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ON RELATIVITY THEORY AND OPENNESS OF THE FUTURE*

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It has been repeatedly argued, most recently by Nicholas Maxwell, that the special theory of relativity is incompatible with the view that the future is in some degree undetermined; and Maxwell contends that this is a reason to reject that theory. In the present paper, an analysis is offered of the notion of indeterminateness (or “becoming”) that is uniquely appropriate to the special theory of relativity, in the light of a set of natural conditions upon such a notion; and reasons are given for regarding this conception as (not just formally consistent with relativity theory, but also) philosophically reasonable. The bearings upon Maxwell’s program for quantum theory are briefly considered.

Nicholas Maxwell (1985) has contended that the special theory of relativity is incompatible with the hypothesis that the future is to some degree open (i.e., not fully determined)—a hypothesis he calls “probabilism”. In defending that claim against a criticism by D. Dieks (1988), Maxwell (1988) refers to a paper by C. W. Rietdijk (1966), called to his attention by Michael Redhead; and credits Rietdijk with “first discovering that SR and probabilism are incompatible” (p. 641).

There is in this a certain inaccuracy, for an argument in all essentials the same as that of Rietdijk was given independently, and almost simultaneously, by Hilary Putnam (1967; but the piece bears the footnote, “The paper was read to a meeting of the American Physical Society in New York, January 27, 1966”). Shortly after the appearance of those papers, I published an article—apparently unknown to the participants in the current discussion—with the following thesis: “[T]he two articles cited [i.e., those of Rietdijk and Putnam] draw their conclusions by means of the same basic argument: a very simple, and in a certain geometrical sense a correct, one; but an argument in both cases seriously misapplied (and in essentially the same way), so that the asserted conclusions do not follow” (1968, 5). That statement, I believe, holds as well for the arguments of Maxwell (which, however different in detail, rest upon the same geometrical facts). Since Dieks has not quite put his finger on what in my

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opinion is the real point, and since Maxwell, reviewing possible objections (in his original paper and in his response to Dieks), has failed to consider that point, a brief restatement seems in order here—with some elaboration of matters that were not treated explicitly in my earlier paper.

I

The issue is whether a notion of “real becoming” can be coherently formulated in terms of the structure of Einstein-Minkowski space-time. Now, such a notion requires that one distinguish “stages” of becoming, in such a way that, at each such stage, the entire history of the world is separated into a part that “has already become”—is “ontologically fixed and definite”, as Maxwell puts it (1985, 24)—and a part that “is not yet settled”. Two questions, therefore, must be answered: (1) What is the (spatio-temporal) nature of the “stages”? (2) By what (spatio-temporal) criterion is one to distinguish, at any given stage, the “definite” from the “not yet settled”?

In order to make possible a decisive attack upon this issue, it is necessary to agree on some general principles that the answers to these questions should be required to satisfy. I believe the following are uncontroversial:

- (i) The fundamental entity, relative to which the distinction of the “already definite” from the “still unsettled” is to be made, is the *here and now*; that is, the space-time point (henceforth simply “point”).
- (ii) If the state at point *b* is “already definite” as of point *a*, then whatever is definite as of *b* is (*a fortiori*) definite as of *a*. (Thus, the notion “is already definite as of” is to be a *transitive relation between points*.)
- (iii) The state at any point *a* is already definite as of *a* itself.
- (iv) For any point *a*, there are points whose state is still unsettled as of *a*.

What can we say of the existence of a relation *R* (with *Rab* to signify that the state at *b* is definite as of *a*), definable in terms of the Einstein-Minkowski geometry of space-time, satisfying conditions (ii), (iii), and (iv)?

Before attacking this question, a point must be clarified concerning the precise notion of Einstein-Minkowski geometry that is presupposed here. Since our issue is the coherence of a notion of *becoming*, we must, of course, postulate a distinguished *time-orientation*: of the two topologically connected components of the set of null and time-like nonzero vectors of space-time, one must be distinguished as the set of “future-pointing”,

and the other as the set of “past-pointing” vectors. Further—this is perhaps less inevitable from the point of view of physics in general and our inquiry in particular—I wish to assume, provisionally, that the space-time geometry does not involve a distinguished spatio-temporal *unit*. From the point of view of the usual algebraic formulation of the theory, this means that, strictly considered, we are given, not a quadratic (or bilinear) form on space-time, but a *class* of such forms (generated from any one member of the class by all possible multiplications by positive real numbers). In more geometrical terms, the structure assigns real-number values, not to (time-like or space-like) distances between points, but only to *ratios* of such distances.

The relation R that we seek, if it is to be definable in terms of the geometric structure, must be invariant with respect to all automorphisms of that structure. Now, the interpretation of the Einstein-Minkowski structure as *spatio-temporal* depends critically upon the principle that the state at any point a is subject to *influence* by the states at all points in the “causal past” of a ; that is, all points b such that the vector from a to b is past-pointing. Therefore the states at all such points should be supposed definite as of a ; in other words, Rab must hold whenever the vector ab is past-pointing. Suppose that Rab holds also for some point b , different from a , not in the causal past of a . There are two cases that we must consider. First, suppose the vector ab is space-like. It is an elementary fact, given the stipulation we have made of the nonexistence of a spatio-temporal unit, that for any pair of space-like vectors there is an automorphism of space-time transforming the first to the second. It follows, by the invariance of R , that Rxy holds whenever xy is a space-like vector; and by the transitivity of R , that Rxz holds whenever there is a point y such that xy and yz are both space-like. But it is also an elementary fact that such a y exists for any given x and z . In this case, then, we are led to the conclusion that R holds universally—in contradiction to our condition (iv). Next, suppose that the vector ab under consideration is not space-like; hence, since it is by assumption nonzero and not past-pointing, that it is future-pointing. There then exists a past-pointing vector bc such that the sum of the two vectors—that is, ac —is nonzero and space-like. We are thus back in the previous case, and therefore have again contradicted condition (iv). It follows that there is only one possible candidate for the relation R we seek:

THEOREM. *If R is a reflexive, transitive relation on a Minkowski space (of any number of dimensions—of course at least two), invariant under automorphisms that preserve the time-orientation, and if Rab holds for some pair of points (a,b) such that ab is a past-pointing (time-like or null) nonzero vector, then for any pair of points (x,y) , Rxy holds if and only if xy is a past-pointing vector.*

I have suggested that the stipulation that there be no distinguished spatio-temporal unit is perhaps challengeable; but a slightly more elaborate argument shows that the theorem remains true even without that stipulation. Indeed, if Rab holds with ab space-like, and if the distance between a and b is s , then the condition of invariance implies that Rxy holds whenever xy is a space-like vector of spatial length s , and the condition of transitivity then implies that Rxy holds whenever xy is the sum of two such vectors. But in a Minkowski space of more than one space-like dimension (i.e., of more than two dimensions in all), every vector can be obtained as the sum of two space-like vectors of length s (for any given positive s); the argument can therefore proceed just as formerly. The case of just one spatial dimension is of no importance for us; but it may be noted that in such a space, the vectors obtainable as sums of two space-like vectors of length s are (1) the zero-vector, (2) every time-like vector, and (3) every space-like vector of length greater than or equal to $2s$. Since every vector is a sum of two time-like vectors, our conclusion once again follows.

II

We must of course consider whether the relation that has proved to be the only candidate to represent the notion “for a, b has already become” in the spatio-temporal terms of special relativity can be regarded as doing so satisfactorily. In the nature of the case, this is not a matter that can be settled by a mathematical theorem; for the conditions of “satisfactoriness” are not *prima facie* clear and definite. We may distinguish two broad classes of relevant considerations: those drawn from physical theory and evidence, and those drawn from (reflection upon) ordinary experience. I do not pretend here to give an exhaustive discussion of the matter, but I should like to offer some observations under both of those heads. I shall begin with some comments upon the view expressed by Maxwell, who dismisses this relation with an argument (1985, 27–28; see also Maxwell 1988, 642) that seems to me to exhibit an extraordinary misunderstanding of the crucial special-relativistic conceptions.

Contemplating two physical events, E_1 and E_2 , with space-like separation from each other, Maxwell asks, “granted . . . that E_1 is here and now” (1985, 25), what the ontological status of E_2 is (definite or indefinite). He takes up in turn four possible answers; that suggested by our theorem is the last of these:

In the fourth place, . . . it may be suggested that E_2 is ontologically indefinite absolutely, like events in the future light cone of E_1 . But this suggestion faces the fatal objection that it postulates not just future alternative *possibilities*, but present alternative *actualities*—a full-

fledged multi-universe view. If E_2 consists of many alternative possibilities from the standpoint of E_1 , then similarly E_1 itself consists of many alternative possibilities from the standpoint of E_2 , and thus from the standpoint of E_1 itself. This fourth suggestion thus commits us to the view that whenever anything probabilistic occurs, there being N equally probable outcomes, three-dimensional space splits up into N distinct three-dimensional spaces, each space containing one of the N outcomes. Any such branching-universe or multi-universe view is, however, far too grotesquely *ad hoc* to be taken seriously. Ontological probabilism combined with Newtonian space-time does not, it should be noted, face this objection since in this case alternative possibilities are all in the future; and they can thus be regarded as alternative *possibilities* only, and not alternative *actualities*. In the relativistic case, this option is not open to us; granted that E_1 and E_2 are outside each other's light cones, and each is ontologically indefinite from the other's standpoint. (Ibid., 27–28; Maxwell's emphasis)

I have quoted this passage *in extenso* because there are a number of things in it that seem to me quite baffling. Why, for instance—supposing it true (as, I shall argue, it is not) that special relativity implies such “branching”—is this, not just *grotesque* (which for argument's sake we may concede), but “grotesquely *ad hoc*”? One might suppose that something implied by a theory that has some claim to be well-grounded should escape *that* imputation. Again, why suppose a *finite* number of possible alternatives; and, indeed, of *equally probable* ones? These, however, are side issues, relevant only as casting a strange light upon the quality of the argument as a whole. To come to the first of a pair of points that are central, by what principle does one infer, from the indefiniteness with respect to one another of two events, that each must be indefinite from *its own* standpoint? Evidently Maxwell assumes that the relation “ x and y are indefinite for each other” is transitive; but why? I see no compelling grounds for such an assumption; and against it, I see what seems to me a most compelling reason: that it renders the notion of becoming incompatible with the special theory of relativity.

Maxwell offers no argument for the assumption in question; indeed, he does not even state it, but appears tacitly to assume it as the principle of the inference he has drawn. I am inclined to think that it is connected with the second, and in my view decisive, of the central points I have mentioned. This concerns Maxwell's phrase “present alternative *actualities*”; and—closely related, if not in effect the very same issue—his remark that “three-dimensional space splits up” into distinct spaces, corresponding to the distinct possibilities. For indeed, if we are to think of the “present” as constituting a “three-dimensional space”—a space-like

hyperplane of space-time—then of course comembership in such a space will be a transitive relation. (Note, however, that we might still wish to claim that distinct events in the same “present” are indefinite for one another, and yet that each is definite for itself; so transitivity of mutual indefiniteness need by no means follow.)

