Stein   5/4 cont.

Maxwell - how he contributed to our understanding of physical reality
(n.b. Prof. Wm Thomson = Lord Kelvin)
Kelvin actually showed how to picture a physical ether which has pot. energy as $|\mathrm{curl}\,\vec{e}|^2$ ⇒ picture each "molecule" of ether as a little gyroscope (= "gyrostatic ether")
but it introduces an anisotropy in ether

What is diff about what he is trying to do in the first paper "On Faraday's Lines" than in 2nd "On Physical Lines"?

~~on~~ (his notions of "physical analogy" in 1st paper → when he says they have "greater applicability to phys probs" (p.160), is he just saying that it yields greater subj facility?
that $\neg\exists$ anything too deep in this idea of analogy

But in 2nd paper, he is describing, a trying to, a real physical phenomenon
in first, $\neg\exists$ anything which he claims to be true or even plausible, but in 2nd paper he believes something is there, but he is not dogmatically asserting ~~...~~ this is right, but that it at least represents something
↓
[illegible] Paper - ~~a~~ stab at describing the kind of thing which might be there

"Physical Lines" - publ. in several parts, over a year
→ he was figuring it out as he wrote it, his ideas changed over the year
(cf. correspondence w/ Kelvin)

He's not describing what he's prepared to assert is really there, but he's attempting a 1st description of what it might be like

⇓

But what is aim of the 1st paper? What does it accomplish?

Historical Background

Bunch of separate phenomena fairly well-described w/ precise principles, but connections between them are quite obscure

⇒ so any new theory is going to have to bring together bunch of disparate phenomena which are all well-described

⇓

Faraday was not a mathematician, sort of held in disdain by mathematicians, who thought he wasn't a theorist

→ but Maxwell thought he was, who conceived of things differently than mathematicians

part of Maxwell's impetus in "Faraday's Lines" is to give a precise account of such theorizing

Faraday had discovered that "galvanism" was really a current of ~~what~~ ~~a~~ electricity, before which the two had no obvious connection

⇓

in 1810's – Oersted discovered connection between galvanic current and magnet

then Ampere set out to describe it quantitatively in 1820's

→ no real connection except analogy between electricity and magnetism before this (Coulomb had "shown" that both statics obeyed an inverse square law – his exp. data really wasn't very good – Cavendish did better)

Ampere advanced theory that magnetism is purely electrical phenomena (or "galvanic") → that "magnetic molecules" were just tiny galvanic currents

Readings: "On Action at a Distance" pp. 185-91 of packet
"A Dynamical th. of EM field" §§1-21, 53-75, 80-81, eq'ns #69 (p. 579),
95-97, 100-101

Stein

5/4 cont.

problem of theory was determination of law of force that galvanic currents exert on each other, worked out quickly by Ampere

⇓

he gave a law for forces between 2 current elements s.t. force was along line connecting the elements

but Grassmann gave equivalent law where force was ~~[illegible]~~ equal and opposite on each element but ~~[illegible]~~ to ~~[illegible]~~ line connecting them (equivalent, cuz all that matters is the net force on the 2 current elements, disregarding the rest of the 2 currents and the ~~[illegible]~~ difference of the 2 laws integrated over both currents gave zero)

⇓

Faraday then finally showed that magnetism wasn't just an electric phenomenon but involved a transport of electricity

⇓

they didn't think of electricity as matter necessarily, but as a state → so all Faraday showed that the state was transported, which implies nothing about matter

and

finally Faraday demonstrated the electric effects of magnets, which his sense of symmetry demanded → induction

⇓

notice that Ampere's formula can't fully explain this, since it's the acceleration of ~~[illegible]~~ electric matter/state which produces a current, which is not really dependent so much on direction ~~[illegible]~~ between current elements

☆ central pt of Maxwell 1st Paper - to show how mathematical results of French/Ger could be re-expressed precisely as representing "something physical" going on in intervening space between electrical matter, as Faraday speculated ~~his~~ lines of force really represented something

☆ **Read**: Maxwell-Thomson Correspondence
in "On Origin of Clerk Maxwell's EM Ideas" by Larmor
+ Daniel Segall in Conceptions of Ether ed. by Geoffrey Cantor

Stein — Cf also John Heilbron Early Histories of Elec

5/11

Maxwell would have had a problem w/ Biot-Savart law (giving force)
cz it gives the $\vec{B}$ field in terms of geometrical relations
tween distant objects

