies of a system.

[12] According to Jakob Bekenstein (private correspondence), Geroch first proposed the example during a colloquium Geroch gave at Princeton in 1970. (Bekenstein tells me that he considers it the first attempt to attribute a temperature to a black hole.) I argue Geroch's counter-example does not work in §6.

negligible tidal forces as it is lowered toward the black hole; draw the piston back through the inside of the box "quasi-statically", so that the process is well approximated as an isentropic process, so filling the box with fluid from a large heat bath consisting of a large quantity of the fluid at fixed temperature T_0; when the piston has withdrawn part but not all of the way, seal the box, leaving the space opened by the piston filled with a mass of the fluid M_0 in thermal equilibrium at temperature T_0, and with entropy S_0; assume the entire energy of the box plus fluid is negligible compared to the mass of the black hole

2. lower the box towards the black hole quasi-statically, using an essentially massless rope; during this process, an observer inside the box would see nothing relevant change; in particular, as measured by an observer co-moving with the box, the temperature, volume and entropy of the fluid remain constant
3. at a predetermined fixed proper radial distance from the black hole, stop lowering the box and hold it stationary
4. very slowly, draw the piston back even further, so lowering the temperature of the fluid to a fixed, pre-determined value T_1 while keeping its entropy the same; the value of the temperature is to be fixed by the requirement that the change in total entropy of fluid plus black hole vanishes during the next step
5. open the box and eject the fluid using the piston, so the fluid falls into the black hole delivering positive mass-energy and positive entropy, and the piston returns to its initial state; *ex hypothesi*, by the previous step, this is an isentropic process
6. pull the box back to infinity (which takes no work, as the box now has zero mass-energy, and so zero weight), returning it to its initial state

Now, let us make the following assumptions: first, that it makes sense to attribute a physical temperature T_{BH} and entropy S_{BH} to a black hole (though we may not know what they in fact are); second, that the entropy of ordinary thermodynamical systems and the entropy of the black hole are jointly additive; and third, that the appropriate temperature at which to eject the fluid into the black hole for the entire cycle to be isentropic (T_1 in step 5) is that one would expect for a thermally equilibrated body in thermal contact with another at temperature T_{BH} sitting the given distance away in a nearly-static gravitational field. It will then follow that the physical temperature must be $8\pi\alpha\kappa$ and the physical entropy $\frac{A}{\alpha}$, where κ is the black hole's surface gravity, A its area, and α is a universal constant, the analogue of Boltzmann's constant for black holes (to be derived below).

Let the stationary Killing field in the spacetime be ξ^a. Let $\chi = (\xi^n \xi_n)^{\frac{1}{2}}$, and $a^a = (\xi^n \nabla_n \xi^a)/\chi^2$ be the acceleration of an orbit of ξ^a. Then a standard calculation shows that

$$\kappa = \lim(\chi a)$$

where the limit is taken as one approaches the event horizon in the radial direction, *i.e.*, near the black hole χa is essentially the force that needs to be exerted "at infinity" to hold an object so that it follows an orbit of χ^a, which is to say, to hold it so that it is locally stationary. Thus χ is essentially the "redshift factor" in a black-hole spacetime.

Let the total energy content of the box when it is initially filled at infinity be E_0 (as measured with respect to the static Killing field). In particular, E_0 includes contributions from the rest mass of the fluid M_0, and from its temperature T_0 and entropy S_{b}; let W_0 be the work done by the fluid as it pushes against the piston in filling the box. By the Gibbs relation and by the First Law of thermodynamics, therefore, the initial quantity of heat Q_{b} in the box is:

$$Q_{\mathrm{b}} = T_0 S_{\mathrm{b}} = E_0 + W_0$$

As the box is quasi-statically lowered to a proper distance ℓ from the event horizon, its energy as measured at infinity becomes χE_0, where χ is the value of the redshift factor at ℓ. Thus, the amount of work done at infinity in lowering the box is

$$W_\ell = (1 - \chi) E_0$$

(Recall that we assumed the box to be so small that χ does not differ appreciably from top to bottom.) This is not standard thermodynamical work, as the volume of the fluid, as measured by a co-moving observer, has not changed. It is rather work done by "the gravity of the black hole". Because we assumed that the mass-energy of the box plus fluid is negligible compared to that of the black hole, we ignore the attendant change in gravitational energy associated with the black hole, just as in the construction of an ordinary Carnot Cycle the heat given up or absorbed by a heat bath is assumed not to change its total heat content (and so not to change its temperature).

