CHRISTIAAN HUYGENS

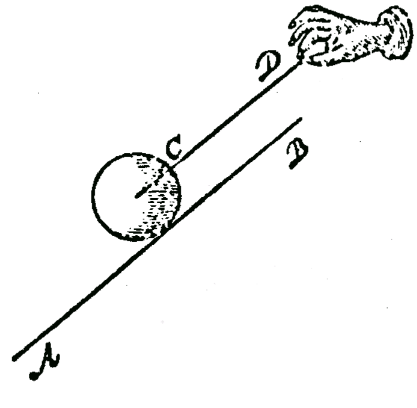
ON CENTRIFUGAL FORCE

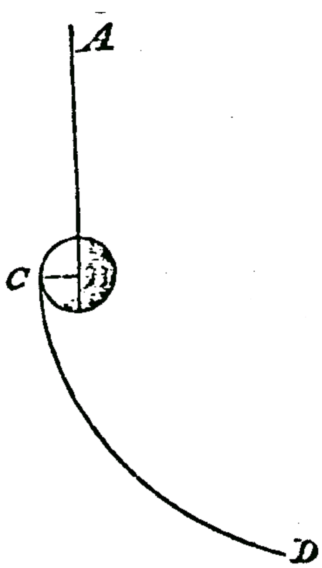
(*De vi centrifuga*, in *Oeuvres complètes*, Vol. XVI, pp. 255-301)
Translated by M.S. Mahoney

Heaviness is a tendency to fall [*Gravitas est conatus descendendi*]. By positing that heavy bodies falling either perpendicularly or along inclined planes are moved with an acceleration such that in equal times equal moments of speed are acquired, one may then most certainly demonstrate that the distances traversed in various times from rest are to one another as the squares of the times. And the latter [result] agrees exactly with experience. Hence, it holds that the former [supposition] is correctly assumed. The experiments of Galileo and Riccioli prove to us that it agrees exactly, unless the resistance of the air causes some slight aberration; but this is so much the smaller as the bodies contain greater weight in relation to surface size and also as we make the trial over shorter distances. Whence it is altogether believable that, unless the resistance of the air poses an obstacle, the same ratio will be most perfectly observed even over the most vast distances. Now, however, just as this [resistance of the air] causes a sphere of cork to reach after a short time a point from which it thereafter continues to fall at a uniform speed (which is also necessarily true of a lead sphere so reduced that it has the same surface in relation to its weight as has the cork, i.e. the diameter of which is to the diameter of that [cork sphere] as the specific weight of cork to the [specific] weight of lead, as I have shown elsewhere), so too I judge that if a sphere of lead, however large, were to continue to fall through the air it would also finally arrive at a uniform motion, albeit after having traversed an immense distance, such that the ratio of acceleration would no longer be valid and consequently would not in fact be maintained with such complete precision. Nonetheless, Galileo's theory concerning this motion should not therefore be thought less outstanding and useful, no more, by Hercules, than all mechanics that treats of weights [should be thought so], because in it one usually assumes falsely that heavy bodies tend to fall along parallel lines, which [bodies] in fact incline toward the center of the earth. But for the demonstration of the things that we will treat here it suffices that over any very small distances from the point of rest acceleration [*sic*] grows according to the odd numbers 1, 3, 5, 7, as Galileo has established.

Thus, when a heavy body is suspended from a string, then the string is pulled, since the heavy body tends to fall away along the line of the string with an accelerated motion of this sort.

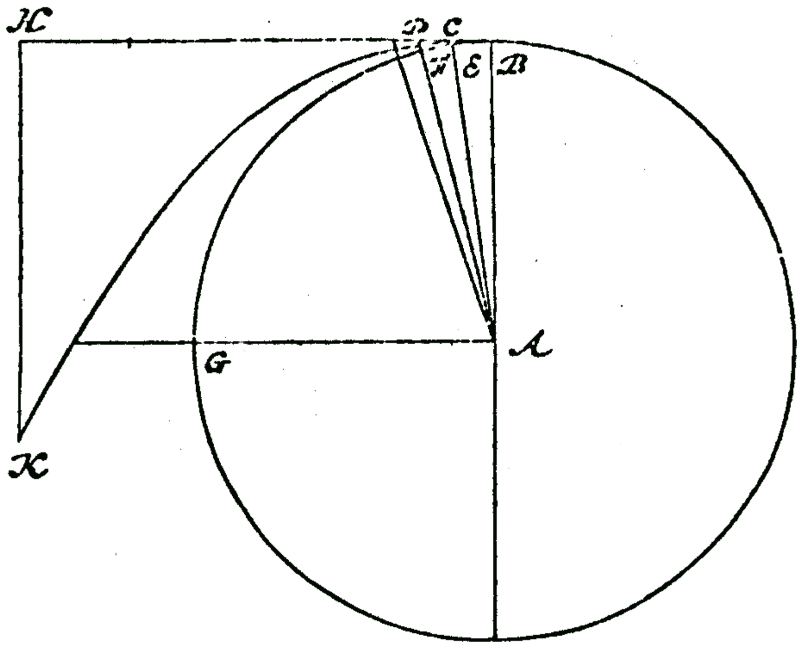
More or less space can, however, be traversed in the same time by an accelerated motion following the said progression; for example, when a heavy body on plane AB is supported by a string CD parallel to this plane. Now here too the heavy body tends to move along line DC with a similarly accelerated motion, but not such that it traverses in a certain particle of time the same space it would traverse in the same particle if it were hung from a perpendicular string. Whence also one feels less a tendency here; that is, it is as much less than that other perpendicular tendency as the heavy body would traverse less space in the same time on the inclined plane than along the perpendicular.





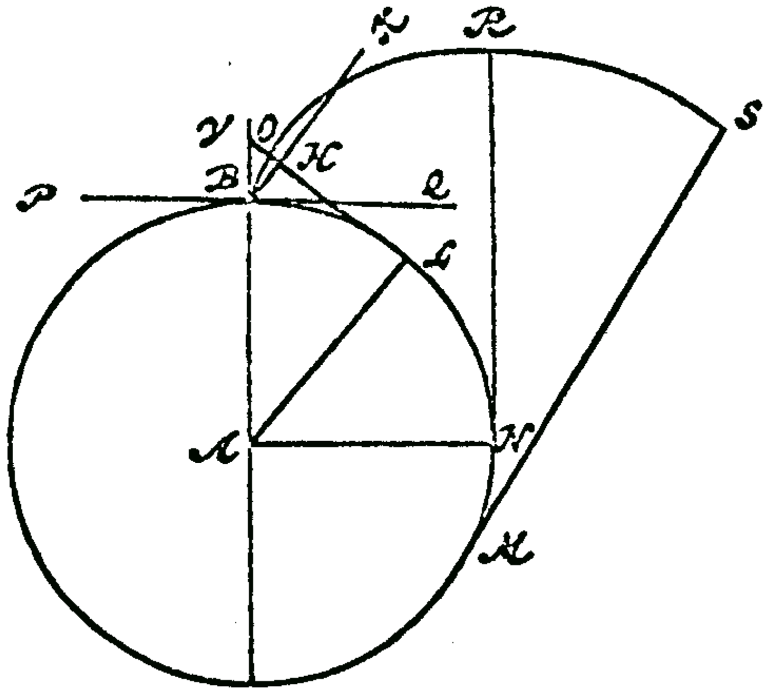
Furthermore, whenever two bodies of equal weight are each restrained by a string, if they have a tendency to fall away along the extension of the string with the same accelerated motion and such that equal spaces are traversed in the same time, we have assumed that one will feel equal pull [*attractio*] on these strings, whether they are pulled downward, or upward, or in any direction whatever. It makes no difference from what cause a tendency of this sort arises, so long as it is present. But the same tendency is present if, given the possibility, or when the tendency is not inhibited, the same motion results. [*Adest autem idem conatus, si data facultate seu non inhibito conatu, idem circa motum continget.*] And that should be considered only at the beginning of motion, in taking some very small part of time. Now, for example, if ball B were to hang from string AB and were to touch on the side the hollow surface CD, such that a line drawn from the center of the sphere B to [the point of] contact were perpendicular to both the string AB and the tangent to the curve, we know then that the globe would be in no way supported by the surface CD, but that cord AB would be pulled equally strongly as [it would] if it did not touch plane CD but were hung freely. But yet, if it were separated from the cord and fell, it would not descend in the same way as [it would] if, freely suspended, it fell away from the cord, but, rolling along surface CD, it would not even accurately observe the proportion of acceleration according to the odd numbers 1, 3, 5, 7. Thus it is clear that one should not consider what happens to the heavy body sometime after separation from the cord, but consider some very small particle of time from the outset of motion, if we want to determine the strength of the tendency.