⇓

he wanted a description of phenomena in which $\vec{B}$ field / force only
depended on what was going on around that point
(inherited from Faraday's viewpoint)

↓

☆ So Biot-Savart could be a derived law from description of
mediated actions, but not a fundamental law

↓

ce EM effects are propagated w/ finite speed
→ thus can't even be that Biot-Savart is a strict derived law
– decent approximation for steady currents
→ if you start moving a charge at t_0, then at distant places
at t_0 $\neg\exists$ EM effects from this, even though geometrical ~~of~~ affairs has changed

Relation of "On Physical Lines of Force" to "Dynamical Theory of EM
Field" → has never been satisfactorily analyzed
Stein thinks ∃ a deep confusion in Maxwell's thought here
→ cf. comment by Boltzmann in his translation of "On Phys Lines
of Force" into German → points to difficulty of understanding Maxwell

Treatise on EM –
ch. 1 description of phenomena – seems to be operational def'ns of concepts
→ his qualifications after description of resin + glass point to difficulties
in operational def'ns (it is not strictly true, for instance, that everything electrified

5/11 cont.

Stein

glass repels resin attracts and vice-versa – polarization of dielectrics)

↓

he has to incorporate some of the theory (viz. 'induction') in the characterization of the fundamentals of the theory -

especially re:

~~the~~ problem of classifying types of electricity into 2 (why not more? why is attraction & repulsion the criterion?)

One can classify 'charged bodies' into 2 classes:

Empirical law: 1) The relation 'A repels B or A is same body as B' is an equivalence relation

- but this oversimplifies, cz of, e.g., polarization of dielectrics

↓

but once we learn enough physics, we understand what this statement 'really means'

→ so the difficulty is, how did the distinctions get started in the first place? delicate dialectic

[cf. Newton's 'deduction' of univ. gravity - had to show that his assumptions, which didn't fit the hypothesis initially, were predicted by it]

→ though Stein says that Newton's prob was more serious than this, cz ∃ far greater wealth of exps on EM than on gravity

also

~~∃~~ ∃ bodies (e.g. magnets) which repel but aren't elec. charged —

↓

but just on repulsion alone ∃ can any # of elec. charges -

so

Empirical law: 2) ∃ exactly 2 equiv. classes

but this still really isn't enough –
we want also

empirical law: 3) charged bodies in the same class "act the same electrically" towards other bodies
⇓
but now "act the same electrically" really needs clarification
but it's good enough for rough work

he then shows, by quasi-thought exps., that charge is an additive magnitude
(positivists thought this was great ex. of exact concept formation from qualitative data to precise quantitative concept
→ but you must keep in mind the diff between thought-exps and what's feasible/practical in lab,
the exactness is specious, but still very helpful in clearing one's thinking)

Faraday's Lines of Force:
appeal to media was *purely* for clarificatory purposes, to clear his thought
but
cf. last ¶ of Part I, p.188, last sentence → seems to imply that Part II is aiming at a real mechanical conception. (but note that Part II is symbolic, not pictorial)

5/11 Oct.

Stein

Part I of paper → Physical Analogy: to

a) represent vividly a set of laws

b) stimulate the search for an underlying theory that might "explain" these laws

↓

in Part II, a) does not appear, but only b) ?

[vacillation by Maxwell on use of the analogy in this paper? this last ¶ really does seem to push for more "explanatory" role – esp. the "readily adopt" of bottom p. 187]

⇓

What Maxwell is really up to is to make Faraday's "intuitive" conception of lines of force "professionally" acceptable

which he really does by a combo of the math and the picture of the fluid lines

We have to be content w/ not precise experimental def'ns of all concepts, but allow them to be refined over time –

↓

viz. def'n of $\vec{E}$ as $\vec{F}/q$ → this is really untenable as a def'n, coz. e.g., in any lighted room the $\vec{E}$ field is changing on the order of 10^{-15} secs

the initial clear rendering is really just schematic for Maxwell → we must have other ways besides measuring the force to "really" define the $\vec{E}$ fields if we want to do so "operationally"

↓

like Hertz's exps in 1888-90 on spark-gaps, which "established" Maxwell's treatise theory (1873)

Faraday's Lines of Force:

Maxwell was showing that you can use Faraday's lines instead of the French (Laplace, Poisson) conception of scalar potential ~~ener~~ energy fields and equipotential surfaces and forces as gradients of the potential (which entirely summarize info about static fields)

the velocity stands for field intensity, pressure stands for potential energy since for incompressible ~~fluid~~ fluid $v = -k \text{ grad } p$

and $\vec{E} = \frac{\vec{F_r}}{b} = -k \text{ grad } V$

this is equivalent to $\oint \vec{v}_u \, ds = 0$

where $\vec{v}_n$ is component of velocity around a very small closed circuit in the fluid equivalent to $\text{curl } \vec{v} = 0$

→ criterion for well-defined potential f= , since this says that work done by field moving around a closed circuit is zero

→ cuz I have a well-defined diff of pot energy just in case the pot energy diff tween 2 pts is path-ind

and

we also get ~~div E~~ from Maxwell's paper

$\text{div } \vec{E} \propto$ density of charge or magnetism

⇓

Thus 2 differential eq'ns entirely summarize info about ~~sta~~ static fields

to include dielectric media in electrostatics, define the displacement vector $\vec{D} = K\vec{E}$, $K =$ dielectric coefficient

then $\vec{D} = -K \text{ grad } p = K\vec{E}$

$\Rightarrow \text{curl } \vec{E} = 0, \quad \text{div } \vec{D} \propto \rho$

same goes for magnetic field $\vec{H}$, magnetic permeability μ, and magnetic induction $\vec{B}$

⑥

Stein 5/11 cont.

in one area, in ~1855, when Maxwell wrote "Faraday's Lines", Faraday's conception of lines of force seemed superior esp. in one field over the (more mathematical, abstract) action at distance conception, and that was in EM induction

↓

cf. pp. 183 "On Action of Closed Currents at a Distance", although this not very explicit here

n.b. for Maxwell "electromotive force" ≃ electric field intensity (i.e. force/unit charge)

but electromotive force does not produce acc. of mass, but only motion of charge

(what we call EMF, the voltage drop, is given by Maxwell as the total EMF)

→ his conception of # of lines crossing circuit delivered the proper quantitative law concerning induction effects

→ so Maxwell not only showed that the abstract math formulation could be rendered by Faraday's picture, but that it actually delivered an important result independent of them

viz. that current produced, viz. total emf, is prop. to rate of change of quantity of lines of magnetic induction passing through surface

→ does not nec imply a principle of individuation of the lines

viz. we have $\int_S \vec{B}\cdot\hat{n}\, dS$ = quantity of lines passing through surface S

~~$\oint \vec{E} \cdot d\vec{s} = \int \text{curl} \times \vec{E}$~~

then the "electro-tonic state" (cf. p. 206) $\vec{\Sigma}$

is such that

any flux $\vec{B} \propto \text{curl}\, \vec{\Sigma}$

since $\oint_{\partial \Sigma} \vec{\Sigma} \cdot d\vec{s} = \int \text{curl}\, \vec{\Sigma}\, d\Sigma$

but $\oint_{\partial \Sigma} \vec{\Sigma} \cdot d\vec{s} \propto \int \vec{B}\, d\Sigma$

⇓

so $\vec{\Sigma}$ is the vector potential of EM field

we have a lot of results, but not yet any of the content of Maxwell's developed later theory

↓

All Maxwell does in "Faraday's Lines" is explain all the known laws, and only those

☆ which Maxwell describes as a virtue (contra Weber)

↓

he will eventually go out beyond ~~known~~ phenomena and go out on a limb

☆ but only after completely clarifying and organizing what is already known

still has to answer what happens when charges at rest are put suddenly in motion

⇓

next paper answers this

read: pp. 180-183

Stein

5/18

self-energy of a system of point charges in electrostatic can is just

$$1/2 \sum_{i \neq j} \frac{q_i q_j}{r_{ij}}$$

but Thomson in 1840's showed this is equal (not literally for point charges) to $\frac{1}{8\pi}\int_{space} \vec{E}\cdot\vec{E}\, dv$ (integral done excludes charge pts)

⇓

very suggestive for Maxwell → energy is not stored up in the charges or a property of them but it is spread out 'elastically' all over space

⇓

Thomson was using charges spread out on conductors, so we never get pt. charges, just charge densities, so we never get singularities in $\vec{E}$ field at points

(→ makes it more plausible to think that all charges are spread out in space that ∃ point charges? → but this raises other probs, e.g. what holds electron together?)

