ENERGY EXTRACTION*

Robert Geroch†

Enrico Fermi Institute
University of Chicago
Chicago, Illinois 60637

1. Introduction

A number of astrophysical phenomena discovered in recent years have the characteristic feature that enormous quantities of energy are extracted from a relatively small system. Examples include the output of electromagnetic radiation from quasars and, possibly, the output of gravitational radiation from the center of our galaxy. Since strong gravitational fields are both an effective source of energy and an effective means for converting energy from one form to another, it is natural to suppose that general relativity will have a significant role in the explanation of such phenomena. Thus, one is led to ask: given a physical system, governed by the equations of general relativity, what is the maximum energy that can be extracted from it?

Of course, if one knows the detailed structure of the system, as well as the mechanism by which energy is to be extracted, one can, at least in principle, actually calculate the energy output. Unfortunately, it is normally the case that neither the structure of the system nor the extraction mechanism is known. In fact, the most significant piece of information about the system is often the magnitude of the energy output itself. Therefore, one is forced to guess the structure and mechanism, calculate the energy output from general relativity, and compare the result with observations. Although this approach has certainly been valuable in the past, it does suffer from certain deficiencies. In particular, it introduces a bias toward conservative explanations: one guesses processes one has seen before. But general relativity is, by comparison with other theories, not well understood. It may well turn out that new and unexpected features of the theory will be important for the interpretation of astrophysical observations.

An alternative approach is to look for general statements within general relativity—statements that are independent of the details of the system and which reflect on the possibilities for energy extraction. This alternative point of view has proved fruitful in the past. For example, although detailed calculations on the collapse of individual systems have certainly been important, the search for broad statements reflecting on collapse phenomena in general led to the singularity theorems. These theorems represent, of course, an enormous advance in our understanding of collapse. Thus, the program is to describe the issue of energy extraction within the context of general relativity and to obtain what general statements one can within this theory. We shall here describe briefly the progress which has been made in this program.

2. Examples

We begin with two examples of the extraction of energy from a highly relativistic system.

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† Sloan Foundation Fellow.

The first[1] involves the extraction of energy from a Kerr black hole. Recall the following facts: 1. The Kerr metric possesses a Killing vector field t^a that, far from the black hole, is unit and (future-directed) timelike. (Physically, this t^a generates the time-translation symmetry of the system.) 2. For $a < m$ in the Kerr metric, there is a timelike three-surface S, the ergosurface, inside of which t^a becomes spacelike. 3. For a free particle with (four-)momentum p_a, the quantity $p_a t^a$ is constant along the particle's path. 4. Momentum is conserved. If a particle with momentum p_a breaks into two particles with momenta p_a' and p_a'', then $p_a = p_a' + p_a''$. That energy can be extracted from the Kerr metric follows from these properties.

It is natural to regard the quantity $E = -p_a t^a$, where p_a is the momentum of a particle, as that particle's energy, for this quantity is constant along the particle's path, and it reduces to the special relativistic energy in the limit far from the black hole. Now let a particle with momentum p_a emit a second particle with momentum p_a', so $p_a'' = p_a - p_a'$ is the momentum of our original particle after the emission. Contracting with t^a, we obtain the equation of energy conservation: $E'' = E - E'$. Suppose first that this emission process is carried out outside the ergosurface, where t^a is timelike. Then $E' = -p_a' t^a$ is positive, since both p_a' and t^a are future-directed timelike. Thus, our original particle's energy decreases as a result of the emission.

However, a new possibility is available in the Kerr metric. Our original particle could first pass through the ergosurface, there emit the second particle, and then return, again through the ergosurface, to the exterior region. Since t^a is spacelike inside the ergosurface, one could, furthermore, arrange matters so that the energy, E', of the emitted particle is negative. (Given a spacelike t^a, choose timelike p_a' so $-p_a' t^a < 0$.) In these circumstances, the emission process is accompanied by our initial particle's actually gaining energy. (The emitted particle, with negative energy, is captured by the black hole.)

By this process, then, energy is, in effect, extracted from the Kerr black hole. (This process fails for $a = 0$, the Schwarzschild solution, because then the ergosurface becomes null, whence the original particle is unable to return to the exterior region after the emission.)