And here body B, after separation from the cord, begins to move in such a way as if it had fallen perpendicularly, since at the beginning it has the determination of motion that is along straight line AB, as this is parallel to the tangent of the curve at C. Now let us see what [sort of] tendency and how great a tendency belongs to bodies attached to a string or a wheel that revolves, such that they recede from the center.



Let BG be a wheel that rotates parallel to the horizon about center A. A small ball attached to the circumference, when it arrives at point B, has a tendency to proceed along the straight line BH, which is tangent to the wheel at B- Now, if it were here separated from the wheel and flew off, it would stay on the straight path BH and would not leave unless it were pulled downward by the force of gravity or its course were impeded by collision with another body. At first glance it indeed seems difficult to grasp why the string AB is stretched so much when the ball tries to move along the straight line BH, which is perpendicular to AB. But everything will be made clear in the following way. Let us imagine some very large wheel, such that it easily carries along with it a man standing on it near the circumference but so

attached that he cannot be thrown off; let him hold in his hand a string with a lead shot attached to the other end of the string. The string will therefore be stretched by the force of revolution in the same way and with the same strength, whether it is so held or the same string is extended to the center at A and attached there. But the reason why it is stretched may now be more clearly perceived. Take equal arcs BE, EF very small in comparison to the whole circumference, say hundredth parts or even smaller. Therefore, the man I spoke of [as] attached to the wheel traverses these arcs in equal times, but the lead would traverse, if it were set free, straight lines BC, CD equal to the said arcs, the endpoints of which [lines] would not, however, exactly [*omnino*] fall on the straight lines drawn from center A through points E, F, but would lie off these lines a slight bit toward B. Now it is clear that, when the man arrives at E, the lead will be at C if it was set free at point B, and when he arrives at F it will be at D. Whence we say correctly that this tendency is in the lead.
But now if points C, D were on the straight lines AE, AF extended, it would be certain that the lead tended to recede from the man along the line drawn from the center through his position; and indeed such that in the first part of the time it would move away from him by the distance EC, and in the second part of the time it would be distant by the space FD. But these distances EC, FD, etc. increase as the series of the squares from unity, 1, 4, 9, 16, etc. Now they agree with this series ever more exactly as the particles BE, EF are taken to be smaller, and hence at the very outset they may be considered as if they differed nothing.



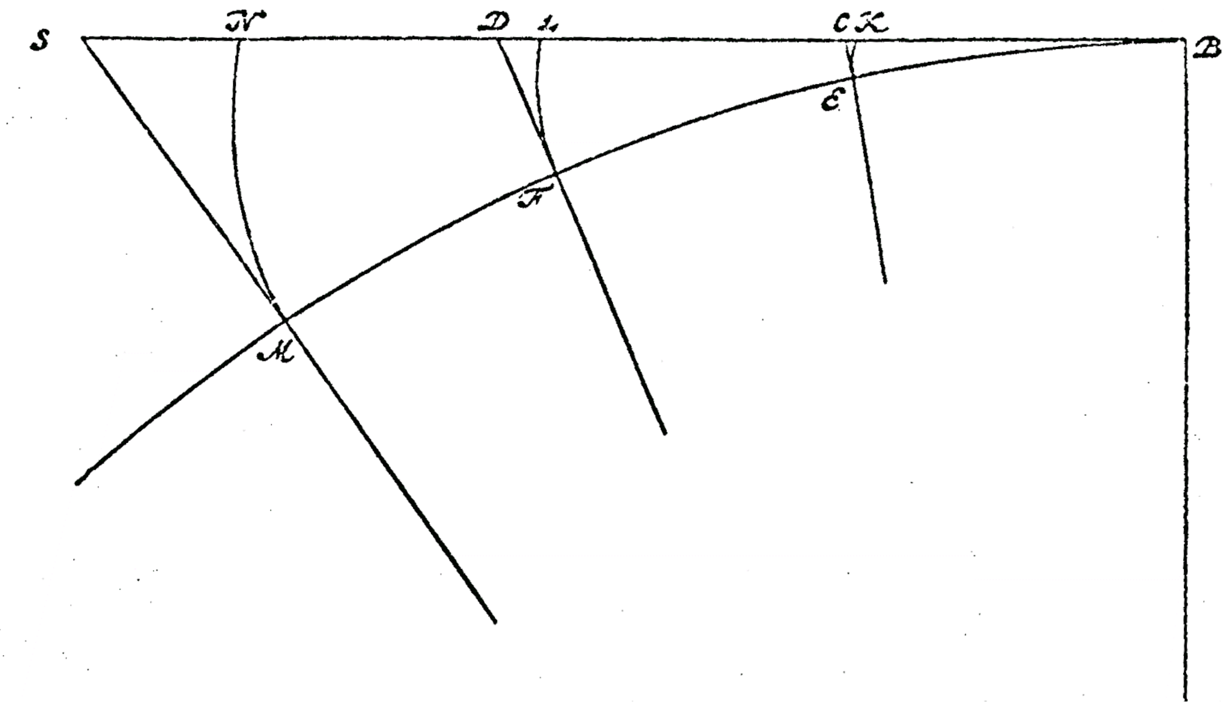
Thus this tendency will clearly be similar to that which is felt when the ball is held suspended on a string, since then too it tends to recede along the line of the string with a similarly accelerated motion, i.e. such that in a first certain period of time it will traverse 1 interval [*spatiolum*], in two parts of time 4 intervals, in three 9, etc. Such, therefore, would be the case if points C, D were on lines AE, AF produced. Now, however, because they lie off the aforesaid lines slightly toward B, it happens then that the ball tends to recede from the man not along the straight line coming from center A, but along some curve that is tangent to the straight line at the point where the man is standing. That is, if plane PQ is tangent to the wheel at B and is attached to it and turns along with it, ball B, if it is separated from the said wheel or plane, will describe with respect to the same plane and the point B, both of which also continue to move, the curve BRS, which is tangent at B to the radius AB moved at the same time and produced. If we wish to describe this curve, we need only place a string around circumference BNM and move its endpoint B toward RS such that the part that leaves the circumference BNM always remains taut [*extensa*]; for by this motion it will describe the said line BRS with its endpoint, which is easy to show. Moreover, the property of this line will be that, if to some point of the circumference, say N, a tangent is drawn which meets the curve at R, this [tangent] NR is equal to arc NB; this is clear from the way the curve is generated. But one must show that the curve and the straight line AB are mutually tangent at B. Let NR be a tangent to the curve, parallel to AB. It is certain that the whole part

BR of the curve lies between the parallel lines AB, NR. For, if on it some point, say 0, is taken, through which is drawn a tangent VOL to the circumference, LO will be equal to arc LB and hence less than the tangent of the same arc, which is LV; whence it is necessary that point O fall between V and L. And the same can be shown for any point taken on BR.

Now, if curve BR is said not to be tangent to straight line BV at B, then some straight line BK can be drawn from B, inclined to BV at an angle small enough such that it does not cut curve BR. Let it be BK. Draw radius AL parallel to BK, and let LH be perpendicular to the same BK, and hence also to AL. LH is therefore equal to the sine of arc BL and therefore less than this arc. But the straight line LHO, lying between the point of contact L and the curve BR, is equal to this arc. Therefore, some part of curve BR, on which point O lies, falls within angle VBK, no matter how small it is supposed to be. Whence it is manifest that the straight line BK cuts the curve and, finally, that it is tangent to the curve at B.

Since, therefore, the ball moving around with the wheel tends to describe a curve with respect to the radius in which it is located, and one, in fact, that is tangent to the radius, it is clear that the string to which it is attached should be stretched no otherwise than if the ball tended to move along the radius itself produced.

