## NOTES ON SEMANTICS

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SEMIOTIC

I. Semiotic and its parts

On object language ($L$) and metalanguage ($M$), see Intr. Sem., sect. 1. An expression in $L$ is a finite sequence of signs in $L$ (Intr. Sem., sect. 2). On sign-events (tokens) and sign-designs, expression-events, and expression-designs, see Intr. Sem., sect. 3.

On the division of semiotic into three parts, viz., pragmatics, semantics, and syntax, see Intr. Sem., sect. 4.

On the distinction between descriptive and pure syntax, see Syntax, sections 2 and 24, and Intr. Sem., sect. 5. On descriptive and pure semantics, see Intr. Sem., sect. 5. We shall here be concerned only with pure syntax and pure semantics.

II. Syntactical signs

Syntactical signs used in $M$ as names (with numerical subscripts, e.g., ‘$A_1$’, ‘$A_2$’, etc.) or as variables (with letter subscripts, e.g., ‘$A_i$’, ‘$A_j$’, etc.) for expressions of the object language: ‘$A$’ for expressions, ‘$s$’ for signs, ‘$c$’ for constants, ‘$v$’ for variables, ‘$in$’ for individual constants, ‘$inv$’ for individual variables, ‘$pr$’ for descrip-
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tive predicates, ‘prv’ for predicate variables, ‘$S$’ for sentential formulas (incl. sentences), ‘$D$’ for designator formulas (incl. designators), ‘$K$’ for classes of expressions (in most cases classes of sentences).

Thus, ‘$in_5$’ is short for ‘the individual constant No. 5’. ‘$\sim pr_2 (inv_5)$’ is short for ‘that expression (of the object language) which consists of ‘$\sim$’, followed by the predicate No. 2, followed by the left-hand parenthesis, followed by the individual variable No. 5, followed by the right-hand parenthesis’.

(I write ‘iff’ for ‘if and only if’. I use as a sign of definition in $M$ either ‘iff’ or ‘$=_{df}$.’)

$A_i$ is an open expression $=_{df}$ $A_i$ contains a free variable.

$A_i$ is a closed expression $=_{df}$ $A_i$ contains no free variable.

LOGICAL SYNTAX

III. Propositional calculus $PC$

A. Rules of formation

1. Signs of $PC$:
   (a) Constants ‘$B$’, ‘$C$’, etc.
   (b) Variables: ‘$p$’, ‘$q$’, etc.
   (c) Parentheses: ‘(‘, ‘)’.

2. Sentences of $PC$:
   (a) Any constant.
   (b) Any variable.
   (c) If $S_i$ is a sentence, $\sim S_i$ is a sentence.
   (d) If $S_i$ and $S_j$ are sentences, $(S_i \lor S_j)$ is a sentence.

B. Rules of transformation.

1. Primitive sentences of $PC$:

   PS1. ‘$\sim (p \lor p) \lor p$’.
   PS2. ‘$\sim p \lor (p \lor q)$’.
   PS3. ‘$\sim (p \lor q) \lor (q \lor p)$’.
   PS4. ‘$\sim (\sim p \lor q) \lor (\sim (r \lor p) \lor (r \lor q))$’.

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2. Rules of inference of $PC$:
   (a) Rule of substitution.
   (b) Rule of modus ponens.

2'. Formulation as definition:
$S_j$ is directly derivable in $PC$ from $K_i$ (or, from the sentences in $K_i$) iff either
(a) for some $S_i$, $K_i$ is \{\{S_i\}\}, and $S_j$ is formed from $S_i$ by substituting any sentence for a variable, or
(b) for some $S_i, K_i = \{S_i, \sim S_i \lor S_j\}$.

IV. The calculus $PC'$ (without variables)

A. Rules of formation.

1. Signs: (a), (c), (d) of $PC$.
2. Sentences: (a), (c), (d) of $PC$.

B. Rules of transformation.

1. Primitive sentence schemata:
   PS1. $\sim (S_i \lor S_i) \lor S_i$.
   PS2. $\sim S_i \lor (S_i \lor S_j)$.
   PS3. $\sim (S_i \lor S_j) \lor (S_j \lor S_i)$.
   PS4. $\sim (\sim S_i \lor S_j) \lor (\sim (S_k \lor S_i) \lor (S_k \lor S_j))$.
2. Rule of inference: modus ponens.