As a second example,[2] consider the collision of two Schwarzschild black holes that, initially, had energies (masses) E_1 and E_2 and were very far apart. Let us suppose that these systems coalesce to form, eventually, a new Schwarzschild black hole with energy E. Since gravitational radiation (which carries energy away) will be produced during the coalescence process, one would expect to have $E \leqslant E_1 + E_2$. It turns out, however, that E, E_1, and E_2 are also related by a second inequality: $E^2 \geqslant E_1^2 + E_2^2$. Thus, we have both upper and lower limits for the energy of the final black hole. If, for example, $E_1 = E_2$, then we have $2E_1 \geqslant E \geqslant \sqrt{2}E_1$. The difference, $2E_1 - E$, represents energy emitted as gravitational radiation. Thus, from 0 to 30% of the energy originally available in the two Schwarzschild solutions will, in this process, be extracted in the form of gravitational radiation.

These examples illustrate the kinds of results we seek. The arguments are largely structure-independent and depend essentially on the presence of strong gravitational fields.

3. Formulation of the Problem

One of the key difficulties in formulating statements in general relativity on energy extraction is the peculiar status of energy in this theory. Energy is not a particularly natural concept in general relativity. The reason, essentially, is that energy in special relativity is associated with the time-translation symmetries of

TABLE 1
COMPARISON OF THE STATUS OF ENERGY IN NEWTONIAN GRAVITATION AND GENERAL RELATIVITY

	Newtonian Gravitation	General Relativity
Energy Density Source	ρ	T_{ab}
Energy Density Field	$-\frac{1}{8\pi G}(\nabla\phi)\cdot(\nabla\phi)$	None
Total Energy Source	$\int \rho \, dV$	None
Total Energy Field	$-\frac{1}{8\pi G}\int (\nabla\phi)\cdot(\nabla\phi)\, dV$	None
Total Energy	$\int \rho \, dV$ $-\frac{1}{8\pi G}\int (\nabla\phi)\cdot(\nabla\phi)\, dV$	Well-Defined

Minkowski space, while no such symmetries are available, in general, in curved space-time. Nonetheless, energy can be defined in general relativity in certain situations. One must so formulate questions on energy extraction that the notion "energy" is required only in these certain situations.

The role of energy in general relativity is compared with that in Newtonian gravitation in TABLE 1. In Newtonian gravitation, the energy densities of the source and of the field, the total energies of the source and of the field, and the total energy are all defined. However, three of these five quantities turn out to be undefined in general relativity. The stress-energy tensor, T_{ab}, plays the role of the energy density of the source in Newtonian theory. There is nothing in general relativity that could reasonably be called the energy density of the gravitational field. (The metric g_{ab} is analogous to the Newtonian potential ϕ. But no tensor field can be constructed from the first derivatives of the metric.) Similarly, neither the total energy of the source nor the total energy of the field is meaningful in general relativity. (The difficulty in defining a "total energy of the source" is that the stress-energy, T_{ab}, is second-rank, and hence cannot be integrated over a spacelike three-dimensional surface.)

It turns out, however, that a quantity that can be interpreted as the "total energy, including the contributions from both the sources and the field" can be defined in general relativity—provided the space-time is asymptotically flat in a suitable sense. To see why this should be the case, we write Einstein's equation, $R_{ab} - \frac{1}{2}Rg_{ab} = 8\pi G T_{ab}$, symbolically in the form $\partial^2 g = T + (\partial g)^2$. One can think of the two terms on the right as the energy density of the matter and the "energy density of the gravitational field", acting as a source for the metric. Thus, if the space-time is asymptotically flat, one might expect to be able to define a quantity, to be interpreted as total energy, in terms of the rate of approach of the metric to flatness in the asymptotic region. The situation is somewhat analogous to that of the modified Newtonian theory, in which the Newtonian field energy is inserted as an extra source term on the right: $\nabla^2\phi = 4\pi G[\rho - (1/8\pi G)(\nabla\phi)\cdot(\nabla\phi)]$. One might reasonably interpret the coefficient of the $1/r$ term in ϕ as the total energy of the system.