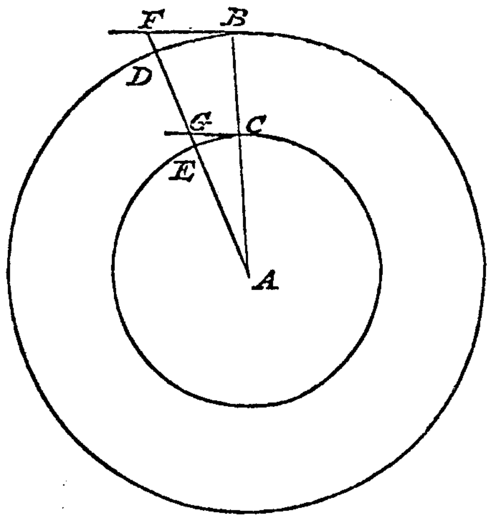
Furthermore, the distances that the ball would traverse on the said curve in uniformly increasing times are as the series of squares from unity, 1, 4, 9, 16, etc., providing that we consider the beginning of motion and minimal distances. This is shown by the next figure, where the equal arcs taken on the circumference of the wheel are BE, EF, FM; and the straight lines on tangent BS and equal to the said arcs are BK, KL, LN; and the lines from the center are EC, FD, MS. Here then, if the ball were to pull away at B from the rotating wheel, then when point B arrived at E the ball would be at K and would have traversed a particle EK of the curve described above. Then, after the second set [*exactum*] time, when B has arrived at F, the ball will be found at L and will then have traversed part FL of the curve. And similarly, when B has arrived at M, the globe will have traversed portion MN of the curve. But these parts of the curved line should be considered, at the beginning of the separation of the ball from the wheel, as if they were the same as the straight lines EC, FD, MS, to which they are tangent, since the arcs from point B can be taken small enough that the differentials [*differentiola*] between these straight lines and curves have a ratio to their lengths smaller than any imaginable ratio.



Consequently, the distances EK, FL, MN should also be looked upon as increasing according to the series of squares from unity, 1, 4, 9, 16. And so the tendency of a ball restrained by a rotating wheel will be in no way different than if it tended to proceed along the straight line drawn through it from the center, that is, with an accelerated motion by which in equal times it traverses increasing distances according to the numbers 1, 3, 5, 7, etc. For it suffices that this progression be observed at the beginning; for, although it may be moved afterwards by some other ratio of motion, that does not in any way affect the tendency before motion has begun. But the tendency of which we have been speaking is clearly similar to that by which heavy bodies hanging on a string tend to fall. Whence we further conclude that the centrifugal forces of moving bodies that are unequal but are moved in equal circles at equal velocities are to one another as the weights, or solid quantities, of the moving bodies. For, just as all heavy bodies tend downward at the same speed of fall and with similarly accelerated motion, and this tendency of theirs has greater moment according as they are greater, so too it should turn out for those [bodies] that tend to move away from the center; their tendency has been clearly shown to be similar to the tendency arising from gravity. But, while the same ball always has the same tendency to fall whenever it is suspended on a string, the tendency of the ball carried around on a wheel is, on the contrary, greater or less according as the wheel turns more quickly or more slowly. It remains that we inquire after the magnitude or quantity of each tendency at different speeds of the wheel. And first we will investigate at what speed it is necessary to rotate the proposed wheel in order that the ball stretch its string with the same strength as when it is suspended perpendicular from it.

PROPOSITION I

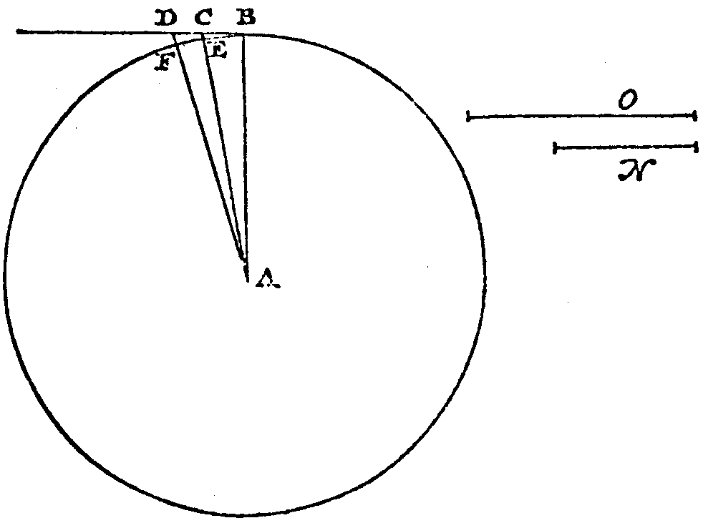
> If two equal moving bodies traverse unequal circumferences in equal times, the centrifugal force in the greater circumference will be to that in the smaller as these circumferences, or their diameters, are to each other.



Let AB, AC be the radii of circles on which two equal moving bodies revolve in equal times. In both circles take similar very small arcs BD, CE, and on the tangents at points B and C take BF, CG each equal to its arc. Thus the moving body carried around in circle BD has a tendency to recede from the center along the extension of its string with a naturally accelerated motion, and by that motion to traverse distance DF in a certain part of time. Moreover, [the body] moving around in circle CE has a similar tendency to recede from the center, but one by which in that same part of time it covers distance EG. Thus, by as much as DF is greater than EG, so the string in the greater circle is pulled with a greater force than [that] in the smaller [circle]; but, it is clear that FD is to GE as BF to CG, i.e. as B~ to AC. Therefore the centrifugal force in the greater circumference is to that in the smaller, as these circumferences, or their diameters, are to each other. Q. E. D.

PROPOSITION II

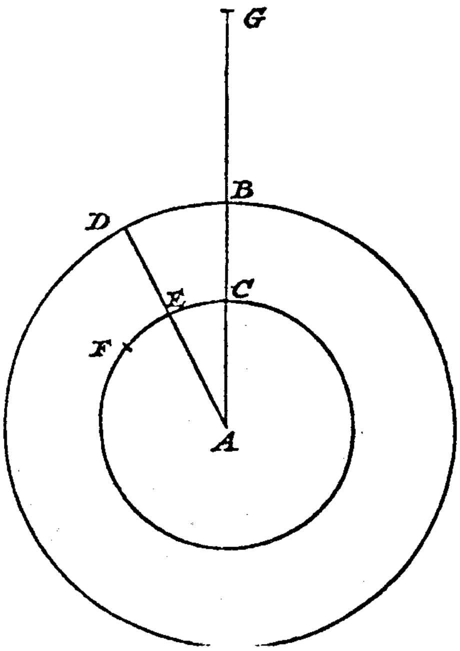
If equal moving bodies revolve in the same or equal circles or wheels at unequal speeds, but both with a uniform motion, the force of recession of the faster body from the center will be to the force of recession of the slower [body] in the duplicate ratio of the speeds. That is, if the strings by which they are restrained are drawn downward through the center of the wheel and support weights by which the centrifugal force of the moving bodies is held in check and exactly counterbalanced [*adaequetur*], these weights will be to one another as the squares of the velocities.



Let there be a circle with center A and radius AB, in the circumference of which a first moving body is moved at a slower speed represented by line N, and then another at a greater speed O. When now one takes very small arcs BE, BF, which are to one another as N to 0, it holds that, in the same part of time in which the slower moving body passes through arc BE, the one that is faster will have traversed arc BF. Let BC, BD taken on the tangent be equal respectively to the arcs BE, BF. Thus it will also hold that each body has a tendency to recede from the center along the extension of its cord with an accelerated motion, but by which motion the moving body that is moved more slowly will have receded from the point of the circumference it occupied by the amount of the distance EC, while that which is faster [will have moved] in an equal time through the distance FD. By as much, therefore, as DF is greater than CE, so much more strongly does the faster body pull than the slower. And, since we have taken arcs BE, BF minimally small, the ratio of DF to CE should be considered the same as that of the square of DB to [the square of] CB, in accordance with what we set forth shortly be fore; as DB is to BC so arc FB is to [arc] BE, i.e. as O to N. Therefore the square of O will be to the square of N as FD to EC, and con sequently as the centrifugal force of the faster body to the force of the slower. Q.E.D.