But the use of such language in discussing special-relativistic space-time is (to borrow Maxwell’s word) grotesque. Or rather: to insist (without supporting argument) upon a notion of “present [spatially distant] actualities”, in assessing special relativity, is simply to beg the question since it is fundamental to that theory that it rejects any such notion. Yet this (as I think) fallacy not only underlies the original arguments of Rietdijk and Putnam but has since occurred repeatedly in the literature. See, for example, Lango (1969), with my reply (1970a); Rietdijk (1976); and not only the papers of Maxwell here under discussion, but the criticism of Maxwell by Dieks as well. For Dieks, discussing the ontological relationship of a pair of events E_1 and E_2 with space-like separation, says that “the subset of space-time diagrams in which E_1 is ontologically definite contains diagrams in which E_2 is definite as well as diagrams in which E_2 is indefinite” (1988, 459); and these “alternative diagrams” (*ibid.*, 458) correspond to alternative ways of coordinating the assignment of “now-points” to the world-lines of different bodies—that is, in effect, to alternative specifications of the “distant present”. The fact that such a fallacy (again, if I am right) not only persists among some writers, but is allowed by referees to find continued publication, is a phenomenon that itself calls for reflection. If this is a misunderstanding, what can account for its persistence?

I have spoken, at the beginning of this section, of two broad classes of relevant considerations—from physics, and from ordinary experience. But it seems to me that the issue I have just raised demands simultaneous attention to both of these kinds. I think that a certain feature of the physical theory—in particular, of its geometric-kinematic part—tends to seduce philosophers into a misconception of its relation to experience; and that this misconception can only be cleared up by a fuller understanding of the bearings of the rest of physics—beyond geometry and kinematics—upon this relation. In the next three sections, I shall attempt to elucidate this point.

III

It is well known that the special theory of relativity involves a definite notion of “relative simultaneity”—relative, namely, to a state of motion (i.e., to a time-like direction): events are “simultaneous relative to” such a direction if and only if the vector they determine is orthogonal to that

direction. This characterization we owe to Minkowski; the conception itself was introduced by Einstein in his original account of the theory, in whose first (kinematical) part it plays a decisive role. There is a large—and, again, well known—literature devoted to the question whether Einstein's definition of simultaneity is in some sense canonical, or whether it is "conventional"—one among a set of alternative possible such notions. Confusion about this question may contribute to the general misunderstanding we are here concerned with, and it therefore seems worthwhile to offer a comment upon it before proceeding.

There are really two distinct aspects to the issue of the "conventionality" of Einstein's concept of relative simultaneity. One may assume the position of Einstein himself at the outset of his investigation—that is, of one confronted by a problem, trying to find a theory that will deal with it satisfactorily; or one may assume the position of (for instance) Minkowski—that is, of one confronted with a theory already developed, trying to find its most adequate and instructive formulation. The issue in its latter aspect has been dealt with—in my opinion, conclusively—by David Malament (1977), who pointed out that the Einstein-Minkowski conception of relative simultaneity is not only characterizable in a direct geometrical way within the framework of Minkowski's geometry (as, of course, Minkowski himself had shown), but is the only possible such conception that satisfies certain very weak "natural" constraints.¹

As to the procedure of Einstein, he of course had no "space-time geometry" *within* which to propose a concept of simultaneity; on the contrary, the task he had conceived was precisely that of *constructing* a suitable space-time geometry—or equivalently, in Einstein's own terms, of devising a suitable new kinematics. Now, Einstein's perception of a need for such a new kinematics derived partly from what he describes as "the unsuccessful attempts to detect any motion of the earth relative to the 'light medium'" (Einstein [1905] 1989, 276, my translation; see also a translation in Lorentz et al. 1923, 37), and partly from a more general conviction on his part that "to the concept of absolute rest there correspond, not just in mechanics but also in electrodynamics, no properties of the phenomena" (*ibid.*). It is to be noted that this conviction is described by Einstein, in that place, explicitly as a *Vermutung*—a conjecture. So his problem was to develop a theory that should embody this conjecture.

¹There is one slightly delicate point to be noted: Malament's discussion, which is concerned with certain views of Grünbaum, follows the latter in treating space-time without a distinguished time-orientation. To obtain Malament's conclusion for the (stronger) structure of space-time with a time-orientation, one has to strengthen somewhat the constraints he imposes on the relation of simultaneity; it suffices, for instance, to make that relation (as in the text above) relative to a *state of motion* (i.e., a time-like *direction*), rather than—as in Malament's paper—to an *inertial observer* (i.e., a time-like *line*).

But it is a feature of the electrodynamics of Maxwell that in it all electromagnetic disturbances, and light in particular, are propagated *in vacuo* with a velocity expressed by a universal constant. It is therefore clear that Einstein required a kinematics compatible with the proposition that this velocity is the same for all inertial observers (and, for each such observer, the same in all directions); and also clear, then, that in such a kinematics the light-signal criterion proposed by Einstein would, ipso facto, correctly characterize simultaneity. (We may set aside the question whether this characterization should be regarded as a “definition”.)

Putting the two aspects together, one sees the following: First, for Einstein, the question (much discussed since Reichenbach) whether the evidence really shows that the speed of light *must* be regarded as the same in all directions and for all inertial observers² is not altogether appropriate. A person devising a theory does not have the responsibility, at the outset, of showing that the theory being developed is the only possible one given the evidence. Second, once Einstein’s theory had been developed, and had proved successful in dealing with all relevant phenomena, the case was quite transformed; for we know that *within* this theory, there is only one “reasonable” concept of simultaneity (and in terms of that concept, the velocity of light is indeed as Einstein supposed); therefore an alternative will only present itself if someone succeeds in constructing, not simply a different empirical criterion of simultaneity, but an essentially different (and yet viable) theory of the electrodynamics of systems in motion. No serious alternative theory is in fact known.³

²Straightforward empirical evidence concerning the velocity of light bore only upon the elapsed time for a round trip, *A* to *B* and back—hence not upon the question of equality of the outward and the return velocities. Moreover, what the evidence tended to show was just that the elapsed times for round trips of equal distance in different (orthogonal) directions were the same—and thus that, for any (inertial) observer, the round-trip velocity of light is the same in all directions. There was no real evidence, apart from electrodynamic theory, that this round-trip velocity is the same in magnitude for all inertial observers. (Reichenbach’s critique is based upon the first point; it ignores the second.)