V. Definitions in general syntax, for any calculus $C$.

1. $R_k$ is a proof in $C = \text{Def} R_k$ is a finite sequence of sentences in $C$ such that every sentence $S_j$ of $R_k$ is either a primitive sentence of $C$ or directly derivable in $C$ from a subclass of sentences which precede $S_j$ in $R_k$.
2. $S_k$ is provable in $C = \text{Def} S_k$ is the last sentence of a proof in $C$.
3. $R_k$ is a derivation with the premise-class $K_k$ in $C = \text{Def} R_k$ is a finite sequence of sentences in $C$ such that every sentence $S_j$ of $R_k$ is either an element of $K_k$ or a primitive sentence of $C$ of directly derivable in $C$ from a subclass $K_i$ of the class of sentences which precede $S_j$ in $R_k$. 

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(4) $S_j$ is derivable from $K_i$ in $C =_{df} S_j$ is the last sentence of a derivation with the premise-class $K_i$ in $C$.

The following concepts are often useful in syntax and semantics.

(5) The class $K$ is closed with respect to the relation $R$ (or the function $f$) = $_{df}$ if $x_1, \ldots, x_n$ are elements of $K$ and $R(y, x_1, \ldots, x_n)$ [or $f(x_1, \ldots, x_n) = y$, resp.], then $y$ is an element of $K$. (Sometimes the following form is used: “if $K' \subseteq K$ and $R(y, K')$, then $y \in K$”.)

(6) The closure of the class $K$ with respect to the relations $R_1, \ldots, R_m$ (or the functions $f_1, \ldots, f_m$) = $_{df}$ the intersection of all classes which contain $K$ as a subclass and which are closed with respect to $R_1, \ldots, R_m$ (or $f_1, \ldots, f_m$, resp.). (Tarski, 1941, sect. 47; Rosser, 1953, pp. 244 ff.)

We can then define (without the terms ‘proof’ and ‘derivation’):

(7) The class of the provable sentences in $C =_{df}$ the closure of the class of primitive sentences with respect to direct derivability.

(8) The class of the sentences derivable from $K_i$ in $C =_{df}$ the closure of the union of $K_i$ with the class of primitive sentences with respect to direct derivability.

VI. Examples for $PC'$ (see IV)

A. Example of a proof in $PC'$.

\[
\begin{array}{c|c}
\text{PS4} & S_i \\
\text{PS1} & \sim (\sim (B \lor B) \lor B) \\
(1)(2) & \sim (B \lor B) \\
\text{PS2} & \sim (B \lor (B \lor B)) \\
(3)(4) & \sim B \lor B \\
\text{PS3} & \sim (C \lor B) \lor (B \lor C) \\
(1)(2) & B \lor C
\end{array}
\]

B. Example of a derivation in $PC'$.

<table>
<thead>
<tr>
<th>Premise</th>
<th>$C \lor B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PS3</td>
<td>$\sim (C \lor B) \lor (B \lor C)$</td>
</tr>
<tr>
<td>(1)(2)</td>
<td>$B \lor C$</td>
</tr>
</tbody>
</table>
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(For further examples of proofs and derivations, see Cooley, 1947, sections 29–31.)

VII. Rules of transformation of PC

Formulated with the null class Λ of sentences (Intr. Sem., sect. 26 ff.)

1. $S_m$ is directly derivable (in PC) from $K_i = \text{Df} =$ one of the following conditions is fulfilled:
   a. $K_i$ is Λ, and $S_m$ is $\sim(v_1 \lor v_1) \lor v_1$;
   b. $K_i$ is Λ, and $S_m$ is $\sim v_1 \lor (v_1 \lor v_2)$;
   c. $K_i$ is Λ, and $S_m$ is $\sim(v_1 \lor v_2) \lor (v_2 \lor v_1)$;
   d. $K_i$ is Λ, and $S_m$ is $\sim(v_1 \lor v_2) \lor (\sim(v_3 \lor v_1) \lor (v_3 \lor v_2))$;
   e. (substitution, as in I);
   f. (modus ponens, as in I).