↓

so what we're really showing is that

$$\frac{1}{8\pi}\int_{space} |\vec{E}|^2 dv = 1/2 \iint_{space} \frac{1}{|p-p'|}\, \rho(p)\rho(p')\, dv\, dv'$$

where $\rho(p)$ = density of charge at pt p

⇓

☆ suggest ∃ might processes which involve redistributions of energy in 'free space' w/o any redistributions of material charges

Maxwell, in Encycl Britt article on 'Attraction', leaves it open that one day it might ~~be~~ turn out that such a view of fields, that it is ~~a~~ modellable by a Newtonian model of mass distribution | action, might turn out to be wrong

⇓

great open-mindedness, which turned out to be prophetic

⇓

contra Kelvin, who like Huygens, demanded mechanical models for intelligibility

⇓

~~you~~ great multi-level thinking, like Newton, in being able to ~~accept~~ produce very elaborate compelling hypotheses (the mech. model) and yet hold out the real possibility that he doesn't have it right yet

Hertz's exps established Maxwell's theory, but ∃ lots of confusion, even in 1890's, 25 yrs after

Physical Lines

we have all 4 Maxwell eq'ns

⇓

but we had everything in Faraday's paper except in eqn 112, p.496, $\text{curl } \vec{H} = 4\pi \vec{i} + \frac{\partial \vec{D}}{\partial t}$

↓

what's new is $\frac{\partial \vec{D}}{\partial t}$ term

③

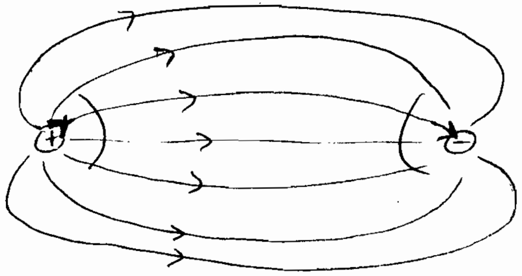
5/18 cont

Stein

What has now changed?

Why is this an important physical advance, as Maxwell ~~thinks~~?

recall Faraday's picture:



imagine ~~position~~ the tension as intrinsic to the lines of force (a taut string) w/ an arrow head ~~at~~ one end, arrow tail at other, ~~so that the~~ ⇒ yields the asymmetry btwn pos. + neg. charges, if we assume that pos. "hooks onto arrow tails" and "neg" "onto arrow heads"

and that the $\overset{\text{attractive}}{\text{the}}$ force on + and − is due to the density of lines increasing between the charges, by some innate property of them to squeeze together, bulge out, and be dragged around

→ opposite picture for 2 +, to get picture of repulsive force

⇓

but it points to a deep problem in this picture on how one really distinguishes + from −

Physical Lines

completely ignores the relation of rolling cells and ball-bearings to regular matter → remember that the one is supposed to completely permeate the other –

if we want actually to calculate the motion of idle-wheels, based on the various rotations of the cells, how quickly they rotate, whether they roll, etc.

→ how does do the things stay in "rolling contact" with the cells?

picture on p. 489

→ can't be rigid rotation, coz the velocity of the swirl in the cell must go around corners,

how to picture it in 3-d space

⇓

but really – how do the calculations he makes hook up with this picture?

→ cf. prop. 5 p. 469

⇓

"calculates" that curl $\vec{H}$ = flow of ball-bearings, so ball-bearings = electric current

Part 3 – the physical model radically changes, from fluid model + elastic model → $\overset{\text{elec.}}{\text{energy}}$ ~~from~~ goes from kinetic energy of fluid to elastic potential of the fluid

☆ and he changes none of the math except for adding $\frac{\partial \vec{D}}{\partial t}$ to curl $\vec{H}$? How can everything still be valid, that swirling fluid can be treated as exactly the same as elastic deformation?

Stein

5/18 cont.