³It is sometimes said that if one rejects “verificationism”, viable alternatives to special relativity are available—see, for example, Sklar (1985): “I can see no way of rejecting the older aether-compensatory theories, originally invoked to explain the Michelson-Morley results, without invoking a verificationist critique of some kind or other” (p. 293). This is not the place to attempt a full discussion of that issue; but some comment seems called for. In the first place, it is not entirely clear what Sklar here means by “the aether-compensatory theories”. The historical reference is, of course, to Lorentz. But it was Lorentz’s hope to *find* such a “compensatory theory”: that is, one in which the change (*a*) in the lengths of quasi-rigid bodies (which—see below in the main text—in effect means a change in the equilibrium configurations of interacting particles) and (*b*) in the rate of clocks (which means a change in the rates of all dynamical processes) should be accounted for by the dynamical interactions of the parts of ordinary matter with the ether through which they move (see, for example, Lorentz 1915, 332–333). Such a theory was in fact never developed. If, on the other hand, all Sklar means by “aether-compensatory theories” is such theories as postulate a distinguished state of inertial motion (to be called “absolute rest”)—without any other modification of the special theory of relativity—then it ought

IV

The points discussed in the preceding section—on the one hand, the motivation for Einstein's empirical criterion of simultaneity; on the other, the mathematical facts about Einstein-Minkowski space-time—surely have at most a tenuous connection with anything like an "experience" or "intuition" of an instantaneous present. To quote myself once again:

[W]hat Einstein's arguments showed was that *a certain procedure of measurement singles out a time axis and gives numerical time differences dependent upon that distinguished axis*; not that an observer's state of motion imposes upon him a special view of the world's structure. This illegitimate metaphysical interpretation of the time coordinate appears perhaps most plainly in Rietdijk's phrase describing *C* and *A*, when at rest with respect to one another, as "experiencing the same 'present'"; there is of course no such "experience": the fact that there is no experience of the presentness of remote events was one of Einstein's basic starting points. (1968, p. 16, n. 15)

In order to clarify just how the geometry of space-time does come to be reflected in experience, it will be instructive to reflect upon the situation in classical physics. In its original conception—in Newton's theory of "absolute space" and "absolute time"—there seems *prima facie* to be the following association: (a) The geometry of space corresponds both to our subjective "sense" or "intuition" of space, and to what we learn by activities that can be described as "spatial measurements" (including under this term not only more or less precise *numerical* measurement, but the vaguer estimates of spatial magnitudes that are continually performed in everyday life—as in reaching for the salt, or writing a line of script); (b) the "geometry" of time corresponds both to our subjective sense of "duration" and the "lapse" of time, and to the results of our activities of

to be noted that there are parallels in an older context, which (I believe) no one would regard as posing serious alternative possibilities at all. For instance, one might suppose that the universe has an absolute center ("pseudo-Aristotelian hypothesis")—simply postulating the existence of a distinguished point (in Newtonian absolute space), with no other modification of Newtonian physics. Or one might assert the existence of a (global) distinction in the universe between the horizontal and the vertical ("pseudo-Epicurean hypothesis")—simply postulating the existence of a distinguished direction, again with no other modification of classical physics. It may well be that to systematize the perception that such "theories" are idle requires a verificationist criterion "of some kind or other" (a question that depends in part upon what one includes under the somewhat vague term "verificationism"); but if that is true, it must count in favor of the appropriate form of verificationism. For the perception that the "pseudo-Aristotelian" and "pseudo-Epicurean" hypotheses are idle is overwhelming, and surely independent of special philosophical theories. The case envisaged, of a "pseudo-ether hypothesis" with a distinguished "state of rest", is in precisely the same position.

temporal measurement. This view agrees, for instance, with Kant's doctrine about space and time: the first parts of (a) and of (b) respectively correspond to the characterization, in the Transcendental Aesthetic, of the forms of outer and of inner intuition; their second parts to the assertion, by the principle of the Axioms of Intuition, of the validity for experienced *objects* of the results obtained by pure mathematics in virtue of those forms.⁴

This very plausible interpretation is quite specious. That might already be surmised from the famous difficulties with the theory of absolute space, recognized by Newton (and of course urged most strongly by his "relationist" or "relativist" opponents); indeed, at the end of his celebrated scholium on this subject, Newton tells us that the problem of the empirical application of the concept of absolute motion is so intricate that it requires the entire central argument of his *Principia* to effect its solution. But the impossibility of this *prima facie* plausible view of the relation of spatio-temporal concepts to experience emerges most clearly when one considers that modification of the Newtonian absolute structure which we have now come to see as the most appropriate framework for Newton's physics (see Stein 1970b). In this modified theory of "Newtonian space-time", the underlying spatial geometry is continually evanescent: spatial distances are defined only for *simultaneous* space-time points, and it is strictly meaningless to assign such a distance between two points separated by a nonzero time-interval, no matter how small. Now, if one thinks in terms of the (naive) interpretation described above, two obvious problems present themselves: First, one has to face the fact that (setting aside here any questions about the notion of simultaneity—a concept that is in fact essential to Newtonian physics) there is certainly no such thing as "instantaneous consciousness" (of anything; therefore, in particular, of spatial relationships); it has long been recognized by psychology that awareness requires some nonzero temporal duration (the so-called "specious present"). So the replacement of Newton's eternally enduring spatial structure by a constantly fleeting one quite undermines the subjective component of the naive interpretation. Second, it is even more obvious that the actual determination of spatial relationships in experience—that is, in a broad sense, by measurement—takes time. How, then, can one measure that underlying fleeting structure?