2. The class of sentences derivable from $K_i$ in PC $= \text{Df}$ = the closure of Λ with respect to direct derivability.

3. $S_m$ is provable in PC $= \text{Df}$ = $S_m$ is derivable from Λ.

SEMANTICS

VIII. Terminological remarks

A. We shall deal here with the designative (or cognitive) meaning component only, leaving aside all others (e.g., the emotive and the motivative meaning components). The designative meaning component is the one that is relevant for questions of truth. Thus our theory is pure, designative semantics. Therefore we consider only declarative sentences (called simply 'sentences') and their parts.

B. For the term 'designator', see M & N, pp. 6 ff. We shall include among designators sentences, individuators (e.g., individual constants and individual descriptions), and predicators (e.g., predicates and lambda-expressions). All designators are closed expressions. Designators and open expressions of similar forms we call 'designator formulas'.
NOTES ON SEMANTICS

C. Terminology for kinds of expressions (compare M & N, pp. 6 ff. and footnote 6).

<table>
<thead>
<tr>
<th>Closed Expressions</th>
<th>Constants</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) designator formulas (incl. (2), (3), (4))</td>
<td>designators</td>
<td>designator constants</td>
</tr>
<tr>
<td>(2) sentential formulas</td>
<td>sentences</td>
<td>sentential constants</td>
</tr>
<tr>
<td>(3) individual formulas</td>
<td>individuators</td>
<td>individual constants</td>
</tr>
<tr>
<td>(4) predicator formulas</td>
<td>predicates</td>
<td>predicates</td>
</tr>
</tbody>
</table>

D. Terminology of intensions and extensions.

<table>
<thead>
<tr>
<th>Designator</th>
<th>Intension</th>
<th>Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>individuator one-place predicator n-place predicator sentence</td>
<td>individual concept property n-adic relation proposition</td>
<td>individual class class of n-tuples truth-value (T, F; or 0, 1)</td>
</tr>
</tbody>
</table>

E. Connectives and operators in M.

In rules and technical statements in M, I shall sometimes write as follows:
1. Parentheses are used as in a symbolic language.
2. ‘Not’ precedes its sentence; e.g., ‘not (it rains)’ for ‘it does not rain’.
3. 'Or' is used in the non-exclusive sense; e.g., 'p or q' for 'p or q or (p and q).
4. I shall sometimes use symbolic quantifiers in M, e.g. '(∃x)' as short for 'there is an x such that'.
5. I shall sometimes use in M the lambda-operator for abstract-expressions, e.g., '(λx) (x is large)' as short for the property of being large' (or, in $M^e$, 'the class of those individuals which are large').

The customary rule of conversion (Church) is used for these $\lambda$-expressions (see M & N, p. 3.)

IX. Semantical systems

A. A semantical system for an object language $L$ contains at least rules of the following two kinds.

(1) The rules of formation define 'sentence in $L$' (as in syntax).

(2) The rules of interpretation give an interpretation for (i.e., specify the meanings of) all designators in $L$. These rules may have various forms. We shall use chiefly two forms;

(a) rules of designation ($Des^i$) or intension,

(b) rules of extension ($Des^e$) including rules of truth.

B. We distinguish two operations or investigations concerning any designator, e.g., a predicate $pr_i$ or a sentence $S_j$ (M & N, pp. 202 ff.):

(1) The question of meaning or interpretation. In technical terms: "what is the intension of the designator?" The question is answered by an interpretation; technically, by the semantical rules of interpretation.

(2) The question of factual application: e.g. "to which individuals does $pr_i$ apply?", "is $S_j$ true or false?". In technical terms, it is the question of the extension of the designator. The answer is (in general) found by an empirical investigation of facts.
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X. The semantical system \( L_1 \)

A. Signs of \( L_1 \)  |  Examples  |  Names in \( M \)
---|---|---
1) individual constants  | '\( a_1 \)', '\( a_2 \)', ...  | '\( \text{in}_1 \)', '\( \text{in}_2 \)', ...  
2) one-place predicates  | '\( P_1 \)', '\( P_2 \)', ...  | '\( \text{pr}_1 \)', '\( \text{pr}_2 \)', ...  
3) connectives  | '\( \sim \)', '\( \lor \)'  |  
4) parentheses  | '(', ')'  |  

\( L_1 \) contains no variables.