PROPOSITION III

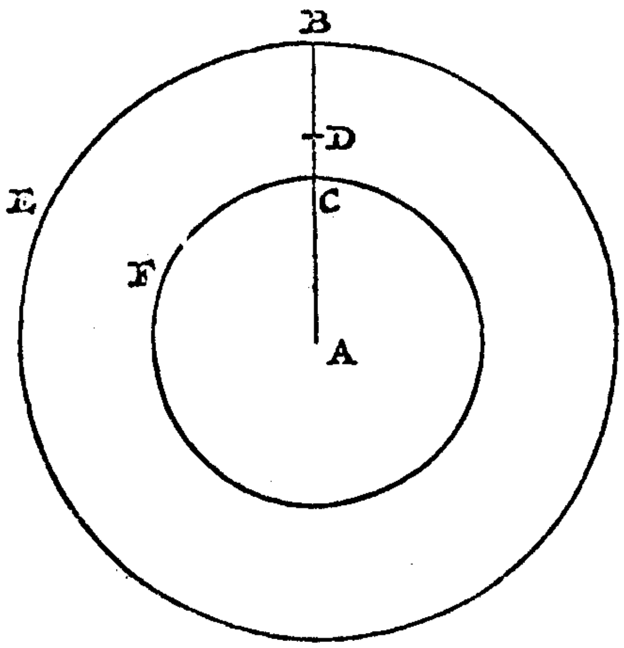
If two equal moving bodies are moved at equal velocities in unequal circles, their centrifugal forces will be in the inverse ratio of the diameters, so that the said force is greater in the smaller circumference.



Let AB, AC be the radii of unequal circles about the same center A, and let equal bodies be moved in their circumferences at equal speed i.e. such that in the same time that some arc BD is traversed in the greater circumference an arc CF equal to this BD in length is traversed in the smaller circumference. I say that the centrifugal force of tile body moving around in circumference BD will be to that which the body moving around in circumference CF has as radius AC to [radius] AB. Draw radius AD, which cuts the smaller circumference at E, and let AG be the third proportional to the two AC, AB. Imagine further some moving body equal to either of the two [given bodies] to be carried around in circumference CF at a speed such that it traverses arc CE in the same time that the two others [traverse] arcs BD and CF. The speed, therefore, of this assumed body will be to the speed of either one of those as arc CE to arc BD, i.e. as AC to AB. And the centrifugal force of the body traversing arc BD will be to the force of the assumed body, which in the same time traverses arc CE, as BA to AC. But the centrifugal force of the assumed body will be to the force of the body that in the same time traverses arc CF in the duplicate ratio of AC to AB, i.e. it will be the same as AC to AG, since we have shown their speeds to be as AC to AB. Therefore *ex aequali* the centrifugal force of the body that traverses arc BD will be to the force of that which in the same time traverses the equal arc CF as BA to AG, i.e. as AC to AB. Q.E.D.

PROPOSITION IV

If two equal moving bodies carried around in unequal circumferences have equal centrifugal force, the time of revolution in the greater circumference will be to the time of revolution in the smaller in the subduplicate ratio of the diameters.

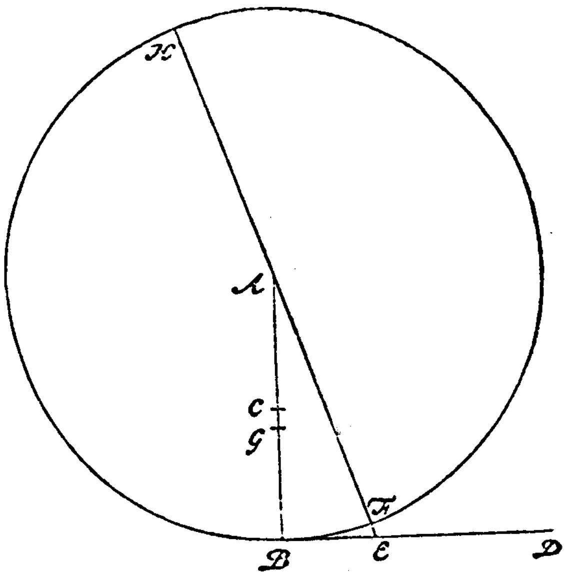


Let BE, CF be unequal circles about the same center A, of which AB, AC are the radii, and in each let a body revolve such that the centrifugal force in each is the same. I say that the time in which the circumference of circle BE is traversed is to the time in which circumference CF is traversed in the subduplicate ratio of AB to AC, i.e. as BA to the mean proportional AD between AB, AC. For, if one imagines a third moving body equal to these others, which traverses circumference CF in the same time that the other covers circumference BE, the centrifugal force of the assumed body will be to the force of the latter as AC to AB. But the centrifugal forces of the first two bodies are assumed to be equal; therefore, the centrifugal force of the assumed body will also be to the force of the body assumed to run in circumference CF as AC to AB. But the centrifugal forces of bodies moved in the same circumference are in the duplicate ratio of the velocities. Therefore, the velocity of the assumed body will be to the velocity of the one that was first posited to revolve in circumference CF as AC to AD, or as AD to AB. But the times of motion along the same circumference correspond in inverse ratio to the velocities. Therefore, the time of revolution of the assumed body, to which the time of revolution of the body moving along circumference BE is equal by hypothesis, will be to the time of revolution of the body that in the beginning was said to move in circumference CF as AB to AD. Q.E.D.

PROPOSITION V

If a body is moved in the circumference of a circle at the speed that it acquires by falling from a height equal to the fourth part of the diameter, it will have a tendency to recede from the center equal to its weight, i.e. it will pull the string by which it is restrained with equal strength as when it is suspended from it.

Let A be the center, AB the radius, of a circle parallel to the horizon, in the circumference of which a body is carried with uniform motion, and at the velocity that it would acquire by falling perpendicularly from a height equal to one half of AB, i.e. CB. I say that the cord by which the body is detained will be pulled by the centrifugal force with equal strength as [would pull it] if the body were suspended freely from the same cord.

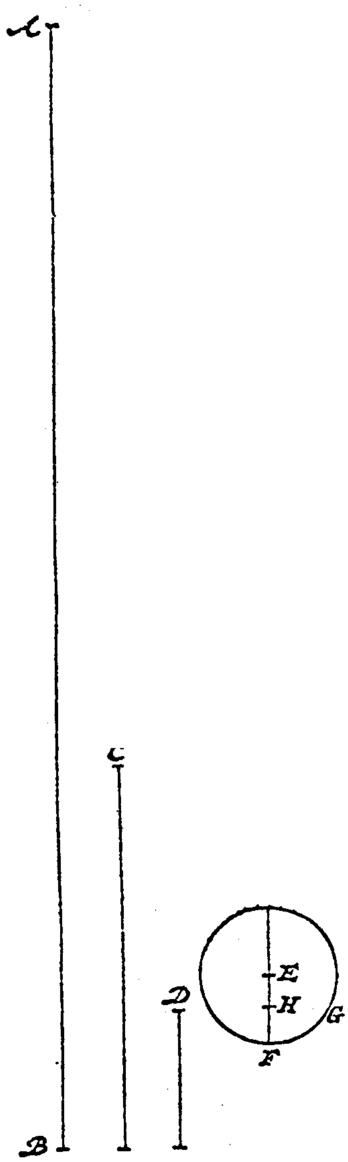


Let the tangent BD to the circle I be equal to the radius AB. Since, therefore, the body moves in the circumference of the circle at the speed that it acquires by falling from height CB, i.e. by which it would traverse with a uniform motion a distance BD twice BC in a time equal to that in which it fell through CB, it follows that, if it is set free at B, it will traverse with a uniform motion the said distance BD in the said time. Take some very small part BE of this BD, and draw through the center the straight line EAH, cutting the circumference at F. Further, let the square of DB be to the square of BE as BC to CG in length. From this, therefore, [it follows] that, if we suppose the time in which it falls with accelerated motion through CB to be represented by line BD, BE will be the time of accelerated motion through CG. But the same BD will also be the time in which it traverses this BD with the uniform motion it has [while] moving in the circumference. Now this time is by hypothesis equal to the time of accelerated motion through CB. Thus, too, BE will be the time in which it traverses this distance BE at the speed of revolution that it has. Whence it holds that in an equal time distance CG is traversed with a motion accelerated from rest, and distance BE with a uniform motion at the speed that the body has been posited to have [while] moving in the circumference. It holds further that, if the body is released at B, it will arrive at E with a uniform motion at the same time that the point B of the circumference reaches F. For, the straight line BE should be considered equal to this arc BF, in that BE is imagined to be infinitely small. Thus we say that there is in it a tendency to recede from point B with a naturally accelerated motion (for it has been shown to be such) through the distance FE in the same time in which it would traverse distance BE with a uniform motion at the speed of its revolution, i.e. in the time in which it would traverse distance CG with a motion accelerated from rest. Wherefore, if it be shown that the distances CG and FE are equal, it will hold that the tendency of the suspended body to fall with an accelerated motion is clearly equal to the tendency of the same body to recede from its string with a similarly accelerated motion when it is moving in a circumference, since the tendency to accelerated motions is the same when equal distances will be traversed by the motions in equal times.