⁴For the "objective" addition provided by the principle of the Axioms of Intuition to the "subjective" results of the Transcendental Aesthetic, see Smith (1950): "This transcendental principle of the mathematics of appearances [*viz.*, the principle of the Axioms of Intuition] greatly enlarges our *a priori* knowledge. For it alone can make pure mathematics, in its complete precision, applicable to objects of experience. . . . Whatever pure mathematics establishes in regard to the synthesis of the form of apprehension is also necessarily valid of the objects apprehended" (p. 200, B 206).

The answer is that one does not “measure” that structure—directly.⁵ And, just as (so Newton told us—and told us truly) some *physics* is required before we can understand how to ascertain the kinematic relationships presupposed by Newtonian theory, so also at least a schematic view of its role in physics is necessary if we are to grasp the connection of the “instantaneous geometry” with our spatial apperceptions and our spatial measurements.

Now, the role of the instantaneous spatial structure in Newtonian physics is patent: That physics assumes the *forces of interaction* among bodies to be determined by their *instantaneous* interrelationships (most characteristically, their instantaneous spatial configuration—conceivably also their instantaneous states of motion). Instantaneous distances and directions therefore enter into the statement of the laws of nature. But this fundamental role is just what explains how the spatial aspect of the underlying structure of space-time reveals itself in spatial measurement. It is not necessary to subscribe to the view of Poincaré, who said that without solid bodies there would be no geometry, to recognize that our ordinary spatial measurements (including those estimates of spatial magnitudes that we constantly make in everyday life) do make essential use of such bodies ([1905] 1952, 61). This granted, what—we must reflect—are solid bodies? If we adopt the point of view of the physics of particles (rather than that of continuous extended media),⁶ we have to regard solid bodies as systems of particles that possess *a configuration of stable equilibrium*; it is that equilibrium configuration that determines the characteristic spatial dimensions of a body, whether the latter is considered as a measuring instrument or as an object to be measured. Since the laws that determine the equilibrium configuration make use of the underlying spatial geometry *at each instant*, it is that (“fleeting”) structure that enters into the determination; and since the resulting configuration is stable, the “fleeting” spatial relationships *of the parts of the body* persist through time.

We see, therefore, in the first place, that there is a certain subtlety in the connection of what are measured as spatial magnitudes with the postulated underlying spatial aspect of the spatio-temporal structure. And in the second place, this physical explanation of what is involved in the processes of objective empirical measurement serves in turn to explain

⁵It is also obvious on brief reflection that one does not in fact measure *that* underlying structure even *indirectly*; obvious, at any rate, if—in contrast to Maxwell—one believes the theory of relativity, which tells us that that structure does not truly characterize the empirical world.

⁶This is the point of view of Newton himself, and (of course) of that branch of (fundamental) classical physics that connects most directly with our present conception of the structure of matter.

what from the (if I may be pardoned for so calling it) “naive Kantian” viewpoint is simply “intuitive”;⁷ for the sensory material upon which we exercise cogitation, or which provides the matter for imagination to work on, derives from objects of the same sort as those concerned in objective empirical measurement.⁸

V

If we turn to the situation in the special theory of relativity, a large part of the foregoing discussion carries over. In particular, the relation of the Einstein-Minkowski space-time structure both to our ordinary experience with ordinary bodies, and to the subjective sense we have of spatial and temporal relations, again can only be understood through the medium of the account given by physics of the processes responsible for the behavior of those bodies (and of our own perceptual apparatus). But in that physical account, there are some significant changes.

In the first place, although special relativity does involve a uniquely distinguished concept of “simultaneity relative to an observer”, this plays a distinctly smaller role in the physics than does the concept of (absolute) simultaneity of Newtonian physics. In the latter, laws of interaction refer to instantaneous (and thus simultaneous) configurations. This cannot be the case in relativistic physics, since there is no distinguished notion of simultaneity for a *system* of particles (whose states of motion will, of course, in general differ). Consequently, the typical form of a law of interaction in relativistic physics is “local” (or more accurately, *infinitesimal*): the behavior of a given particle of matter, or a given position in a field, is influenced only by the state of affairs “infinitely nearby”.⁹ In accordance with this fact, the relationship that does play a fundamental role in the physics is that of what may be called “infinitesimally nearby relative simultaneity”: the force on a particle is represented by a vector orthogonal to the world-line of that particle. (Because this is a strictly infinitesimal relationship, the statement continues to hold without change

⁷The “naive Kantian” viewpoint is not that of Kant (see footnote 8).

⁸This remark, I believe, would be fully endorsed by Kant; according to him the mere manifold of intuition, with whose pure form the Transcendental Aesthetic deals, can never come to consciousness except through those modes of synthesis which, in his view, take place through the categories and are constitutive of objective experience. Although the doctrine is in many ways obscure, one thing is clear: it posits a quite complex relation between the empirically objective and the subjective, in which the latter is in a significant sense dependent upon the former.

⁹I here ignore the (rather awkward) forms of distance-action law that have been considered within special relativity—the first of which were the theories of gravitation suggested by Poincaré ([1906] 1954, 538–550) and Minkowski ([1908] 1911, 401–404; see also Minkowski [1909] 1911, 443–444, trans. in Lorentz et al. 1923, 90). No such proposals have proved fruitful.

in *general* relativity, where there is no good analogue to the Einstein relation of distant relative simultaneity.)

A second difference particularly affects the question of the “intuition” of simultaneity, and therefore touches the heart of the issue with which we are here chiefly concerned. Let us consider a “specious present” π of some percipient being; and let us call an event e “contemporaneous” with π if signals—interaction—influence—can occur *mutually* between e and π . In the Newtonian case, the spatial extent of the set of events contemporaneous with a given specious present is infinite; and it is rather natural to see in this fact the precise correlate, in the physical theory, of the “intuitive” notion of a “present” throughout all of space. The situation in the relativistic case is significantly different; but before proceeding to discuss it, a little further reflection on the Newtonian case is in order.