B. Rules of Designation (\( \text{Des} \)) for \( L_1 \). The relation \( \text{Des} \) holds in all and only those cases which are determined by the following rules:

R1. Individual constants.
(a) \( \text{Des}(\text{in}_1, \text{Los Angeles}), \)
(b) \( \text{Des}(\text{in}_2, \text{the desk in Royce Hall 242}), \) etc.

R2. Predicates.
(a) \( \text{Des}(\text{pr}_1, (\lambda x) (x \text{ is large})), \)
(b) \( \text{Des}(\text{pr}_2, (\lambda x) (x \text{ is red})), \) etc.

(a) If \( \text{Des}(\text{pr}_1, F) \) and \( \text{Des}(\text{in}_p, x) \), then \( \text{Des}(\text{pr}_1 \text{in}_p, F(x)). \)
(b) If \( \text{Des}(S_p, p) \), then \( \text{Des}(\sim S_p, \text{not } p)). \)
(c) If \( \text{Des}(S_p, p) \) and \( \text{Des}(S_q, q) \), then \( \text{Des}(S_p \lor S_q, p \text{ or } q). \)

(In the above rules, '\( \text{Des} \)' is used in three different types. An exact formulation which complies with the rule of types can be obtained either by attaching type indices to '\( \text{Des} \)' or by assigning '\( \text{Des} \)' to a transfinite level; see Intr. Sem., p. 51.)

C. Examples of consequences from the \( \text{Des} \)-rules for \( L_1 \).
From R1(a), R2(a), and R3(a):
(1) \( \text{Des}(\text{pr}_1 \text{in}_1, (\lambda x) (x \text{ is large}))(\text{Los Angeles}), \)
hence by conversion:
(2) \( \text{Des}(\text{pr}_1 \text{in}_1, \text{L.A. is large}). \)
Further, again with conversion:
(3) \( \text{Des}(\sim(\text{pr}_1 \text{in}_1 \lor \sim \text{pr}_2 \text{in}_2), \text{not (L.A. is large or not (the desk R. H. 242 is red}})). \)

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The first argument expression in (3) is a spelling description (i.e., a description specifying each sign) for the sentence \( \sim(P_1a_1 \lor \sim P_2a_2) \) in \( L_1 \).

XI. Some definitions in general semantics for a semantical system \( L \)

A. Conditions of adequacy for designation in \( L \).

A two-place predicate \( 'D' \) in \( M \) is an adequate predicate for designation in \( L \) only if the following condition is fulfilled:

For every designator \( A_i \) in \( L \), a sentence in \( M \) of the form \( 'D(..., \ldots)' \), with a spelling description of \( A_i \) in the place of \( '...' \) and a translation of \( A \) into \( M \) in the place of \( '\ldots' \), follows from the definition or the rules for \( 'D' \). (Intr. Sem., pp. 53 ff.)

On the basis of the rules in \( XB \), \( 'Des' \) fulfills this condition as an adequate predicate for designation in \( L_1 \); see the examples in \( XC \).

B. General definitions of truth and falsity.

\( 'Des' \) is supposed to be an adequate predicate for designation in \( L \).

1. \( A_i \) is true (in \( L \)) = \( \text{df} \) there is a \( p \) such that \( \text{Des}(A_i, p) \) and \( p \).
2. \( A_i \) is false (in \( L \)) = \( \text{df} \) there is a \( p \) such that \( \text{Des}(A_i, p) \) and \( \neg p \).

Theorems.

3. \( A_i \) is a sentence in \( L \) iff there is a \( p \) such that \( \text{Des}(A_i, p) \).
4. \( A_i \) is true in \( L \) or \( A_i \) is false in \( L \), iff \( A_i \) is a sentence in \( L \).