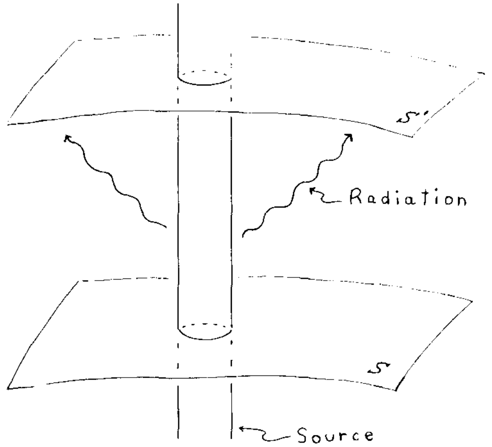

FIGURE 1. The Arnowitt-Deser-Misner energy. The energy is defined in terms of asymptotic behavior on a spacelike, three-dimensional surface S. Since radiation emitted between two such surfaces is incident on the second, the energy is invariant under translations of S.

There are, in fact, two distinct regimes in which a total energy can be defined in general relativity. The first is at spatial infinity,[3,4] i.e., in the limit as one approaches the asymptotic region along spacelike directions (FIGURE 1). More precisely, the Arnowitt-Deser-Misner energy is defined in terms of the behavior of initial data on a spacelike three-dimensional surface S, which, asymptotically, approaches the behavior of a t = const. plane in Minkowski space. The Arnowitt-Deser-Misner energy is invariant under translations of the surface S. This is a result one would have expected physically, for, if a system emits energy, say, in the form of gravitational radiation, between two such surfaces, S and S', then the emitted radiation crosses S', and hence makes a contribution to the Arnowitt-Deser-Misner energy of S'. Thus, in this rough picture, one would expect the decrease in the energy of the system between S and S' to be compensated for by the gravitational radiation that crosses S' and not S.

The second regime[5,6] in which total energy can be defined is at null infinity. The Bondi energy is defined in terms of the rate of approach of the metric to flatness along a null surface that, asymptotically, looks like a null cone. Since gravitational radiation can escape between two such null surfaces, one would expect that the Bondi energy will decrease as one moves from one null surface to the next (FIGURE 2). This has been shown to be the case.[5,6]

No relation is known between the Bondi energy and the Arnowitt-Deser-Misner energy. However, these physical considerations suggest the following, very attractive, conjecture: In the limit as the null surface is moved to the distant past, the Bondi energy approaches the Arnowitl-Deser-Misner energy.

A second difficulty involves the meaning of the term "extract." Roughly speaking, one wishes to regard energy as having been extracted from a system if the energy is removed in the form of kinetic or potential energy, but not, for example, if it is removed as mass energy. For example, if a galaxy throws off entire stars, one would not normally regard the mass energy of these stars as energy extracted from the galaxy. But, as we have seen above, this Newtonian distinction between different types of energy is simply not available in general relativity. Apparently, the only possibility consistent with the constraints imposed by general relativity is to regard energy as having been extracted from a system if it can be converted into radiation that reaches null infinity. That is to say, extracted energy refers to changes in the

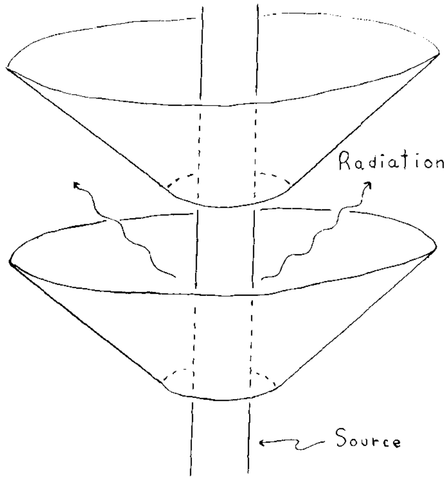

FIGURE 2. The Bondi energy. The energy is defined in terms of asymptotic behavior on a three-dimensional null surface. Since radiation can escape between successive surfaces, the energy decreases under translation of the surface into the future.

Bondi energy. Physically, this point of view is perhaps not unreasonable, for one might expect intuitively that energy extracted, e.g. as kinetic or potential energy, could then be converted into gravitational radiation.

Now consider an asymptotically flat solution of Einstein's equation, and let E be its Arnowitt-Deser-Misner energy. One might imagine assigning to this system a second quantity $E_0 \leqslant E$, called the bound energy. We require that this E_0 be so chosen that the system is incapable in principle of radiating away more energy than $E - E_0$. Thus, we would require that E_0 be less than or equal to the smallest value assumed by the Bondi energy. For the Schwarzschild solution, we would have, presumably, $E_0 = E$, while the example of Section 2 suggests that we should choose[7] $E_0 < E$ for the Kerr solution.