The concept of the “spatially unbounded present” is not, in fact, as “natural”, or (primitively) “intuitive”, as our entrenched habits of thought and speech lead us to suppose. It is at least a plausible anthropological hypothesis that the primitive notion—the notion that first arises “naturally” in the course of human development and socialization—is that of contemporaneity *with respect to communication or influence*. At any rate, it is clearly a fact about the historical etymology of our language that the original meaning of the word “present” was not *now*, but *here-now* (i.e., “nearby now”). Indeed, the original explicit meaning would seem rather to have been the spatial one (Latin *praesens*, present participle of *praesesse*—“to be in front of”, “to be at hand”)—although, of course, with the temporal component understood (indeed, implied by the tense). That remains a current usage: When a soldier at roll call responds “Present!” upon hearing his name, he is not merely announcing that he still exists; he means that he is on the spot.

Now, in the theory of relativity, the only reasonable notion of “present to a space-time point” is that of the mere identity-relation: present to a given point is that point alone—*literally* “here-now” (see Stein 1970b, 15). On the other hand, the set of events contemporaneous with a specious present will always be a spatially extended one. And it is, I think, of very great relevance to the misconception I am trying to dispel, that this spatial extent—although finite—is in fact *and in principle, as a matter of physics*, always, in a certain sense, immensely large. I must explain in what sense “large”, and on what basis in physical principle.

The point is analogous to one made long ago by Schrödinger when he posed, as the point of departure for his celebrated discussion of the physical basis of life, “the odd, almost ludicrous, question: Why are atoms so small?” (1945, 4). Having first translated this into the less ludicrous question, why we are so large (compared to atoms)—and, more particularly, the question: “Why should an organ like our brain . . . of neces-

sity consist of an enormous number of atoms, in order that its physically changing state should be in close and intimate correspondence with a highly developed thought?" (*ibid.*, 7)—he answered as follows:

The reason for this is, that what we call thought (1) is itself an orderly thing, and (2) can only be applied to material, i.e. to perceptions or experiences, which have a certain degree of orderliness. This has two consequences. First, a physical organization, to be in close correspondence with thought (as my brain is with my thought) must be a very well-ordered organization, and that means that the events that happen within it must obey strict physical laws, at least to a very high degree of accuracy. Secondly, the physical impressions made upon that physically well-organized system by other bodies from outside, obviously correspond to the perception and experience of the corresponding thought, forming its material, as I have called it. Therefore, the physical interactions between our system and others must, as a rule, themselves possess a certain degree of physical orderliness, that is to say, they too must obey strict physical laws to a certain degree of accuracy. (*Ibid.*, pp. 7–8)

The conclusion, Schrödinger says, is that the requisite stability and orderliness, in the face of thermal disorder at the atomic/molecular level, demands that the relevant systems be composed of enormously large numbers of atoms; except—except—that this confident expectation (attributed by Schrödinger to “the naive physicist”) is belied at the level of the structures that have proved to be basic to all life, including our own: the genes. And this remark initiates his argument that the true physical basis of life must be found in the properties of large and complex molecules, whose stability and regularity of interactions can only be accounted for through recourse to quantum mechanics.

What is of interest for our purpose is not Schrödinger’s important observation that the view of the “naive physicist” fails at the fundamental level of biological process, but just the first part of his discussion—the account of the “naive” physicist’s view itself (which in fact remains correct, so far as our psychophysical interactions with a stable environment are concerned). Let me pose an analogue of Schrödinger’s “odd, almost ludicrous question”, in the terms in which I once heard it raised in a colloquium discussion: Why is it that, in the geometry of space-time, we are so long and thin?

In order to make that question reasonably clear, let us take “we” to mean our bodies during some short but perceptible interval of time—say, a specious present. As to “long and thin”, that is to be understood as follows: The Minkowski metric can be taken to assign a ratio of lengths not only to a pair of space-like intervals or a pair of time-like intervals,

but also to a space-like interval and a time-like one. The ratio, obtained in this way, of the spatial extent of our bodies to the temporal length of a specious present is exceedingly small: we are temporally long and spatially thin. And the same is true of all the ordinary objects with which we deal—including the earth. Why should this be so?

A little reflection will show that the question could have been put another way: Why is the velocity of light so great? Of course, if we express ratios of space-like to time-like intervals in direct Minkowski-metric terms, as above, the velocity of light is in fact unity; but this does not (as might at first appear) show that the question of “great” or “small”, “thin” or “wide”, turns on mere conventional choices. To say (in this connection) that the velocity of light is “great” must be understood in the sense, not that this velocity is expressed by a large number, but that during a specious present, light travels a spatial distance that bears a very large ratio to the spatial extent of our bodies or of ordinary objects.

The question therefore has a clear content. It also has a simple answer—and almost the same answer as does that of Schrödinger. For although we know little about the physiological conditions required for consciousness to occur, one thing is pretty certain: these conditions involve the coordinated functioning of some part of the central nervous system. And it is clearer still, so far as perceptions of our surroundings are concerned, that the things we perceive must possess a degree of stability (and must interact with us in stable patterns); this, indeed, is close to a repetition of the point already discussed in connection with spatial measurement from the Newtonian point of view. But according to relativity theory, interactions are not instantaneous; they are propagated with a time delay—with a speed at most equal to that of light. Now, for stable configurations of particles to be established, and for processes with stable patterns to occur (or for orderly *readjustments* of such patterns to occur), it will in general be necessary for very many interactions back and forth to take place throughout the system in question; roughly speaking, many signals must pass in both directions to establish a degree of regular coordination.¹⁰ And from this it immediately follows that the “graining” of time with respect to which a percipient organism can experience conscious interaction with its environment must be such that the “moments” of time (the specious presents) are long enough to allow such signals—and, therefore, light signals—to travel very many times the maximum spatial dimensions of the organism *together* with its (relevant) environ-

¹⁰Note that this argument does not presuppose that the orderly configuration of the system is established by *long-range* forces. Long-range order will typically result from short-range forces, by the intermediation of intervening particles; but it is clear that the speed of propagation of such intermediated actions, or “relayed signals”, cannot be greater than that of the direct signals—that is, than the speed of light.

ment. On any more fine-grained subdivision of time, one will have to expect rather complicated fluctuations—which, however, average out to steady patterns over a specious present.