C. Example of a derivation in \( M \) for 'true in \( L_1 \').

<table>
<thead>
<tr>
<th>Premise</th>
<th>( \text{L.A. is large} )</th>
<th>( \text{Des}(pr_{1in_1}, \text{L.A. is large}) )</th>
<th>( \text{Des}(pr_{1in_1}, \text{L.A. is large}) ) and ( \text{L.A. is large} )</th>
<th>( \exists p ) ( \text{Des}(pr_{1in_1}, p) ) and ( p )</th>
<th>( \text{pr}_{1in_1} \text{ is true in } L_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>rules ( XB ) for ( L_1 )</td>
<td>( \text{Des}(pr_{1in_1}, \text{L.A. is large}) )</td>
<td>( \text{Des}(pr_{1in_1}, \text{L.A. is large}) ) and ( \text{L.A. is large} )</td>
<td>( \exists p ) ( \text{Des}(pr_{1in_1}, p) ) and ( p )</td>
<td>( \text{pr}_{1in_1} \text{ is true in } L_1 )</td>
<td></td>
</tr>
</tbody>
</table>
NOTES ON SEMANTICS

XII. Interchangeability in sentences with 'Des'

A. The three Des-relations.

1. Suppose that 'Des(...,---)' is directly obtained from the rules for 'Des in L' and that (in accordance with XIA), '...' is a spelling description for a designator $A_i$ in $L$ and '---' is a translation of $A_i$ into $M$. Under what condition shall we say that another sentence 'Des(...,-..-)', with another designator (in $M$) '-..-' in the place of '---', holds likewise? The answer depends upon what is meant by 'Des'.

2. We shall distinguish three semantical relations: Des*, Desi, and Des%, characterized as follows. The derivative sentence holds
   (a) with 'Des*', iff '---' and '-..-' have the same extension,
   (b) with 'Desi', iff '---' and '-..-' have the same intension,
   (c) with 'Des%', iff '-..-' and '-..-' have the same sense.

3. Two designators are said to have (a) the same extension iff they are materially equivalent, and (b) the same intension iff they are logically equivalent (see $M$ & $N$, sections 3 and 5).

4. We say that two designators have the same sense or are synonymous iff the one can be obtained from the other by transformations of the following kind:
   (a) replacement of a definiendum by its definiens or vice versa (or replacement of corresponding substitution instances);
   (b) rewriting of a bound variable;
   (c) lambda-conversion.

B. Examples for $L_1$.

In each of the subsequent examples (1) and (2), the sentence (a) is directly obtained from the rules XB for $L_1$ (in (2), (a) is supposed to be the rule for $pr_5$). Therefore, (a) holds for Des*, Desi,
and $Des^s$. The second argument expression in (b) is synonymous with that in (a) (assuming suitable definitions for ‘desk’, ‘featherless’, and ‘biped’). Therefore, (b) holds likewise for all three relations. The expression in (c) is logically equivalent but not synonymous with that in (a). Therefore, (c) holds for $Des^e$ and $Des^i$, but not $Des^s$. Finally, the expression in (d) is materially equivalent to that in (a). Therefore, (d) holds for $Des^e$ only.

1. For a sentence in $L_1$

<table>
<thead>
<tr>
<th></th>
<th>$Des^e$</th>
<th>$Des^i$</th>
<th>$Des^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\forall x (x \text{ is large})$ $\text{L.A.}$ or $(\exists x (x \text{ is red}))$ (the desk...)</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(b)</td>
<td>$\forall x (x \text{ is featherless and } x \text{ is a biped})$</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(c)</td>
<td>$\forall x (x \text{ is a biped and } x \text{ is featherless})$</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(d)</td>
<td>$\forall x (x \text{ is human})$</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

2. For a predicate in $L_1$

<table>
<thead>
<tr>
<th></th>
<th>$Des^e$</th>
<th>$Des^i$</th>
<th>$Des^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>$\forall x (x \text{ is featherless and } x \text{ is a biped})$</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(b)</td>
<td>$\forall x (x \text{ has no feathers and } x \text{ has two feet})$</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(c)</td>
<td>$\forall x (x \text{ is a biped and } x \text{ is featherless})$</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>(d)</td>
<td>$\forall x (x \text{ is human})$</td>
<td>T</td>
<td>F</td>
</tr>
</tbody>
</table>

With $Des^s$ both in (1) and in (2), (a) is true but (d) is false, although the interchanged expressions have the same extension. Thus, sentences with $Des^s$ are bot extensional ($M & N$, sect. 11). The same holds for $Des^s$. Therefore, $Des^e$ and $Des^s$ require non-extensional metalanguages. The extensional metalanguage ($M^e$) can accommodate only $Des^e$.