✱ How are the eq'ns justified now in part 3 of paper?

cf. eq'n 112 p.496 → one part of eq'n is from old model, other part is from new model !

p. 491

→ the displacement is in opp direction of emotive force

→ so em force is restoring force of the electric medium

⇓

but this is not what the em force started out as —

what is going on?

what besides $\vec{E}$ puts the medium under stress, to push the particles one way?

if ∃ something like this, then the restoring force will be force on the already displaced particles

⇓

but then displacement of particles here cannot be analogous to displacement of elec particles in ordinary dielectric, for this displacement is in direction of $\vec{E}$

⇓

this is analogous to problem in the Faraday picture

→ what causes the stress in the stretched cords?

Maxwell seems to think here also that $\vec{E}$ is both the stress of the cords and what causes the cords to stretch

↓

his later eq'ns push forward the interp that $\vec{E}$ is the restoring force of medium

⇓

but what stresses it?

Later in the paper, prop. 15, p. 457,
elec. potential is introduced ~~to~~ as ~~what is~~ a measure of what stresses the field, + solve problem of force

⇓

but ~~a what~~ it was originally ~~put~~ introduced only as a constant of integration, w/o any connection to his "physical model"

⇓

[he seems, despite himself, pushed further and further away from his "physical model"
→ but then what do the equations represent/mean?
what is he calculating?]

⇓

all related to probs of comprehension when Maxwell's theory came out

⇓

[why today do we take it for granted that we understand Maxwell's theory, what the equations "represent/mean"? Related to the problem of whether or not QM needs an "interp" –]

while vortex lines "reasonably" seem to produce $\vec{B}$ lines, as polar vortex lines, but what is $\vec{E}$ in electrostatics? what produces the thg?

⇓

How did this nonsensical stuff help Maxwell find $\frac{\partial \vec{D}}{\partial t}$, the only really new important elec phenomenon?

5/18 cont.

Stein

Hertz's comment really just means that —
Maxwell's eq'ns make perfect sense in abstraction from all models,

→ perspicuous mathematical relations among well-defined physical quantities,

no matter what, if any, "explanation" we give of what goes on —

$\frac{\partial \vec{D}}{\partial t}$ added on p. 456

→ how do we know 'which side' of eq'n to add it to?

→ ∃ so much ambiguity in the model —

~~→ can be seen as to add it as if the displacement is in the direction of E~~

what is r now? since $\partial h/\partial t$ is added into it —

~~of particles moving as electric current~~

curl $\vec{H} = 4\pi\vec{i}$ implies that all currents are steady currents,

since div curl $\vec{H} = 0 = \mathrm{div}\,\vec{i}$

→ can't be true in general, so long as we conceive of conducting currents leading to distribution of charges

→ leads to need for $\frac{\partial \vec{D}}{\partial t}$, since div $\frac{\partial \vec{D}}{\partial t}$ = div $\frac{\partial \rho}{\partial t}$, which is a current

(this only works since we now think of $\vec{i}$ as $\rho\vec{v}$, a flow of charged matter — all we really need is continuity eq'n, $\mathrm{div}\,\vec{i} = -\frac{\partial \rho}{\partial t}$)

⇓

but where did Maxwell get it?

so the displacement current has a magnetic effect

⇓

total current has div = 0, like incompressible fluid, which is how he always talks of it later

but here we would have to have that

→ something ~~about~~ like. the displacement would have to be in same direction as ~~E~~ force

How does Maxwell calculate velocity of light?

he needs density, elastic coeffs. of medium —

from magnetic theory, he gets info about density of medium, since magnetic energy is kinetic energy of swirl

electrostatic energy is from elastic properties of medium

⑤

5/25 cont.

Stein

~~[illegible]~~

terminology of two papers – sign changes

Dyn. Theory		Phys Lines
(p, q, r)	=	(p, q, r)
(f, g, h)	=	$-(f, g, h)$
(F, G, H)	=	$-(F, G, H)$
$(P, Q R)$	=	(P, Q, R)
(α, β, γ)	=	(α, β, γ)
μ	=	μ
e	=	e

Maxwell gets mixed up on signs in eq'n G, p. 561

⇓

leads to absurd conclusions:

(G) $\text{div } \vec{F} = -e$

$\Rightarrow \text{div } \frac{\partial \vec{F}}{\partial t} = \frac{-\partial e}{\partial t}$

but by (H) $-\frac{\partial e}{\partial t} = \text{div } \vec{p}$

but by (A) $\vec{p} = \vec{p}\,' - \frac{\partial}{\partial t}\vec{F}$

and by (C) $\vec{p}\,' = \text{curl } \vec{B}$

so $\text{div } \vec{p} = -\text{div } \frac{\partial}{\partial t}\vec{F}$

~~a contradiction~~ $\Rightarrow \text{div } \vec{p} = 0$

and thus that $\frac{\partial e}{\partial t} = 0$

⇒ all distribution of charge at any pt of space is constant over all time

EM theory of light in 'Dynamical Theory'