This notion of the bound energy would not be very useful unless one obtained a prescription for the behavior of the bound energy when two systems are combined. Consider two asymptotically flat solutions of Einstein's equation, with Arnowitt-Deser-Misner energies E' and E'', and with bound energies E_0' and E_0''. It would presumably be possible to introduce a new asymptotically flat solution of Einstein's equation in which, in the limit of the distant past, the original two systems are seen infinitely far apart. These systems would, in this solution, interact, and, among other things, emit gravitational radiation. This would be our notion of "combining systems."

The behavior of the Arnowitt-Deser-Misner energy under this combining of systems would be $E = E' + E''$. But how would the bound energies behave? The equation $E_0 = E_0' + E_0''$ would certainly not do, for, as we saw in Section 2, two Schwarzschild black holes can interact to produce gravitational radiation. But this example also suggests a plausible equation for the addition of bound energies: $E_0^2 = E_0'^2 + E_0''^2$.

Thus, the proposal is that asymptotically flat solutions in general relativity be assigned, in addition to a Arnowitt-Deser-Misner energy E, a bound-energy E_0 with the following properties: (i) E_0 is less than or equal to the smallest value assumed by the Bondi energy, and (ii) when systems are combined, the bound energies add by the sum of squares. It is intended that both singular (black-hole) and nonsingular solutions be included within this framework. This proposal is intended merely to represent an example of the kinds of results on energy extraction that one might obtain from general relativity. Of course, to state precisely and prove theorems along this line would be a difficult task.

4. Ultimate Limits on Energy Extraction

Is it conceivable that certain systems in general relativity have the property that unlimited amounts of energy can be extracted from them? (In the language of Section 3, this is the question of whether some systems must have a bound energy of minus infinity.) If, indeed, this were possible in principle, then one would face no difficulties, in principle, in supplying the energy needs in astrophysical applications. On a more practical level, the mere existence of such general relativistic systems would, at least, suggest alternative explanations for certain astrophysical observations.

The answer to our question depends crucially on the ground rules. If, for example, matter with negative local energy density were permitted, then there would certainly be no limits in principle on the energy which could be extracted from a system. The continued extraction of energy would simply leave behind a system of negative total energy. However, since no matter with negative local energy density has ever been observed, it seems reasonable to rule out this possibility.

A second possibility arises when singularities are permitted. Consider the Schwarzschild solution with mass parameter m negative—a typical "negative energy singularity." Suppose it were possible to create such a singularity, starting with a nonsingular space-time and using only matter with positive local energy density. Then, just as above, one would expect to be able to extract energy from a system indefinitely, leaving behind a singularity whose energy becomes more and more negative.

There are, however, reasons to believe that, although negative energy singularities exist as solutions of Einstein's equation, such singularities cannot be created as described above. The Schwarzschild solution with negative total energy has the property that observers in the asymptotic region can see the singularity itself—it has what is called a naked singularity. The Schwarzschild solution with positive energy, on the other hand, does not have this property. But there is evidence for the following conjecture, called the naked-singularity conjecture: it is impossible to obtain a naked singularity by evolution via Einstein's equation starting with nonsingular initial data and introducing only matter with positive local-energy density. Thus, should this conjecture turn out to be true, it would probably also be true that it is impossible to extract unlimited energy from a system, leaving behind a negative-energy singularity.

There is still a third possibility. It is conceivable that, after one has extracted energy from a system, there remains a system without singularities in which the local energy density of matter is everywhere positive but whose total energy is, nonetheless, negative. This possibility is not so absurd as it might at first appear to be. In Newtonian gravitation, for example, the total energy (the last entry in Table 1) can become negative, even for a system with positive-energy density of matter. Indeed, for a body of mass m and radius r, the total Newtonian energy becomes negative for r less than the order of Gm/c^2, i.e., for r less than the order of the Schwarzschild radius of the body. But this is precisely the regime in which Newtonian gravitational theory must be replaced by general relativity.[8] Thus, it is certainly conceivable that, in the presence of strong gravitational fields, nonsingular systems with positive-energy density of matter have negative bound energy. One would like to know whether or not this possibility actually occurs within Einstein's theory.