And that CG, FE are equal is shown thus: HE is to EB as EB to EF, and thus the square of HE is to the square of EB as HE to EF in length. Whence, if the subquadruples of the antecedents be taken, the square of AF will be to the square of EB as the fourth part of HE, which should be considered equal to 1/411F, or BC, is to FE. But the square of AF is to the square of BE, or the square of DB to the square of BE, as BC to CG in length, by construction. Therefore, BC will be to CG as the same BC to FF, and thus FE, CG are equal to each other; whence the proposition holds.

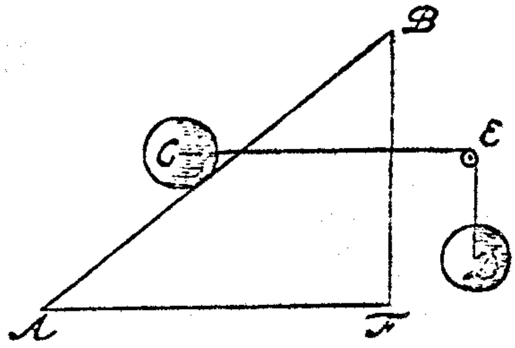
PROPOSITION VI

Given the height that a moving body traverses in a certain time, say a second, in falling perpendicularly from rest, to find the circle in the circumference of which a body moving around horizontally and completing its revolution also in a second has a centrifugal force equal to its weight.



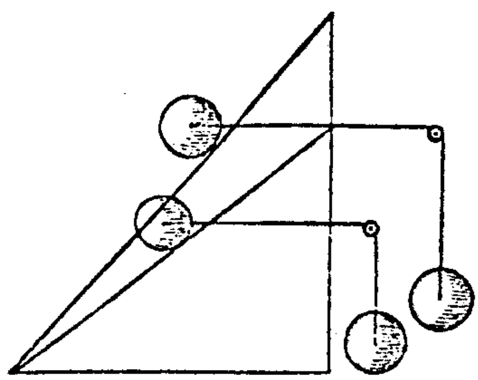
Let there be given a height AB, along which glides a body falling from rest in the time, e.g., of one second. Let the circumference of the circle be to its diameter as AB to the line C, and as the latter to a third [line] D. Draw circle EFG with a diameter equal to this D. I say this [circle] is the one that was demanded. For divide the radius EF in two at H. Then, if the body moves in circle FG at the velocity it acquires in falling from height HF, and with a uniform motion, it will have a centrifugal force equal to its weight. If, then, we show only that the whole circumference of FG is traversed once at the said velocity in the time of one second, then it will hold that circle EFG satisfies the proposition. It holds that the body, with a uniform motion and at the speed that it has acquired at the end of the fall through HF, will traverse a distance twice this HF in the same time in which it fell through HF. If, therefore, at the said acquired speed it is moved with uniform motion through the circumference of FG, the time in which it completes it will be to the time of fall through HF as the circumference of FG to twice HF, or to EF. And, if the doubles of the consequents are taken, the time of uniform motion through the circumference of FG will be to the double time of fall HF, i.e. to the time of fall through D (for D is four times IIF), as the circumference of FG to twice FE, or to D; that is, as C to D (for C is necessarily equal to this circumference of FG); that is, as AB to C. But AB is to C as the time of fall through AB, i.e. the time of one second, to the time of fall through D, since AB to D is the duplicate ratio of that of AB to C. Therefore, the said time of uniform motion through the circumference of FG will be to the time of fall through D as the time of one second to the same time of fall through D. Therefore the said time through the circumference of FG will be equal to the time of one second, which it was necessary to show. Since calculation shows that the height AB that a falling body traverses in one second is 15 Rhenish feet, 7 1/2 thumbs, and since AB is to C as the circumference to the diameter, i.e. as 22 to 7, according to Archimedes, and also as C is to D, or the diameter of circle FG, this diameter will be approximately 19 ounces, of which the half is 9 ounces, 6 lines. Thus, if some body completes in the time of one second each of its revolutions in a circumference, the radius of which is 9 1/2 ounces, the centrifugal force will be equal to its weight.

LEMMA I



If weight C is supported on inclined plane AB by a freely hanging weight D, and cord CE is parallel to the horizon, the weight of D will be to the weight C as the perpendicular BF to the base FA. This holds from the Mechanics. From this [it follows] that, if BF is assumed equal to FA, the weight D should be equal to this C.

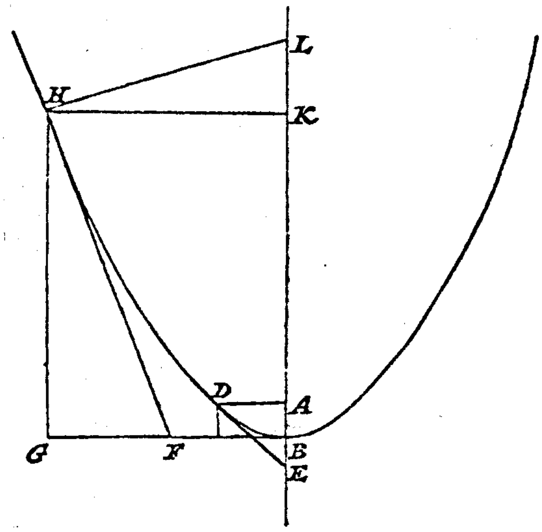
LEMMA II



If equal weights are supported on differently inclined planes, [being] retained along lines parallel to the horizon, the supporting powers will be to one another as the tangents of the angles at which the planes are inclined to the plane of the horizon.

PROPOSITION VII

On the curved surface of a parabolic conoid, which has its axis erected perpendicularly, all revolutions of a body traversing circumferences parallel to the horizon, be they large or small, are completed in equal times, which times are each equal to two oscillations of the pendulum of which the length is one-half the latus rectum of the generating parabola.

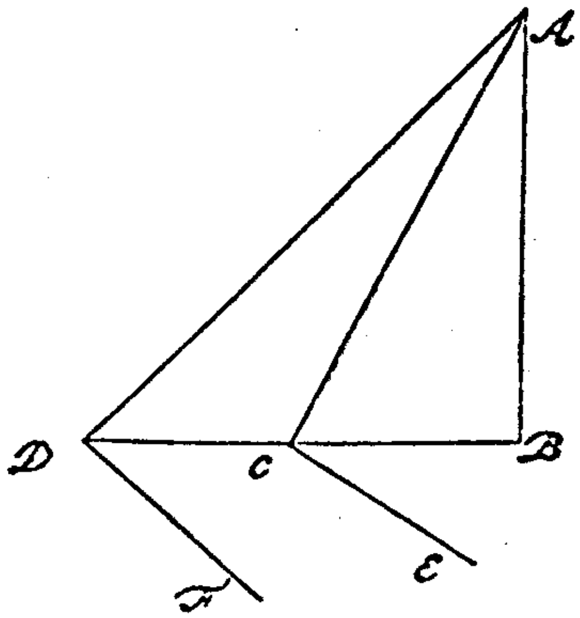


Let HDB be a parabola, the revolution of which about axis BK forms a parabolic conoid. In that axis I take BA equal to 1/4 of the latus rectum; the ordinately applied [line] AD will be equal to one-half the latus rectum. Suppose also that a body at D revolves around the axis AB at such a velocity that the centrifugal force is equal to its weight. This force will, therefore, since the angle ADE is semiright, support the body at point D. And, if the body is rotated elsewhere, say at H, with center K and radius KH, the centrifugal force by which it is sustained at point H will be equal to the force by which the body could be held on the plane HF tangent to the paraboloid by the straight line HK parallel to the horizon. But, by the first lemma, this force will be to the force of gravity as HG to GF, or, by similar triangles (since HL is supposed normal to HF), as HK to KL, or as HE' to AD, since by the nature of the parabola KL is always equal to one-half the latus rectum. Therefore, the centrifugal force by which the rotating body is held at H is to the weight of the body, or to the centrifugal force at D, as HK to DA. Whence, by the converse of the first [proposition], they complete their circumferences in the same time.