Now, when the box is held at the proper distance ℓ from the black hole and the piston slowly pushes or pulls so as to change the temperature of the fluid from T_0 to T_1 (as measured locally), the piston does work (as measured at infinity)

$$W_1 = \chi(E_0 - E_1)$$

where E_1 is the locally measured total energy of the fluid after the fluid's (locally measured) volume has been changed by the piston. When the fluid has reached the desired temperature T_1, the box is opened and the piston pushes the fluid quasi-statically out of the box, so it will fall into the black hole; in the process, the piston does work W_2 (as measured at infinity). Now, by the First Law, the total amount of energy the fluid has as it leaves the box is

$$E_1 - \frac{W_2}{\chi} = T_1 S_{\mathrm{b}} \tag{5.2.2}$$

as measured locally.

In order to compute the total amount of energy and the total amount of heat dumped into the black hole as measured at infinity, we must compute the temperature of the box as measured from there. It is a standard result (Tolman 1934, p. 318) that the condition for a body at locally measured temperature T to be in thermal equilibrium in a strong, nearly static gravitational field is that the temperature measured "at infinity" be χT. Thus the temperature of the box as measured from infinity will be χT_1. It thus follows from equation (5.2.2) that the total amount of heat dumped into the black hole is

$$\chi T_1 S_{\mathrm{b}} = \chi E_1 - W_2$$

But $\chi E_1 = \chi E_0 - W_1$ and $\chi E_0 = E_0 - W_\ell$, so

$$\chi T_1 S_\mathrm{b} = E_0 - W_\ell - W_1 - W_2$$

The expression on the righthand side of the last equation, however, is just the total amount of energy in the box as measured at infinity, and so $\chi T_1 S_\mathrm{b}$ is the total amount of energy the black hole absorbs, as measured from infinity, which is entirely in the form of heat.

Now, because we have assumed that the entropy for the fluid and for the black hole is additive, the total change in entropy is

$$\Delta S = -S_\mathrm{b} + \frac{\chi T_1 S_\mathrm{b}}{T_{\mathrm{BH}}}$$

For the process to be isentropic,

$$\Delta S = 0$$

and so

$$\frac{\chi T_1 S_\mathrm{b}}{T_{\mathrm{BH}}} = S_\mathrm{b} \tag{5.2.3}$$

It follows immediately that $T_1 = \frac{T_{\mathrm{BH}}}{\chi}$, precisely the temperature one would expect for a thermally equilibrated body in thermal contact with another body at temperature T_{BH} a redshift distance χ away. Write Q_{BH} for the amount of heat the black hole absorbs ($= \chi T_1 S_\mathrm{b}$), so equation (5.2.3) becomes

$$\frac{Q_{\mathrm{BH}}}{T_{\mathrm{BH}}} = S_\mathrm{b}$$

Now, in the limit as the box, and so the heat and entropy it contains, becomes very small (while the temperature remains constant), we may think of this as an equation of differentials,

$$\frac{\mathrm{d}Q_{\mathrm{BH}}}{T_{\mathrm{BH}}} = \mathrm{d}S_\mathrm{b} \tag{5.2.4}$$

This expresses the well known fact that temperature plays the role of an integrating factor for heat. Since $\mathrm{d}Q_{\mathrm{BH}}$ is the change in mass of the black hole, $\mathrm{d}M_{\mathrm{BH}}$, due to its being the entirety of the energy absorbed, there follows from the First Law of black-hole mechanics[13]

$$\frac{8\pi \mathrm{d}Q_{\mathrm{BH}}}{\kappa} = \mathrm{d}A \tag{5.2.5}$$

Thus, κ is also an integrating factor for heat. It is a well known theorem that if two quantities are both integrating factors of the same third quantity, the ratio of the two must be a function of the quantity in the total differential, and so in this case

$$\frac{T_{\mathrm{BH}}}{\kappa} = \psi(A) \tag{5.2.6}$$

[13]At least two conceptually distinct formulations of the First Law of black-hole mechanics appear in the literature, what (following Wald 1994, ch. 6, §2) I will call the physical-process version and the equilibrium version. The former fixes the relations among the changes in an initially stationary black hole's mass, surface gravity, area, angular velocity, angular momentum, electric potential and electric charge when the black hole is perturbed by throwing in an "infinitesimally small" bit of matter, after the black hole settles back down to stationarity. The latter considers the relation among all those quantities for two black holes in "infinitesimally close" stationary states, or, more precisely, for two "infinitesimally close" black-hole spacetimes. Clearly, I am relying on the physical-process version, for the most thorough and physically sound discussion and proof of which see Wald and Gao (2001).