There is a conjecture in general relativity that would shed considerable light on these possibilities, the positive-energy conjecture:[9] A nonsingular, asymptotically flat solution of Einstein's equation in which the local-energy density of matter is everywhere nonnegative has nonnegative total energy. Actually, there are two conjectures, depending on whether "total energy" refers to the Bondi energy or the Arnowitt-Deser-Misner energy. The Bondi case is the more important, for a proof in this case would imply immediately that, for a system with Bondi energy E at some epoch, no more energy than E could be extracted without creating singularities. On the other hand, one might expect a proof in the Arnowitl-Deser-Misner case to have essentially the same consequence. An argument in support of this positive-energy conjecture is given in the Appendix. Apparently, general relativity itself places upper limits on the energy that can be extracted from systems satisfying its equations.

5. Summary

Given a physical system in general relativity, how much energy can be extracted from it? This question appears to be both difficult to formulate and difficult to answer. It seems likely, nonetheless, that the theory will eventually yield general statements that bear on this question. In fact, several recent conjectures and theorems in general relativity can be interpreted as giving information on the possibilities for energy extraction. Unfortunately, it is perhaps too early to say whether or not this information will ultimately be useful in astrophysical applications.

Notes and References

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7. That is, the example shows that energy can be extracted from a Kerr solution if a single additional particle is admitted. Thus, we are effectively requiring that the bound energy be, in some sense, a continuous function of the solution. Note, incidentally, that this example shows that the bound energy will not in general be the smallest value assumed by the Bondi energy.
8. One can get an idea of the direction in which the general relativistic effects will act by considering the modified Newtonian theory of Section 3. Note that the equation can be written in the form $\nabla^2(e^{1/2\phi}) = e^{1/2\phi}2\pi G\rho$. Thus, in this theory, if ρ is non-negative then the total energy (defined in terms of the $1/r$ term in ϕ) will also be nonnegative.

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10. This condition $\pi = 0$ means, geometrically, that our surface S has maximal area relative to nearby surfaces. This rather unphysical additional condition is often made to simplify proofs. The positive-energy theorem is probably true without it.
11. One must, of course, verify that such a choice exists.

Appendix

Positive-Energy Theorem

Consider initial data for Einstein's equation. That is, consider a three-dimensional manifold S with positive-definite metric g_{ab} and extrinsic curvature π^{ab}, subject to the constraint equations:

$$\nabla_b(\pi^{ab} - \pi g^{ab}) = J^a \tag{1}$$

$$R = 2\mu + \pi^{ab}\pi_{ab} - \pi^2 \tag{2}$$

where J^a and μ are, respectively, the momentum and the energy density of the source, where R is the scalar curvature and ∇_a the covariant derivative on S, and where we have set $\pi = \pi_a{}^a$. For an asymptotically flat initial-data set,[3] the Arnowitt-Deser-Misner energy E is defined in terms of the asymptotic behavior of g_{ab}.

We shall outline briefly the steps of an argument that, if such an initial data set is singularity-free, and if $\mu \geqslant 0$ and $\pi = 0$,[10] then $E \geqslant 0$.

Introduce a function t on S such that the two-dimensional surfaces $t =$ const. in S are nested topological two-spheres, with the innermost sphere, $t = -\infty$, reducing to a point. Denote by h_{ab} and p^{ab} the induced metric and extrinsic curvature, respectively, of these surfaces. Then we have the Gauss-Codazzi equation

$$\mathcal{R} = p^2 - p^{ab}p_{ab} + R - 2R_{ab}\xi^a\xi^b \tag{3}$$

where ξ^a is the unit normal to our family of surfaces, $\mathcal{R}$ is the scalar curvature of the surfaces, and we have set $p = p_a{}^a$.

Define a scalar field ϕ on S by $\phi\xi^a\nabla_a t = 1$. A dot, affixed to a quantity, will denote its rate of change with respect to t (i.e., its Lie derivative by $\phi\xi^a$). Then

$$\dot{h}_{ab} = 2\phi p_{ab} \tag{4}$$

$$\dot{p} = -D^aD_a\phi - \tfrac{1}{2}\phi p^2 - \tfrac{1}{2}\phi p^{ab}p_{ab} + \tfrac{1}{2}\phi\mathcal{R} - \tfrac{1}{2}\phi R \tag{5}$$

where D_a denotes the intrinsic derivative within our surfaces.