Moreover, the time in which the revolutions are completed will be determined thus. Since we have supposed the body D to be rotated such that it has a centrifugal force equal to its weight, it will be rotated at the velocity that it would acquire by a perpendicular fall from one-half AD. But at that velocity it would, in the time of this descent, traverse line DA with a uniform motion. Therefore, the time of revolution is to the time of descent through one-half DA as the circumference of the circle to the radius DA. But the time of a very small oscillation is to the time of perpendicular fall from one-half the height of the pendulum as the circumference of the circle to the diameter, and thus the time of two very small oscillations of the pendulum DA is to the time of perpendicular fall from one-half the height DA as the circumference of the circle to the radius, i.e. as the time of the whole revolution to the same time of perpendicular fall from one-half DA. The time, therefore, of revolution in a parabolic conoid is equal to the time in which two oscillations of a pendulum are completed, the length of which is DA, one-half the latus rectum of the generating paraboloid.

PROPOSITION VIII

If two moving bodies suspended from unequal strings are revolved, such that they traverse circumferences parallel to the horizon while the other end of the string is held fast, and tile axes or altitudes of the cones whose surfaces are described by the strings in this motion are equal, tile times in which each body completes its circle will also be equal.

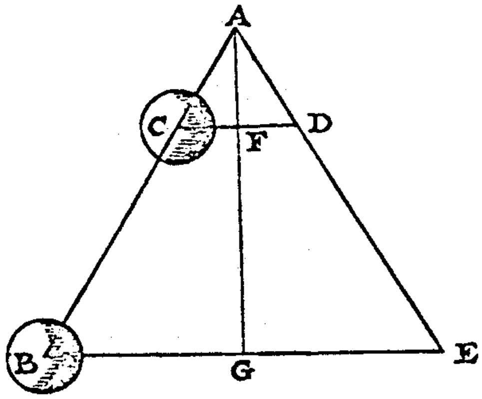


Let AC, AD be strings joined at a common vertex A, and let there be moving bodies attached at C and D respectively, which are revolved in horizontal circles of which the radii are BC, BD. Moreover, let AB be the same axis for both cones described by strings AC, AD in their revolutions. I say that the times of revolution are equal to one another. Posit first that the moving bodies are equal. Let CE be perpendicular to AC and DF perpendicular to AD. It holds therefore that there is a centrifugal force of the bodies that supports the strings extended obliquely in such a way. And, since the body C has by its weight the same tendency to fall as [it would] if it lay in plane CE, and the centrifugal force by which it tends to recede from axis AB along BC inhibits this tendency of weight, it is necessary that the said centrifugal force be equal to the power by which body C would be supported on the inclined plane CE by a line BC parallel to the horizon. For the same reason it is necessary that the centrifugal force by which body D is supported be equal to the power by which this same [body] would be supported on plane DF also by a straight line parallel to the horizon. But this latter power is to that former one, which has been said to sustain body C, as the tangent of angle DAB to the tangent of angle CAB, i.e. as DB to CB. Therefore, the centrifugal force that body D has in its circle will also be to the centrifugal force of body C in its circle as radius DB to radius CB. Whence, from the converse of Proposition I, it follows that the times of revolution are equal.

If, however, the moving bodies are unequal, the equality of the times applies nonetheless. For, if, for example, body C is assumed to be heavier than it was formerly, it will, according as it is heavier, require a correspondingly greater power by which it is sustained on the inclined plane CE by a line parallel to the horizon, and consequently will require a correspondingly greater centrifugal force. But, in order to have that force, it must traverse the circle in the same time in which it was assumed [to traverse its circle] earlier, when it was lighter, as is clear from what we have said above. Therefore the proposition holds.

PROPOSITION IX

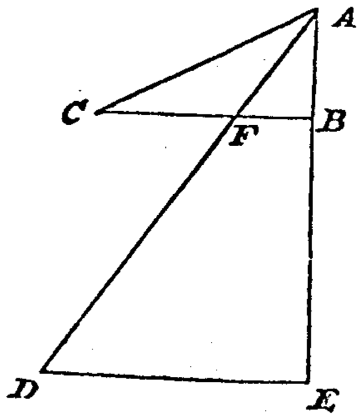
The times of revolution along horizontal circles CD, BE, given the same angle of gyration CAD, are in the subduplicate ratio of the lengths of the strings, AC to AB.



For in both revolutions of this sort the centrifugal force [required] to sustain the same obliquity of the string is the same. And, if the said force is the same, then the squares of the times in which the circles are completed must be as the distances from the axis of revolution, by the converse of IV. Therefore, here CF will be to BG, i.e. AC to AB, as the squares of the times of revolution. Q.E.D.

PROPOSITION X

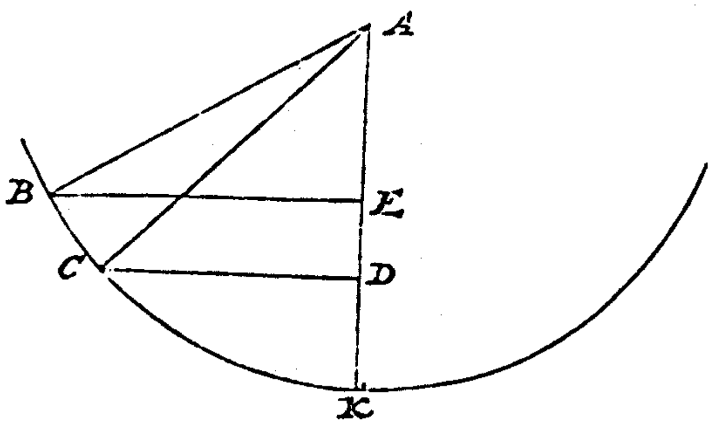
If any two moving bodies suspended on strings describe by revolution circles parallel to the horizon, the times of revolution will be in the subduplicate ratio of the altitudes of the cones whose surfaces are described by by the strings.



Let AC, AD be strings, attached to which bodies C and D describe horizontal circles, while the ends of the strings remain fixed at A. And let C draw string AC over the conical surface of which the axis is AB, and D the string DA over the surface of a cone of which the axis is AE. I say the time of revolution of body C is to the time of revolution of body D in the subduplicate ratio of AB to AE. For imagine some other moving body attached to string AF to form by its revolution a cone of side AF, axis AB. Its time of revolution is therefore equal to the time of revolution of body C. But the time of revolution of body F is to the time of revolution of body D in the subduplicate ratio of AF to AD, or of AB to AE. Therefore, the time of revolution of body C will also be to the time of body D in the subduplicate ratio of AB to AE. Q.E.D.

PROPOSITION XI

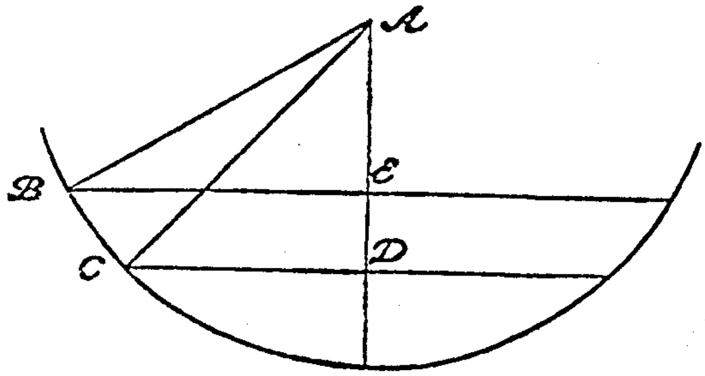
If a moving body suspended on a string describes by its motion, when the upper end of the string is at rest, unequal circles parallel to the horizon, the times of revolution along the said circles will be in the subduplicate ratio of the sines of the angles at which the string is inclined to the plane of the horizon.



Let AB be a string attached at A. And let a moving body suspended from it and revolved horizontally stretch it first along the straight line AB and then along the straight line AC. And, parallel to the horizon draw BE, CD, which meet the perpendicular AD at E and D. Therefore, because AB, AC are equal, AE will represent the sine of angle ABE, and AD the sine of angle ACD. I say then that the times of revolution along the circles of radii BE, CD will be to one another in the subduplicate ratio of AE to AD. This is manifestly clear from the above proposition.