for some ψ. (It is also the case that $\frac{T_{\text{BH}}}{\kappa} = \phi(S_{\text{b}})$ for some ϕ, but we will not need to use that.) It follows from equations (5.2.4) and (5.2.5) that

$$\frac{1}{8\pi}\psi(A)\text{d}A = \text{d}S_{\text{b}} \tag{5.2.7}$$

and so integrating this equation yields the change in the black hole's area, ΔA as a function of S_{b}, say $\Delta A = \theta(S_{\text{b}})$. (From hereon, we fix some arbitrary standard value for A, and so drop the 'Δ', as is done in analogous calculations in ordinary thermodynamics.)

In order to complete the argument, and make explicit the relation between A and S_{b}, and at the same time fix the relation between κ and T_{BH}, consider two black holes very far apart, and otherwise isolated, so there is essentially no interaction between them. Perform the Geroch-Carnot cycle on each separately. Let A_1 and A_2 be their respective areas, θ_1 and θ_2 the respective functions for those areas expressed using $S_{\text{b}1}$ and $S_{\text{b}2}$, the respective entropies dumped into the black holes by the cycles, and let $\theta_{12}(S_{\text{b}12})$ be the function for the total area of the black holes considered as a single system, expressed using the total entropy $S_{\text{b}12}$ dumped into the system. Both the total area of the black holes and the total entropy dumped in are additive (since the black holes, and so the elements of the Carnot-Geroch cycles, have negligible interaction), *i.e.*,

$$\theta_1(S_{\text{b}1}) + \theta_2(S_{\text{b}2}) = \theta_{12}(S_{\text{b}12}) = \theta_{12}(S_{\text{b}1} + S_{\text{b}2}) \tag{5.2.8}$$

Differentiate each side, first with respect to $S_{\text{b}1}$ and then with respect to $S_{\text{b}2}$; because θ_{12} is symmetric in $S_{\text{b}1}$ and $S_{\text{b}2}$,

$$\frac{\text{d}\theta_1}{\text{d}S_{\text{b}1}} = \frac{\text{d}\theta_2}{\text{d}S_{\text{b}2}}$$

Since the parameters of the two black holes and the two cycles are arbitrary, it follows that there is a universal constant α such that

$$\frac{\text{d}\theta}{\text{d}S_b} = \frac{\text{d}A}{\text{d}S_b} = \alpha$$

for all Schwarzschild black holes. It now follows directly from equations (5.2.6) and (5.2.7) that

$$T_{\text{BH}} = 8\pi\alpha\kappa$$

and from equation (5.2.3) that

$$S_{\text{BH}} = \frac{A}{\alpha}$$

up to an additive constant we may as well set equal to zero.[14] α is guaranteed by construction to have the proper dimensions to give T_{BH} the physical dimension of temperature (mass, in geometrized units), and S_{BH} the physical dimension of entropy (dimensionless, in geometrized units).

As a consistency check, it is easy to compute that the total work performed in the process,

$$W_T = W_0 + W_\ell + W_1 + W_2$$

[14] In contradistinction to classical thermodynamical systems, geometrized units for the entropy of black holes can be naturally constructed: let a natural unit for mass be, say, that of a proton; then one unit of entropy is that of a Schwarzschild black hole of unit mass. Why does classical black-hole thermodynamics allow for the construction of a natural unit for entropy when purely classical, non-gravitational thermodynamics does not?

equals the total change in heat of the box during the process, $Q_\mathrm{b} - \chi T_1 S_\mathrm{b}$, exactly as one should expect for a Carnot-like cycle. One can use the total work, then, to define the efficiency of the process in the standard way,

$$\eta := \frac{W_T}{Q_\mathrm{b}} = 1 - \frac{\chi T_1 S_\mathrm{b}}{Q_\mathrm{b}}$$

from which it follows that

$$\eta = 1 - \frac{8\pi\alpha\kappa}{T_0}$$

Thus, one can use the standard procedure for defining an absolute temperature scale based on the efficiency of Carnot cycles, and one concludes that the absolute temperature of the black hole is indeed $8\pi\alpha\kappa$.