XIII. Three metalanguages: $M^e$, $M^i$, $M^s$

A. Three identity signs.

We take $M^e$ as a metalanguage containing $Des^e$: likewise $M^i$ with $Des^i$, and $M^s$ with $Des^s$.
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We shall use ‘\(=^e\)’ in \(M^e\) as a sign of identity of extensions; likewise ‘\(=^i\)’ in \(M^i\) for identity of intensions, and ‘\(=^s\)’ in \(M^s\) for identity of senses. Thus, if ‘\(...)\)’ and ‘\(-\)’ stand for designators (either sentences or predications or individuators), then

(1) ‘\(\ldots =^e - -\)’ is true in \(M^e\) iff ‘\(\ldots\)’ and ‘\(-\)’ are materially equivalent;
(2) ‘\(\ldots =^i - -\)’ is true in \(M^i\) iff ‘\(\ldots\)’ and ‘\(-\)’ are logically equivalent;
(3) ‘\(\ldots =^s - -\)’ is true in \(M^s\) iff ‘\(\ldots\)’ and ‘\(-\)’ are synonymous.
(This use of ‘\(\; =^s\)’ in \(M^s\) is like that of ‘\(\; =\)’ in the symbolic object language in \(M & N\), sect. 3).

B. Principles of interchangeability.

In all three cases, if ‘\(\ldots = ^o - -\)’ holds in \(M^o\) (the superscript \( ^o \) stands for ‘\(e\)’ or ‘\(i\)’ or ‘\(s\)’), then ‘\(\ldots\)’ and ‘\(-\)’ are interchangeable in any context, in accordance with the following principle:

(1) If \(A_i\) and \(A_j\) are designators in \(M^o\), \(-- A_i --\) is a designator containing \(A_i\), and \(-- A_j --\) is formed from \(-- A_i --\) by replacing \(A_i\) by \(A_j\) then from \(A_i = A_j\), \(-- A_i -- = ^o -- A_j --\) is deducible in \(M^o\).

Therefore:

(2) If \(-- A_i --\) and \(-- A_j --\) are sentences in \(M^o\), the following inference is valid in \(M^o\):
\[
A_i =^o A_j
\]
\[
\rightarrow A_i --
\]
\[
\rightarrow A_j --
\]

Hence, as a special case:

(3) The following inference is valid in \(M^o\):
\[
A_i =^o A_j
\]
\[
Des^o (\ldots, A_i)
\]
\[
Des^o (\ldots, A_j)
\]

Thus the desired transformations of sentences with ‘\(Des^o\)’ (XII A2) are obtained.
C. General characterization of the three metalanguages for an object language $L$.

<table>
<thead>
<tr>
<th></th>
<th>$M^e$</th>
<th>$M^i$</th>
<th>$M^s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Designators in $M^{oo}$ are interchangeable (a) iff they are materially equivalent (in $M^e$)</td>
<td>logically equivalent (in $M^i$)</td>
<td>synonymous (in $M^s$)</td>
</tr>
<tr>
<td></td>
<td>(b) iff they have the same extension</td>
<td>intension</td>
<td>sense</td>
</tr>
<tr>
<td>2.</td>
<td>The values of the variables in $M^{oo}$ are extensions</td>
<td>intensions</td>
<td>senses</td>
</tr>
<tr>
<td>3.</td>
<td>The semantics of $L$, formulated in $M^{oo}$, can be based on a relation Des$^e$</td>
<td>Des$^i$</td>
<td>Des$^s$</td>
</tr>
<tr>
<td>4.</td>
<td>$Des^{oo}$ assigns to each designator in $L$ an entity, namely an extension</td>
<td>an intension</td>
<td>a sense</td>
</tr>
<tr>
<td>5.</td>
<td>$Des^{oo}$ assigns the same entity to two designators in $L$ iff they are materially equivalent (in $L$)</td>
<td>logically equivalent (in $L$)</td>
<td>synonymous (in $L$)</td>
</tr>
</tbody>
</table>