PROPOSITION XII

If a pendulum carried in a conical motion makes very small revolutions, their individual times will have the same ratio to the time of perpendicular fall from twice the height of the pendulum as the circumference of the circle to the diameter; and consequently they will be equal to the time of two very small lateral oscillations of the same pendulum.

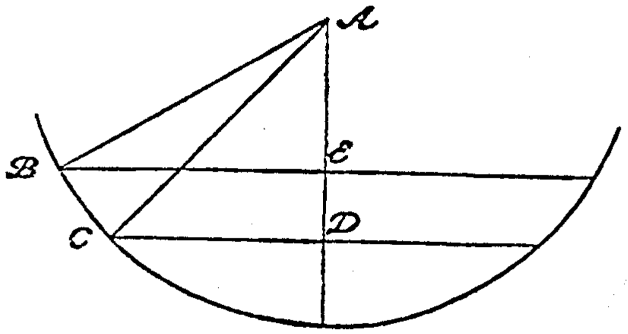


Let AC be a string attached at A, suspended from which a moving body by revolving describes a horizontal circle, the radius of which, DC, is equal to this DA, such that angle CAD is semiright. The centrifugal force at C will be equal to the weight of the body, and thus it will traverse the circumference described by radius DC at the velocity that the body would have acquired by perpendicular fall from the height of one-half DC, or its equal DA. But DC is to CA as 1 to /2, and thus the time of perpendicular fall from one-half DC to the time of perpendicular fall from one-half CA, which times are in the subduplicate ratio of DC to CA, will be in the ratio of 1 to //2. Whence the time of perpendicular fall from one-half DC to the time in which it falls from twice AC, which is twice the time of fall from one-half AC, will be as 1 to 2//2, or as any radius to twice the same radius multiplied by /2.

But the time of perpendicular fall from one-half DC is to the time of revolution through the circumference described by radius DC as the radius to the circumference. And the time of revolution through the circumference DC is to the time of a very small revolution in the subduplicate ratio of AD to AC, or as 1 to //2. Therefore the time of perpendicular fall from one-half DC is to the time of a very small revolution as the radius to the circumference multiplied by //2, and the time of a very small revolution of pendulum AC to the time of perpendicular fall from twice the height of the pendulum [is] as the circumference multiplied by //2 to twice the radius multiplied by //2, or as the circumference to the diameter. Since, however, the time of a very small lateral oscillation of pendulum AC is also to the time of perpendicular fall from one-half AC (or, if the doubles of each are taken, the time of two very small lateral oscillations of pendulum AC is to the time of perpendicular fall from twice AC) as the circumference to the diameter, the time of a very small revolution of pendulum AC will be to the time of perpendicular fall from twice the height of pendulum AC as the time of two very small lateral oscillations of pendulum AC to the same time of perpendicular fall from twice AC. Therefore, the time of a very small revolution of pendulum AC will be equal to the time of two very small lateral oscillations of the same pendulum AC. Q.E.D.

PROPOSITION XIII

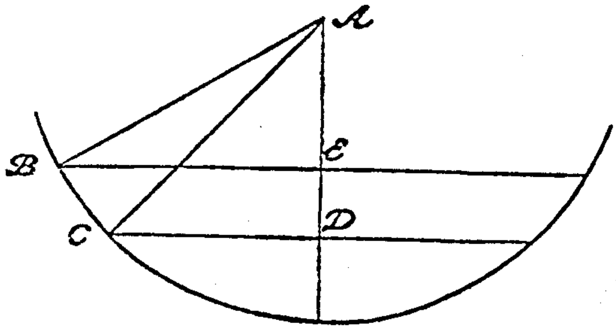
If a moving body is carried in a circumference, and it completes its individual revolutions in the time in which a pendulum having the length of the radius of its circumference would, by conical motion, complete a very small revolution, or twice a very small lateral oscillation, it will have a centrifugal force equal to its weight.



Let string AC be equal to the radius of the circle in which the body is moved, such that angle CAD is semi-right, and let the time of revolution along CD be 1. The time of a very small revolution of the same pendulum will be //2. But, by hypothesis, the time of revolution through the circumference of which the radius is AC is the same. Therefore, the time of revolution through CD is to the time of revolution through AC as 1 to //2, or in the subduplicate ratio of CD to AC. Whence, by the converse of the fourth [proposition], these two bodies so moved will have equal centrifugal forces, and thus, since in CD the centrifugal force is equal to the weight, the same will hold true in the revolution through the circle of which the radius is AC. Q.E.D.

PROPOSITION XIV

The times of revolution of any pendulum carried in a conical motion will be equal to the time of perpendicular fall from a height equal to the string of the pendulum, when the angle of inclination of the string to the plane of the horizon is approximately 2 parts, 54 scruples; to be precise, if the sine of the said angle is to the radius as the square inscribed in the circle to the square of its circumference.

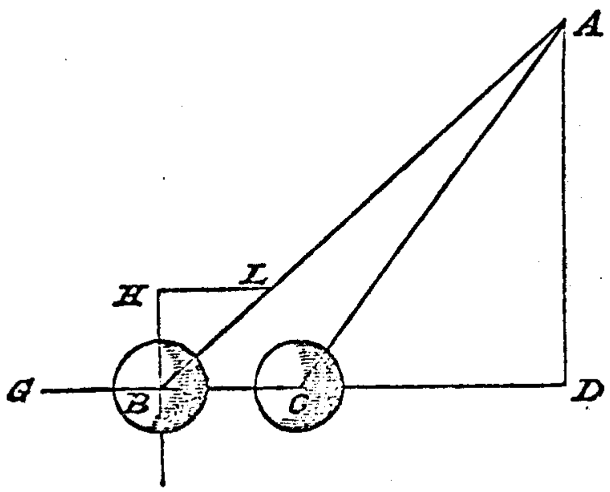


Let AD = DC = a, AE = b; let the circumference of the circle be to the radius as c to r; let the time of perpendicular fall through one-half CD be 1. The time of fall through one-half AC will be //2. But the time through one-half AC is to the time through AC as 1 to /2; therefore, the time through AC will be as /8. But the time of revolution through CD is to the time of fall through one-half CD as c to r. Thus, the time of revolution through CD will be CD = c/r. But the time of revolution at C is to the time of revolution at some other point B in the subduplicate ratio of AD to AE, or as /a to /b. Therefore, the time of revolution at B = And, if we now suppose AE, the sine of angle ABE, to be to the radius AB as the square inscribed in the circle to the square of its circumference, b will be to a/2 as 2rr to cc, or bcc/arr = 2/2, or = //8 And since is the time of revolution at B, and //8 the time of descent through AC, this time of revolution at B will be equal to the time of perpendicular fall from a height equal to the string of the pendulum.

Moreover, since 2r is to c as 7 to 22, 4rr will be to cc as 49 to 484, or 2rr to cc as 49 to 968. From this it follows that 968 is to 49 as a/2 (the radius = 100,000) to 5062, the sine of angle ABE, approximately 2 degrees 54'. Q.E.D.

PROPOSITION XV

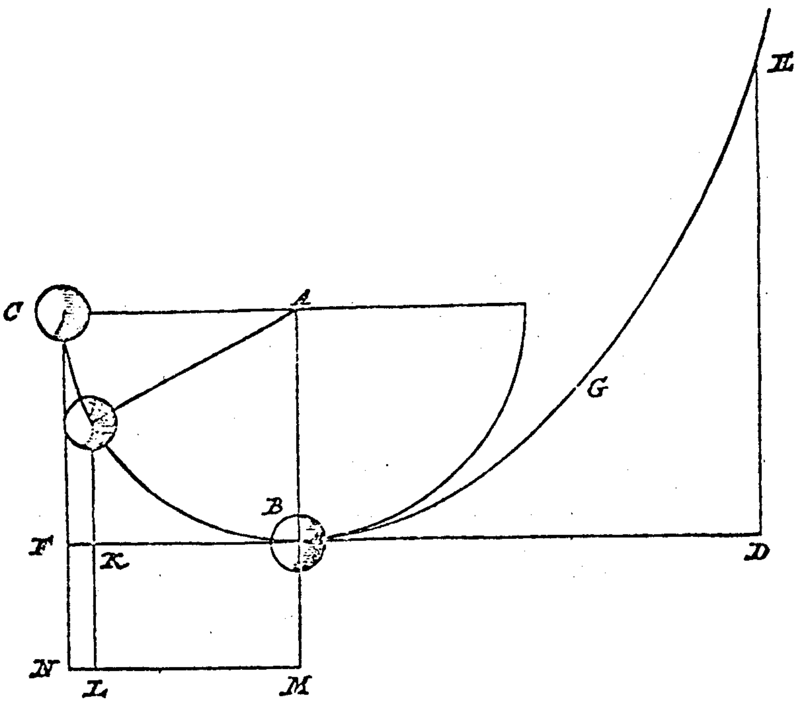
If two pendulums of equal weight but of unequal length of strings are revolved in a conical motion, and the altitudes of the cones are equal, the forces by which they stretch their strings will be in the same ratio as that of the lengths of the strings.