Essentially the same construction and calculations can be carried out for Kerr black holes, with only a few slight modifications needed (*e.g.*, to hold the box stationary outside the event horizon now means to hold it co-rotating with the event horizon, so the angular work-term for that must be accounted for in the calculation of total work done). The same results hold; in particular, one derives the same constant α, in this case by considering three Carnot-Geroch Cycles performed separately on one Schwarzschild and two Kerr black holes, each at a great distance from wach other, and then performing the essentialy same calculation that starts at equation (5.2.8) above.

5.3 The Clausius and Kelvin Postulates for Black Holes

The standard arguments in favor of the Clausius and Kelvin postulates (as given, *e.g.*, in Fermi 1956, ch. 3), which rely on the impossibility of constructing a *perpetuum mobile* of the second kind, do not translate straightforwardly into the context of general relativity, where there is no general principle of the conservation of energy. Remarkably enough, however, as with the Second Law, both Postulates follow directly from theorems of differential geometry.

Postulate 5.3.1 (Clausius Postulate for Black Holes) *A transformation whose only final result is that a quantity of gravitational heat is transferred from a black hole at a given surface gravity to a system at a higher temperature is impossible.*

Assume that such a transformation as described in the antecedent of the postulate were possible. Then the change in irreducible mass of the black hole would have to be strictly greater than the change in its total mass during the interaction, with no other change in the spacetime than that another system absorbed heat. In particular, its irreducible mass must increase. However, it follows from equation (5.1.1) and the Second Law that an increase in irreducible mass must yield an increase in the black hole's area, and so its entropy, violating the assumption that nothing else thermodynamically relevant in the spacetime changed.

Postulate 5.3.2 (Kelvin Postulate for Black Holes) *A transformation whose only final result is that a quantity of gravitational heat is extracted from a stationary black hole and transformed entirely into work is impossible.*

The argument is essentially the same as for the Clausius Postulate for black holes. Again, for such a process to occur, the irreducible mass of the black hole would have to increase, but that would necessitate a change in the area of the black hole, violating the conditions of the postulate.

6 Discussion

It seems as though we can attribute heat to a Schwarzschild black hole only when it is being perturbed. In fact, however, one can extract both "heat" and work from a Schwarzschild black hole, by perturbing it; indeed, this is in perfect analogy with ordinary thermodynamical systems that have reached heat death, from which heat and work can be extracted only if one perturbs them properly.[15] In fact, the analogy is even better than that brief remark suggests: stationary classical black holes do not "radiate heat", but neither do ordinary classical thermodynamical systems in equilibrium; they exchange heat only when they are in direct contact (contiguous) with another system at a different temperature, but the same holds for stationary classical black holes, in so far as their immediately contiguous environment is "at the same temperature", *viz.*, has essentially the same effective surface gravity as measured at infinity as the black hole does.

An obvious complaint against the argument based on the construction of the Carnot-Geroch Cycle is that it is circular: why assume a classical black hole has an entropy in the first place? The best answer to this is implicit in the series of thought-experiments Wheeler initially posed in the late 1960s that inspired the entire field of black-hole thermodynamics in the first place: if we don't assume black holes have entropy, then we would, with effortless virtuosity, be able to achieve arbitrarily large violations of the Second Law of thermodynamics. The world external to a black hole is isolated from the interior of the black hole. So, take your favorite highly entropic system and throw it into a black hole: the entropy of that system vanishes from the external world, so lowering the total entropy of an isolated system. The only escape from this possibility is to assign the black hole itself an entropy in such a way that, when an ordinary entropic system passes into a black hole, then the black hole's entropy increases at least as much as the entropy of the system entering it. This postulate is generally referred to as the Generalized Second Law: the total entropy of the world, *viz.*, the entropy of everything outside black holes plus the entropy of black holes, never decreases (Bekenstein 1973; Bekenstein 1974).