→ Maxwell sees that it can be deduced directly from his eq'ns, instead of indirectly as he had done in 'Physical Lines' (though the apparatus was available to him then)

as Faraday's law was usually written

$\frac{\partial \vec{B}}{\partial t}$ = curl $\vec{E}$, we need some convention to allow

$\frac{\partial \vec{B}}{\partial t}$ = electromagnetic units to be expressed as curl $\vec{E}$ = electrostatic units

→ a field that exerts ~~heavens~~ 1 unit of force on 1 em unit of charge only exerts $1/c$ units of force on 1 electrostatic unit

so c e.s.u. = 1 em.u.

$$\Rightarrow \quad 1/c \, \frac{\partial \vec{B}}{\partial t} = \text{curl } \vec{E}$$

and

we want $4\pi \vec{i} + \frac{\partial \vec{D}}{\partial t} = -\text{curl } \vec{H}$, so we want

D measured in emu

$$\Rightarrow \quad 4\pi \vec{i} + 1/c \, \frac{\partial \vec{D}}{\partial t} = -\text{curl } \vec{H}$$

so

consider space free of charges, currents (k and μ = constant)

$$\Rightarrow 1) \; 1/c \, \frac{\partial \vec{B}}{\partial t} = \text{curl } \vec{E} \quad \Rightarrow \quad \frac{\mu}{c} \frac{\partial \vec{H}}{\partial t} = \text{curl } \vec{E}$$

$$2) \; 1/c \, \frac{\partial \vec{D}}{\partial t} = -\text{curl } \vec{H} \quad \Rightarrow \quad \frac{k}{c} \frac{\partial \vec{E}}{\partial t} = -\text{curl } \vec{H}$$

$$3) \; \text{div } \vec{H} = 0$$

$$4) \; \text{div } \vec{E} = 0$$

differentiate 2) wrt t $\Rightarrow \frac{k}{c} \frac{\partial^2 \vec{E}}{\partial t^2} = -\text{curl } \frac{\partial \vec{H}}{\partial t} = -\frac{c}{\mu} \text{curl curl } \vec{E}$

since div $\vec{E} = 0 \Rightarrow$ curl curl $\vec{E} = -\Delta \vec{E}$

(Δ = Laplacian = $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$)

Stein

5/25 cont

Lagrange freed scientists even *more* from the necessity of giving explicit mechanical models of phenomena and Maxwell ~~on~~ understood and exploited the power of this method

→ understand the methodological ramifications of such a formalism

⇓

almost a "phenomenological" account

→ *not* to purge metaphysics, but *only* so as not to go beyond what was *in fact* known, what one has a warrant to claim to know –

He feels warranted, e.g., to make the *categorical* claim that ∃ 2 energy densities, electric + magnetic and the *probable hypothesis* that

electric = elastic energy of strain of medium

magnetic = kinetic energy of medium

(c.f. 'Dynamic Theory' pp. 563-4)

↓

∴ Notice that Maxwell himself just had the latter as a *prob hypothesis*

So he's giving a 'mechanical' treatment of EM field w/o *trying* to give any specifics

→ cf. intro of generalized mechanic quantities in Part II

and how he shows that magnetic field can be represented generally by distribution of currents in wires in space ~~in~~, *constrained* to fixed paths

→ but this is only a *limited* ~~en~~ case of EM

→ he never treats *general* case of unconstrained currents, e.g.