But then it is entirely clear why we should have developed “intuitions” of something like “cosmic simultaneity”, or a “cosmic present”: in all our ordinary experience, the time that we experience as a “moment”—a specious present—is in the exact sense already explained contemporaneous with events as far distant, spatially, as we ever normally have to do with at all. (The truth of this, of course, depends only upon the *fact* that the velocity of light is very great; the point of the analysis sketched above is to indicate that this fact is no merely accidental one.) But these intuitions are quite as illusory as those that once made—and still make, I think, at some stage of every person’s individual development—the notion of the earth unsupported in space, or that of people living at the antipodes (“upside down”), appear paradoxical. And I suggest that it is a defect in the degree to which the real content of relativity theory has been assimilated by many philosophers—even philosophers with considerable scientific sophistication (Rietdijk, Putnam, Maxwell, Dieks)—that is responsible for the persistent misconception I am examining.

VI

I hope the foregoing suffices to show the deficiency of Maxwell’s argument about “alternative present actualities”. That, however, does not exhaust the considerations he brings to bear; some further attention is in order to the points he raises that concern quantum mechanics.

Maxwell advocates an interpretation of quantum mechanics that he calls a “‘microrealistic, propensity’ version” (1985, 36), or a “‘propensiton’ version” (*ibid.*, 37), of the theory. This is simply the view that collapse of the wave-packet is a genuine physical process that occurs instantaneously over a distance (namely, the full extent of the wave-packet itself—which, strictly speaking, will usually mean all of space). It is, of course, quite clear that such a view is hard to reconcile with relativistic space-time theory (although, I shall argue presently, it is not strictly *impossible* to reconcile the two). Maxwell himself says:

Despite [its] impressive credentials, propensiton QT [quantum theory] may well be judged to be wholly unacceptable for one reason alone. The theory is irreparably incompatible with special relativity. For propensiton QT postulates that, in appropriate physical conditions, propensitons—which may be smeared out in space over large volumes—collapse *instantaneously* into very small volumes; and this contradicts special relativity.

The standing of propensiton QT changes dramatically however if the main argument of this paper is correct, and *probabilism in general* contradicts special relativity. For in this case *any* fundamentally probabilistic physical theory must contradict special relativity. In particular, all interpretations and versions of QT which hold QT to be fundamentally probabilistic must contradict special relativity. Thus the fact that propensiton QT contradicts special relativity in the way indicated . . . cannot tell in any way at all against the theory. (Ibid., 38–40; Maxwell’s emphasis)

We see, therefore, that the metaphysical argument of the paper—concerning the (conceivability of) “ontological indefiniteness” of the future—is intended to support a particular view of quantum mechanics against what is *prima facie* a formidable objection. Now, there is something a little odd and disconcerting about the way that argument is deployed to this end. For early in the paper, Maxwell remarks:

[O]ne might suppose that the argument of section 1 establishes that special relativity (realistically interpreted) must be false if the basic laws of nature are probabilistic in character, and not deterministic. [But] this is not correct. . . . The truth is that two distinct versions of probabilism need to be distinguished. On the one hand there is probabilism as this has been defined above, a view which asserts that the basic laws are probabilistic *and that the future is now in reality open with many ontologically real alternative possibilities* whereas the past is not. This view may be renamed *ontological probabilism*. On the other hand there is *predictive probabilism* . . . , a view which asserts that the future, like the past, is now in reality entirely fixed and determined even though the basic laws are probabilistic and not deterministic. . . .

This difference . . . crucially affects the question of the compatibility of special relativity and probabilism. The argument of section 1 presupposes *ontological* probabilism. It fails if *predictive* probabilism is presupposed. We may thus conclude that special relativity is incompatible with ontological probabilism, but compatible with predictive probabilism. (Ibid., 24–25; Maxwell’s emphasis)

That is a very clear and straightforward acknowledgment; but how does it square with the remarks quoted from the later part of the paper? There, as we have seen, the force of the objection from the (alleged) incompatibility with relativistic space-time theory is met with the claim that “*any* fundamentally probabilistic theory must contradict special relativity”. The passage just quoted, however, makes it plain that this claim is made only for theories that are, fundamentally, “*ontologically* probabi-

listic". Since "propensiton QT" seems to be the only (sketch of a) theory on the scene that is clearly of this latter type,¹¹ it is hard to see how the claim that "any" such theory must contradict special relativity can destroy the force of the objection or establish that the alleged incompatibility "cannot tell in any way against" propensiton QT.¹² (Of course, this point is of no substantial consequence if, as I have argued, the alleged contradiction between special relativity and "ontological probabilism" is specious; yet it is perhaps of some importance in assessing the general position.)