Let AB, AC be two pendulums of different length, and let two equal weights suspended at the endpoints B and C be rotated about the common axis AD. I say that the force by which string AB is stretched is to the force by which string AC is stretched in the ratio of the strings, AB to AC. For, if we suppose the weight B located in it to be sustained by a power at A pulling the string AB and by another power at G, equal to the centrifugal force and pulling along the straight line BG, it holds from the Mechanics that, if BH is drawn perpendicular to the horizon and HL parallel to the same, the force at A stretching string AB will be to the gravity of weight B as LB to BH, or as AB to AD. Likewise, the force by which string C is stretched will be to the gravity of weight C, or to the gravity of weight B, which has been posited equal to this C, as AC to AD. Therefore, the force by which string AB is stretched in revolving will be to the force by which string AC is stretched as AB to AC. Q.E.D.

PROPOSITION XVI

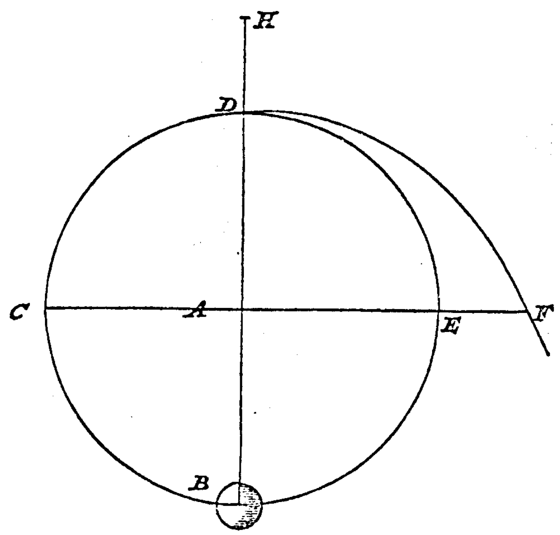
If a simple pendulum is set in motion with the maximum lateral oscillation, i.e. if it descends through the whole quadrant of the circle, when it reaches the lowest point of the circumference it will pull its string with a force three times as great as if it were simply suspended from it.



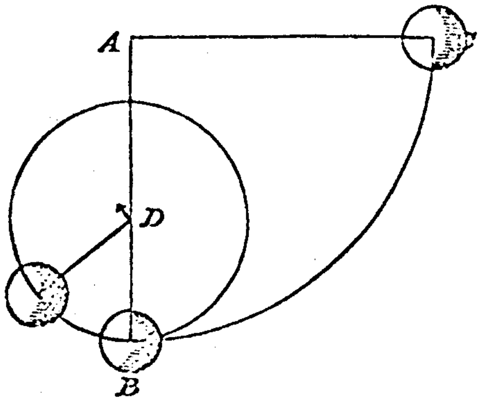
If ball C attached to A by string AC descends through a quadrant of circumference CB, when it arrives at B it will pull string AB with a force three times as great as if it were hung by its weight alone. For, first, the velocity at which it would continue to move along the straight line BD, if the string were released at B, is the same as that which it would have at point F, if it were to fall perpendicularly through CF. But, in that case, it would acquire just enough speed [*tantem celeritatem*] to traverse twice the distance of this CF with a uniform motion in the same time in which it fell from C to F. Therefore, at B the ball has the tendency to traverse the line BD, which is twice AB, in the same time in which it would fall from A to B (not considering, that is, the force of its gravity, by which it would in the meantime also have descended and described some parabola). Let BGE be a parabola, of which B is the vertex and AB one-half the latus rectum. Since, therefore, the recession of ball B from circumference BC while it is moved along BD with a uniform motion may, at the beginning, close to point B, be taken as the same as the recessions from parabola BGE, it holds that the centrifugal force that the ball has at B from revolution alone is a tendency to recede from center A, or from circumference BC, with a motion accelerated according to the numbers 1, 3, 5, 7, etc., and consequently is similar to that tendency by which the body tends to fall, which we call gravity. But this tendency in ball B is as much as would be in a body equal to it that would traverse with accelerated motion the distance DE in the same time in which it would traverse distance BD with a uniform motion, i.e. in a time equal to that in which the ball would fall from A to B with a likewise accelerated motion. Therefore, because DE is twice BA, the centrifugal tendency of the ball at B is twice its gravity. But another tendency is added by gravity here, by which ball B (in the same time in which it would fall from A to B) now also tends to traverse the same amount of distance with a naturally accelerated motion downward. Therefore, with both tendencies taken together, it tends to traverse, with a motion accelerated according to 1, 3, 5, 7, a distance equal to both DE and AB, i.e. three times this AB. Wherefore, the force with which the body descending from C pulls at point B is three times that which arises from the simple weight of ball B hanging freely. Which also agrees exactly with experience.

PROPOSITION XVII

A globe hung on a string from the center of a circle perpendicular to the horizon cannot be revolved around the circumference of this circle unless the string can support six times the weight hung [on it].



Let BCDE be a circle standing perpendicularly to the horizon, and from the center A let ball B be suspended. I say that, in order that this ball be able to revolve along the circumference BCDE, it is necessary that the string be able to support six times the suspended weight B. For, in order that the string remain extended when the ball passes through point D and descends through arc DE, the velocity of the ball there must be such that, if it were released, it would describe parabola DF of which AD is one-half the latus rectum. Whence, it must have as great [a velocity] as a body falling from height HD, one-half of this DA, would have at D. Therefore, in order that [the ball] ascending from B through semicircle BCD have the said velocity left over at D, the speed at B must be so much as to enable it to ascend perpendicularly to point H. For, having this speed at B, by whatever path it reaches height D, it will always retain so much speed as to enable it further to ascend perpendicularly, or by any other path, to H; that is, as much speed will be left to it as it would acquire falling from height HD, which we said it needed at point D. Furthermore, the speed, by which it would ascend perpendicularly from B to H, or which it would have [by] falling from HB, is to the speed that it would acquire falling from AB in the subduplicate ratio of these distances, i.e. in that of Ö 10 to 2. But it has been shown in the preceding [proposition] that, if it revolves in the circumference at the speed which it acquires falling from AB, or through arc EB, the centrifugal force alone will be twice the weight of the simple ball. And the speed at which it here revolves in the same circumference is to that [speed] as 10 to 2, and consequently the centrifugal force is in the duplicate ratio, i.e. 10 to 4, or 5 to 2. Therefore, the centrifugal force here will be to the gravity of the ball as 5 to 1. But, to this centrifugal force, when the ball passes through B, must be added the force of gravity by which it tends to fall downward, which has been said to be to the said centrifugal force as 1 to 5. Therefore, the whole force, or pull [*attractio*], that the string feels when the ball passes through B will be six times the weight of the ball.



From this I find: if a ball attached by a string from AB is released at C, of the same height as point A, and B is divided at D such that DB is 25AB, and at D a nail is fixed, which the string hits when the ball falls from C, then, in turn, the ball will be able to turn about the nail D and describe a circle. If the nail D is fixed any higher, it will not be able to do so. For, since the speed of the ball at B should, in order to complete a whole revolution, be to the speed that it would acquire falling from DB as 10 to 2, as has just been shown, hence the heights should be in the duplicate ratio of this, to wit, as 10 to 4, or 5 to 2; by falling from [DB] it acquires these speeds. Therefore AB is to DB as 5 to 2.