"displacement currents" in space or 3-d currents in electrolytes

↓

he never really shows that the whole system is subsumeable under Lagrangian dynamics, but only that certain specific cases are (1864)

and

he never really got beyond this in the Treatise either (1873)

but in 1878 Fitzgerald published treatment of reflection, refraction, semi-refraction, and found that he could use Hamiltonian system (which is usable w/ ∞ degrees of freedom, as opposed to Lagrange, which is only really applicable to finite degrees)

⇓

and he found what he could use in McCulloch's paper of 184?, that the elastic energy of the medium could be expressed $|\text{curl}\,\vec{d}\,|^2$, $\vec{d}$ = displacement, and erase all difficulties and yield correct formulas for refracted vs reflected waves (viz Fresnel's formulas) and could deal w/ double refraction

→ and Fitzgerald saw that this formulation could be applied to Maxwell's case, where energy density = $\frac{k}{8\pi}\vec{E}\cdot\vec{E} + \frac{\mu}{8\pi}\vec{B}\cdot\vec{H}$

↓

and Fitzgerald showed that using McCulloch like expression for potential energy, and suitable kinetic energy, then the Lagrangian $\mathcal{L} = T - V$, subsumed under principle of least action, satisfies Maxwell's Laws, and allows one to solve reflection and partial refraction, and polarization

Maxwell Stein

5/25

the notion 'field' was very young in science at Maxwell's time

→ introduced ca. 1843 by Thomson as 'Magnetic Field'

↓ a 'field of flowers'

originally more like our 'meadow' or 'area' → thus 'field' for Maxwell was strongly correlated w/ the area in which the activity took place – cf. (4), p. 527 'Dynamical Theory of EM Field'

→ better formulation in (3)

Note that notion of force operative in Maxwell that all there is is force from bodies impinging on each other implies an important notion of energy operative here, that ∃ only kinetic energy, and that 'potential' energy just has to do with changes of forces –

Maxwell's theory inspired some, most notably J.J. Thomson, and Hertz, to ~~the~~ propose that all physics could be reformulated in terms of constraints (as due to Lagrange & D'Alembert) as opposed to forces (and thus do away with potential energy –)

(recall that constraints do no work)

→ Hertz was skeptical about 'ϕal' notion of force (– of course 'constraint' is no clear beauty either –)

⇓

inspired by Maxwell's success ~~£~~ in giving a theory of EM w/ an ether about which no one knew anything about its particular structure, just in terms of generalized ('Lagrange') coordinates and a what were in effect constraints

"Physical Lines" paper was detailed sketch of how such an ether might just be (he was not wedded to it) physically underlying

⇓

but in "Dynamical Theory" he posits underlying medium w/ no express content except

1) field quantities express state of underlying medium (thus they're Lagrangian generalized coords)

2) these quantities are related by laws which can be subsumed under Lagrange's laws of motion, for some system underlying this exposition

(cf. p.184 of packet, p.309 of Maxwell, 'Proof of the Eq'ns of Motion of a Connected System")

⇓

a way to express just what is 'known' or 'have evidence for' about a subject w/o prejudging anything about the specifics of the phenomena

w/o going into "unwarrantable detail" ~~(as he had~~

→ viz. a method which would allow him to not rely on methods like speculation in 'Physical Lines" paper

↓

the way Newton investigated light, w/o prejudicial mechanical hypotheses

→ Lagrange codified some methods for this kind of investigation, but Maxwell wanted even more

5/25 cont.

Stein

⇒ $\frac{k\mu}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = \Delta \vec{E}$ the wave eq'n

simply and directly deducible

↓

great confidence booster to him, that w/ no physical specifics, he could derive that the medium supported waves of speed ~~[illegible]~~, so speed = c = ratio of esu to emu (since k=μ=1) ~~only~~ by only looking at the relations he has derived between these perfectly observable quantities

⇓

and we get extended result of what velocity of light should be in other medium

(which is actually pretty wrong for many indices of refraction, e.g. water

→ but this is no cause for worry, since the eq'ns on which they're based, $\vec{D} = k\vec{E}$ and $\vec{B} = \mu\vec{H}$, k and μ are coefficients of responses to ~~k~~ media of fields. viz to ~~stat~~ steady-state response after initial field upset previous state

and so is an inapplicable approximation to light waves (w/ extraordinary light frequency) interacting w/ matter → no steady state

↓

need an 'effective' value of k and μ, which will be much smaller, since complete polarization, etc., won't have time to set

→ need detailed account of the response of the medium

thus a detailed account of internal structure of dielectrics

→ relation of optical properties of bodies to their electrical properties

⇓

thus, for one thing, good reflectors, opaque, should be good conductors, and they are
(much successful, for complicated reasons, with dielectrics and transparency)

⇓

so Maxwell could see that the diff in predictive power of his theory wrt dielectrics vs metals pointed to very diff internal structures in them leading to electrical phenomena