Though I use the mechanism of Geroch's infamous thought-experiment to construct the Carnot-Geroch Cycle, nothing seems to preclude Geroch's original use of it to argue that, were classical black holes to have physical temperature, it would have to be absolute zero independently of what value its surface gravity had, and so naturally lead to violations of the Second Law of thermodynamics. If one arranges matters just so, the weight lifted by the lowering of the box will have extracted *all* the energy content of the box exactly when it reaches the event horizon; one can then dump into the black hole the stuff in the box, which still has its original entropy but zero mass-energy; thus, one will have converted thermal energy into work with 100% efficiency, implying the black hole must have

[15] This line of thought is strongly buttressed by Kiefer (2004), who shows that the spectrum of the quasi-normal modes of a perturbed Schwarzschild black hole yields its Bekenstein-Hawking temperature.

temperature absolute zero. Because the matter dumped into the black hole has no mass-energy, the area of the black hole does not increase; because the matter still has its original entropy, however, the total entropy of the world outside the event horizon has decreased, thus violating the Generalized Second Law. I think the proper answer to this problem is to note that one must admit the possiblity of arbitrarily precise measurements in order to make the mass-energy of the box plus fluid exactly zero exactly at the event horizon. If one allows the possibility of such measurements, however, then it is not a justified idealization to ignore the stress-energy contained in the rope holding the box above the black hole. One may be justified in treating the rope as having *initially* zero stress-energy, as an idealization, but once the box approaches the black hole, the internal tension in the rope will become a non-trivial momentum flux (as different parts of the rope, at different distances from the horizon, pull on each other with different force), and so one has to take account of that stress-energy.[16] One will, therefore, never be able to get the internal energy of the entire system, box plus rope, exactly to zero before one dumps the entropic stuff into the black hole.

Another possible problem with the arguments is that they still seem to allow other possible mechanisms for producing arbitrarily large violations of the Generalized Second Law. Put a Schwarzschild black hole in a reflecting box and pervade the box with thermal electromagnetic radiation at a lower (Planck) temperature than the classical Bekenstein-Hawking temperature of the black hole. The black hole will eventually absorb the thermal radiation: heat, it seems, will spontaneously flow from a system at a lower temperature to one at a higher temperature, a seeming violation of the Generalized Second Law. First, one should note that this is *not* a violation of the Generalized Clausius Postulate, as the irreducible mass, and so the area of the black hole, increases after absorption. If one takes the Generalized Clausius Postulate as the appropriate formulation of the Generalized Second Law in the context of classical black-hole mechanics and thermodynamics, as the ordinary Clausius Postulate is in classical thermodynamics alone, then there is no violation of the Generalized Second Law (Maxwell 1888; Fermi 1956). The most decisive reason to think this is not truly a counter-example, however, comes from the fact that the appropriate notion of "thermal coupling" for interactions involving black holes is the transfer of *gravitational* heat, not just the transfer of ordinary thermodynamical or electromagnetic heat. Consider an analogous case, with electromagnetic heat and ordinary heat: that a colder liquid flows into an area of hotter electromagnetic radiation does not imply that heat has flowed from a colder to a hotter system. What is relevant is the exchange of heat appropiately understood between the two systems, and in particular the way that the energy of the electromagnetic radiation is transformed into the liquid's ordinary heat. In the same way, in the case at issue for us, the mere fact that thermalized radiation falls into the black hole does not imply that it is carrying an excess of gravitational heat into the black hole.

Still, one might respond to this line of argument, that blackbody radiation in a reflective box at a lower temperature than a Schwarzschild black hole also in the box will necessarily lead to a lowering of total entropy afer the black hole absorbs the radiation (when quantum effects are ignored), and that is surely a violation of the Generalized Second Law if anything is. A straightforward plausibility

[16] See Price, Redmount, Suen, Thorne, MacDonald, and Crowley (1986) for detailed calculations taking account of the rope's stress-energy during such a lowering process.

argument, however, shows that that argument is a little more delicate than its simple statement might indicate.

The entropy of thermalized electromagnetic radiation is $\frac{4}{3}$ times the radiation's total internal energy U divided by its temperature. Its total internal energy, however, is proportional to T^4 (its temperature raised to the fourth power, the coefficient of proportionality given by the volume occupied by the radiation, and a fixed constant depending on $\hbar$, k and c). Therefore, its entropy is proportional to its T^3. Its entropy, therefore, is always strictly numerically lower, in geometrized units, than U, no matter what it's temperature is, unless its temperature is less than 1K. Since the entropy of a Schwarzschild black hole (area) is directly proportional to its mass-energy, it follows that, no matter what the temperature difference between the black hole and the radiation is, the total entropy of the system will necessarily increase after the black hole absorbs the radiation, unless the temperature of the radiation is less than 1K, in which case the result is unclear. Thus, that is the only case of potential interest.