For each value of t, set

$$W = \int (2\mathcal{R} - p^2)\, dA \tag{6}$$

where the integral extends over the sphere $t =$ const. It follows from Equation 3 that, for a small sphere about a point, $W = 0$. For a large, asymptotically spherical surface in an asymptotically flat space, $W = E/r$, where E is the Arnowitt-Deser-Misner energy, and r is any typical radial distance. Using Equations 4 and 5 and the Gauss-Bonnet equation, the rate of change of W with respect to t is

$$\dot{W} = \int [2pD^aD_a\phi + \phi pp^{ab}p_{ab} - \phi p\mathcal{R} + \phi pR]\, dA \tag{7}$$

We now choose[11] our two-dimensional surfaces so that $\phi = p^{-1}$. Substituting into Equation 7 and integrating the first term by parts,

$$\dot{W} = -\tfrac{1}{2}W + \int [R + (p^{ab} - \tfrac{1}{2}ph^{ab})(p_{ab} - \tfrac{1}{2}ph_{ab}) + 2\phi^{-2}(D^a\phi)(D_a\phi)]\, dA \tag{8}$$

It follows from (2) that the integral on the right is non-negative, so (8) becomes $\dot{W} \geqslant -\frac{1}{2}W$. But $W = 0$ for $t = -\infty$, so this equation implies $W \geqslant 0$ for all t. Hence, $E \geqslant 0$.

Discussion of the Paper

JOHN COCKE (*University of Arizona, Tucson, Ariz.*): Could you be a bit more specific about the naked-singularity conjecture? We know that bodies can collapse asymptotically into a singularity. Doesn't this statement depend on the coordinate system used?

DR. GEROCH: Let us take a Schwarzschild collapse as an example. A system that has collapsed to form a Schwarzschild black hole is not a naked singularity. One cannot look into a Schwarzschild black hole and actually see the singularity. Only negative-mass Schwarzschild black holes are naked singularities. So, the conjecture has a chance because, presumably, one cannot collapse a body composed of positive matter density to form a negative-mass Schwarzschild singularity. But there are other arguments. For example, the Kerr solutions are naked singularities when a, the angular momentum parameter, exceeds m, the mass parameter. What we would like to do is to start with a Kerr solution with a less than m and toss in a particle, like a gyroscope, or some other clever object in order to push a up above m. If you try to throw in such a body it will always be ejected. So, a Kerr black hole that is not a naked singularity will try to prevent itself from becoming one.

UNIDENTIFIED SPEAKER: Could you explain for those of us who are not working in general relativity how an external observer can see a particle fall through a surface of infinite redshift that should correspond to an infinite time dilation.

DR. GEROCH: Let us take Minkowski space, and draw a certain Killing vector, that is, a certain time symmetry in this space. By choosing an origin and drawing two null planes through it, I can generate a boost Killing vector in the xt plane. If you are following the symmetry of this Killing vector, then this surface is already a surface of infinite redshift. Yet there is nothing spectacular about this surface; it is just a null surface in Minkowski space. If you are located near $R = 2M$, the situation is no different from that near any null plane in the space. The Killing vector changes from being timelike in one region to being spacelike in other regions. When you say that a surface has an infinite redshift you really mean that it has an infinite redshift for those people who choose to move along this Killing vector. For those who move in other ways, there is no infinite redshift at all.

UNIDENTIFIED SPEAKER: A particle going through the ergosphere will encounter matter that is already within the sphere and perhaps collide with or come into direct physical association with this matter. It seems peculiar that, with respect to an external observer, the original particle that fell through the sphere is fundamentally different from those particles which were already inside.

DR. GEROCH: . . . It is simply not true that one cannot go through the ergosphere. Once you go through the surface you can stay inside, or, if you choose, come back out again. I am not saying that you can do this with geodesics—you might need a rocket ship, but you can travel in and out. Usually, when Killing vectors change from timelike to spacelike, they create a certain null surface such that once you travel through the surface you cannot come back. This Killing vector, however, creates a timelike sufrace across which it changes from being timelike to being spacelike. In this case, you can go back and forth as much as you like.

UNIDENTIFIED SPEAKER: After you return you can, presumably, relate your experiences. Does that not violate the basic principle that one cannot receive information from the inside of a surface of infinite redshift?

DR. GEROCH: Not only is that not a basic principle, it is also false. In the Kerr solution one can indeed go inside the ergosphere. This ergosphere is the surface of infinite redshift for those particles that happen to choose to travel along the timelike Killing vector. The reason why it is called the surface of infinite redshift is that it becomes impossible for a particle to continue to travel along the direction of the timelike Killing vector after it goes through the surface. The surface has an infinite redshift only for those people who choose to follow a particular trajectory.