Maxwell was also able to deduce Fresnel's result on double refraction, by showing that one must write
$\vec{D} = k_{ij} \vec{E}$ where k_{ij} is now a symmetric tensor
⇒ ellipsoid of polarization for $\vec{E}$ inside the crystal

since $\operatorname{Div} \vec{E} = 0 \Rightarrow \vec{E} = \operatorname{curl} \vec{d}$
⇒ energy density $= |\operatorname{curl} \vec{d}|^2$
= M^c Culloch's form, as Fitzgerald shows
(for free field, of course, this could be true of either, since $\vec{H}$ and $\vec{E}$ are perfectly interchangeable)

⇓

still need though interaction ~~energy~~ of field w/ charges and currents, and ordinary matter

What we've learned since Maxwell: not that "there is no ordinary matter/ether in free space", but that free space has some of the properties of ordinary matter, not others, wrt em phenomena

5/25 cont.

Stein

cf. eqn's (5), p. 566 "Dynamical Theory"

⇓

Maxwell gets it wrong, coz of his conception that "free space" is really filled w/ "matter in motion"

free space displacement current differs from ordinary current in that they effect $\vec{H}$ through Maxwell's eq'n, but magnetic field "exerts no force on it", like it does matter currents

→ still needed to separate $\vec{D}$ into $\vec{E}$ and $\vec{P}$

⇓

Further clarification was needed, to distinguish the relations which constitute the field quantities from the non-field effects which the field produces

but Maxwell did ~~not~~ deduce ~~the~~ that light would exert pressure on non-transparent body

→ completely new result for wave-theory of light

⇓

but if ∃ an ether, then what does the body exert a counter-force on as Newton's law demands?

→ need a conception of the momentum of the field itself in abstraction from "ordinary matter/ether"

Maxwell conceived of it as forces exerted by the stressed and moving medium

Lorentz, 1892 paper,

begins the real attack on EM phenomena for moving bodies

↓

only $\vec{E}$ & $\vec{B}$ characterize fields through all space, and that $\vec{D}$ & $\vec{H}$ are "artifacts," approximate quantities produced by polarization, but not fundamental quantities, produced by interaction of fields w/ ordinary matter

→ charges are associated w/ particles having mass and currents are just motions of these particles

↓

then interactions of fields and matter are just the interaction of the "true" fields and the particles/currents of matter

$$\Rightarrow \quad \vec{F} = \rho\vec{E} + \rho\vec{v}\times\vec{B}$$

→ but no reaction force here, which was seen as a problem for Lorentz {though very successful at accounting for other EM, mechanical, optical and even heat phenomena} → so why should it be a concern? why should we not simply think that conservation of momentum ~~& energy~~ applies only to particle mechanics, and we've moved past this — {the problem is not action-at-a-distance, cuz we think that EM phenomena propagate at finite rate} → this is why we seem pushed to save conservation of momentum → from where does the momentum come that the light carries off from the candle?

⇓

the source transfers momentum to the field which propagates in time, but not "Newtonian" momentum ⇒ it's the flow of energy in the field, the Poynting vector, $\vec{E}\times\vec{H}$ → that it completely served to conserve momentum in light/matter interactions → pointed out by Poincaré — who thought it completely uninteresting, w/o physical content

Lorentz dropped the notion of "motion of ether" entirely
→ thus the notion of "frame of reference of ether" → cuz things will be diff if you are in motion wrt ether → Lorentz takes Maxwell eq'ns to hold everywhere w/ respect to state of motion since he explicitly says "globally ether is at rest" → almost = distinguished motion drops out, but not quite, since distinguished state is frame of ether rest → but then how to ascribe energy flow etc. to it?

→ Einstein says doesn't even make sense to say Lorentz's ether is "at rest" since it will be same for all observers → but still problem of coherence the notions of "energy flow"? → related to earlier pt that "ether" has some properties of ordinary matter (energy flow) but not others (~~no~~ state of motion at pts) → Maxwell's stress tensor of EM field turns into stress-energy tensor of relativistic theory → measures momentum flow, but w/o real ind. notion of force → a new concept of "force" – internal redistribution of momentum in the field w/o associated accelerations

→ final move away from Newton → basic reformulation of notions of mechanics)

⇓

fields are elementary constituents of nature w/o anything underlying to support them