The entropies of the black hole itself and of the electromagnetic radiation, however, are not the only relevant quantities to consider. As the electromagnetic radiation falls into the black hole and perturbs it, it will emit gravitational radiation. The Area Law guarantees that this emitted radiation will not cause the entropy of the black hole ever to decrease. The question now, therefore, is whether or not entropy of the emitted gravitational radiation will counter-balance whatever potential entropy difference there might be between the sub-1K electromagnetic radiation that might otherwise lead to a violation of the Generalized Second Law. Based on a simple heuristic argument, there seems to be some reason to believe that it will, thus avoiding a violation of the Second Law. Since the gravitational radiation will, at most, have a thermalized temperature equal to that of the electromagnetic radiation, and since it seems plausible that the entropy of gravitational radiation at a given temperature will always be much greater than the energy of electromagnetic radiation at the same temperature (think how much more difficult it is to extract usable energy from gravitational as opposed to electromagnetic radiation), it seems plausible that the entropy of the emitted gravitational radiation will always more than counterbalance whatever entropy difference there may be between the electromagnetic radiation and the black hole itself. Since, again plausibly, below 1K the entropy of the gravitational radiation will increase much more quickly with decrease of temperature than that of the electromagnetic radiation, this argument seems even a little more secure for the case of interest to us. Of course, this argument requires a more rigorous calculation to be made secure, but it seems plausible enough to put the onus of the debate back on the opponent, at least until such time as a more rigorous calculation shows it to be wrong.

My arguments open the possibility for real insight into existing questions about black-hole mechanics and thermodynamics. Black holes have enormous entropy, far more than any reasonably conceivable material system that could result in one on collapse (Penrose 1979). There must, therefore, be a correspondingly enormous and discontinuous jump in entropy when a collapsing body passes its Schwarzschild radius. How can one explain that? More modern characterizations of entropy, whether of a Boltzmannian or Gibbsian form founded on statistical considerations, have no explanation for this jump. If, however, one conceives of entropy in a purely classical way, *à la* Clausius and Maxwell, as a measure of how much work it takes to extract energy from a system, how much free energy a system has, what forms its internal energy (as opposed to free energy) are in, then black holes have enormous energy, only a very small amount of which is extractible, and there is a clear physical discontinuity in extractability of energy when an event horizon forms.

If my arguments were correct, they would lend support to the idea that general relativity is only an effective field theory, and the Einstein field equation only an equation of state, *à la* Jacobson's (1995) derivation of the Einstein field equation from a statistical analysis of an underlying ensemble of "quantum spacetime atoms". If that is true, then the entire program of "quantizing gravity" may be misguided from the start, at least in many of the ways it's currently being pursued, just as it would be equally mistaken to try to quantize the classical thermodynamics of an ordinary gas rather than the elements ("classical atoms") of its statistically modeled molecular dynamics.[17] To push that analogy a little farther, if black holes were to have a viable and interesting thermodynamics in the absence of quantum effects, then that may suggest that we should be looking for a reduction of classical curvature and metrical structure in general relativity to an underlying "ensemble of classical spacetime atoms", with the classical relativistic behavior recovered through an appropriate classical statistical treatment of the dynamics of such an ensemble. The elements of such an ensemble of classical spacetime atoms would then be the appropriate target for attempts to quantize gravity.

Yet another possibility, independent of but perhaps complementary to that just mentioned, is that one may take my arguments as showing that the signature of quantum gravity, in particular the traces of whatever statistical quantities it may give us for making traditional sense of the thermodynamical phenomena I discuss here, show up already in purely classical, non-statistical theory, in the same way Einstein (1926) conceived of and treated classical thermodynamical phenomena in his epochal work on Brownian motion in 1905. Finally, and I think most importantly, my arguments lend *prima facie* support to projects (especially in cosmology) that want to attribute entropy generically to classical "gravitational degrees of freedom", as in the work of Clifton, Ellis, and Tavakol (2013), and as required by Penrose's Conformal Curvature Hypothesis (Penrose 1979), and several programs in quantum gravity (Amelino-Camelia 2013; Ashtekar, Reuter, and Rovelli 2015; Elvang and Horowitz 2015; Rovelli 2008).

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[17] Of course, I do not mean to suggest that all current programs of quantum gravity are misguided or doomed to failure. For what it's worth, I personally think Loop Quantum Gravity is, in fact, amazingly interesting and successful, probably the single most successful program around today. Even it, however, still has serious conceptual and technical problems, enough that it is still worth looking for and working on alternative possibilities in parallel to work on it, and other programs in quantum gravity.

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