I have argued, then, both (a) that special relativity is perfectly compatible (in general) with "ontological probabilism", and (b) that Maxwell's attempt to draw the sting of the alleged incompatibility with his preferred interpretation of quantum mechanics would fail in any case. There remains the question of the true relationship between relativity theory and Maxwell's propensiton QT. Here another quotation from Maxwell himself will be helpful. Immediately following the remark that instantaneous collapse contradicts special relativity, he adds:

This contradiction is not merely because of the fact that propensiton QT postulates a faster-than-light collapse of wave packets, or propensitons. Many have argued that faster-than-light particles—tachyons—are permitted by special relativity, as long as it is conceded that such particles move in one direction in some reference frames, and in the opposite direction in others. Much more seriously, it is the demand that propensiton collapse be *instantaneous* which irreparably contradicts special relativity. For special relativity asserts that all inertial reference frames are physically equivalent. In only one reference frame, however, will any given probabilistic collapse of propensiton state be instantaneous; in other, relatively moving inertial reference frames the collapse will not, according to special relativity, be instantaneous (though always faster-than-light). (Ibid., 38; Maxwell's emphasis)

But why, one must ask, would not "tachyonic" collapse suffice? Maxwell refers to the experiment of Aspect, Grangier and Roger as apparently confirming "that wave packet collapsing events, associated with measurement, occur in a faster-than-light way" (ibid., 39); but this does not show that such a "faster-than-light way" need be somehow invariantly

¹¹The view that reduction of the wave-packet is a real process, precipitated by an act of observation—that is, by some specifically *mental* intervention—can, I think, be regarded as a particular variety of the "propensiton" interpretation.

¹²In the central part of Maxwell's paper (1985, 28–36), arguments of a metaphysical nature are offered in favor of ontological probabilism. I am here taking it for granted—perhaps illegitimately—that in the absence of a serious issue within physics, such arguments against a theory like that of special relativity are futile.

instantaneous. One must admit that there is something odd about the notion of an extended event (the collapse), whose parts are, on the one hand, “ontologically indefinite” from each other’s perspectives, and yet, on the other hand, mutually coordinated. But here it is pertinent to take note of a peculiarly relativistic indeterminacy—which may seem to contrast oddly with the alleged commitment of special relativity theory to determinism—that significantly mitigates this apparent paradox. Let us adopt the view that, for any event e , what is definite (besides e itself) is just its “causal past”. Then even if the whole of physics were deterministic (in the usual sense of that word: the state of affairs over any space-time region is completely determined by that on a full space-like cross-section of the causal past of that region), it would nevertheless be true that the specification of everything that is “already definite” for e would not suffice to determine the state of affairs at a space-time point x outside the past light-cone of e ; for the past cone of e contains no full cross-section of the past cone of x . And this will hold of our tachyonically related points of collapse: although the correlations implied by quantum mechanics from the collapsed “propensiton” must obtain, other relevant circumstances at x —for example, for something like the Aspect experiment perhaps, instrument settings that determine what is being measured—will not be determined by the past of e . It therefore remains not only formally consistent, but quite reasonable, to regard conditions at x as not yet definite from the point of view of e .¹³

There is, to be sure, a fundamental difficulty with this tachyonic hypothesis: namely, just what may determine, and according to what law, the space-time locus of tachyonic collapse? Maxwell in effect proposes a solution to this difficulty: that there is a unique globally defined system of space-like hypersurfaces that partition space-time into the loci of all possible such collapses. This system, then, would constitute his re-established objective simultaneity-relation.

The trouble with that neat solution is that there is absolutely no evidence for it (any more than there is for the more general “tachyonic” hypothesis). Indeed, in the first place, there is no evidence for the pro-

¹³This statement may seem to conflict with a criterion employed earlier in the paper, when, from the fact that the physical state at any point is influenced by the states at all points in its causal past, it was inferred that one is forced to regard the latter as “definite as of the former”. However, the (supposed) tachyonic relation is different in a fundamental respect from that of “causal” influence in the sense of relativity theory. If a and b are two points with space-like separation, and if a wave-packet reduction affects them both, it is in the first place impossible (in the relativistic theory) to say that this reduction is precipitated at the one point rather than the other; and in the second place, it is crucial to the theory that the outcome of an observation (or other interaction) at b is not influenced in any detectable way by anything done at a (and, of course, conversely). This is the reason why reduction of the wave-packet (if it is indeed a genuine physical process) cannot be exploited to send a signal with superluminal velocity.

propensiton interpretation itself. This interpretation implies some departures from the predictions of standard quantum mechanics. The results of an enormous number and variety of experiments—including (of course) those of Aspect et al.—are in agreement with the predictions of standard quantum mechanics, and none in contradiction therewith. (Maxwell's claim that the Aspect experiment "is a great experimental success for propensiton QT" is, to say the least, odd in view of these facts.) And in the second place, if that interpretation is after all correct, it would still seem premature to assume that the loci of collapse are fixed once for all, and uniformly for all "propensitons"—rather than somehow dependent (whether deterministically or probabilistically) upon the particular characteristics of the collapsing packet. Should the latter prove true, relativity theory would obviously not have been impugned.

In the closing section of his paper, Maxwell suggests that the first step towards a resolution of the conflict between *general* relativity and quantum theory is to render the former compatible with probabilism, by introducing a system of space-like hypersurfaces of simultaneity, in some way related to the distribution of matter (1985, 40–41). It is well known that in the present form of the general theory of relativity, there are cosmological solutions that do, and others that do not, have such systems of hypersurfaces. If one were to accept Maxwell's suggestion, the easiest course would be simply to postulate that such a determinate system exists. And if Maxwell were to propose a way to exploit such a postulate for the development of a quantum theory of gravitation, that would undoubtedly be a proposal worth exploring.

But the difficulties here are extremely great. The general theory of relativity, as it is understood at present, is in principle a *global* theory—global with respect to all of space-time; and "ontological probabilism" implies, in principle, that the future distribution of cosmic mass-energy, with which are entangled the metric and even the topological structure of the entire space-time manifold, is not yet fully determined. It is consequently not at all clear how the legitimation of a notion of cosmic simultaneity could even achieve the compatibility with probabilism that Maxwell seeks—much less how it might facilitate the development of the desired quantization of the theory.

In short, Maxwell's metaphysical considerations appear defective; their connection with the positive scientific program he advocates is questionable; and the program itself, although it is concerned with deep and important matters, looks at this stage exceedingly tenuous.

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