D. Example for C5 in $M^i$.

$pr_1 in_1 \lor pr_2 in_2$ ($S_1$) and $pr_2 in_2 \lor pr_1 in_1$ ($S_2$) are logically equivalent in $L_1$. From the rules of $L_1$ (XB):

$Des^i (S_2, \text{the desk . . . is red or L.A. is large})$.

By (XIIB) 1(C): $Des^i (S_1, \text{the desk . . . is red or L.A. is large})$.

Thus $Des^i$ assigns the same entity to $S_1$ and to $S_2$. 

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XIV. The vocabulary of the semantical metalanguage

A. Semantical metalanguage $M$.

B. The vocabulary of the semantical metalanguage $M$ consists of the following four parts (the above diagram shows only some constants of the parts (2), (3), and (4)):

1. **The logical vocabulary**: logical constants ('not', 'or', 'every', etc.) and general variables ('$x$', '$F$', '$p$', etc.).

2. **The syntactical vocabulary**: Names of the signs in $L$, and a notation for concatenation. Thus a spelling description for any expression in $L$ can be formulated. Further, syntactical variables (e.g., '$pr_i$', '$A_i$', etc.).

3. **The non-semiotical vocabulary** (translation vocabulary): descriptive constants referring to non-linguistic entities (e.g., things in the world). This vocabulary, together with (1), must be sufficient for a translation of all sentences in $L$.

4. **The semantical vocabulary**. The semantical terms are defined on the basis of the terms of the three other parts.

C. **The semantical theory** (pure semantics) contains only those sentences of $M$ which

(a) contain at least one term of the semantical vocabulary (4), and
(b) are logically true. (Thus the theory does not include 'pr \_i in \_i is true in L \_1'. This sentence is not a theorem of semantics but one of geography; it is factually true and logically equivalent to the sentence 'L.A. is large' in M and to 'P \_1 a \_1' in L \_1.

XV. The uniqueness of the designatum

A. The rules of Des\(\infty\) for L \_1 are as in XB. But now we can replace the formulation “Des holds in all and only these cases ...” by a more exact one with 'x \(\infty\)', in M\(\infty\), adding the following rules to R1, R2, and R3.

R1 (\(\infty\)). For any in \_i, x, and y, if Des\(\infty\) (in \_i, x), then Des\(\infty\) (in \_i, y) only if x = \(\infty\)y.

R2 (\(\infty\)). For any pr \_i, F, and G, if Des\(\infty\) (pr \_i, F), then Des\(\infty\) (pr \_i, G) only if F = \(\infty\)G.

R3 (\(\infty\)). For any A \_i, p, and q, if Des\(\infty\) (A \_i, p) then Des\(\infty\) (A \_i, q) only if p = \(\infty\)q.

B. Theorems. based on the above rule for L \_1.

(1) For any in \_i, x, and y, if Des\(\infty\) (in \_i, x), then Des\(\infty\) (in \_i, y) iff x = \(\infty\)y.

(2) For any pr \_i, F, and G, if Des\(\infty\) (pr \_i, F), then Des\(\infty\) (pr \_i, G) iff F = \(\infty\)G.

(3) For any A \_i, p, and q, if Des\(\infty\) (A \_i, p), then Des\(\infty\) (A \_i, q) iff p = \(\infty\)q.

These theorems together with the rules XB say that, for each designator in L \_1, there is exactly one designatum \(\infty\) (i.e., entity assigned to it by Des\(\infty\)).

C. Sufficient and necessary condition of adequacy for designation in L.

A two-place predicate 'D' in M\(\infty\) is an adequate predicate for designation \(\infty\) in L iff the following two conditions are fulfilled:

(1) the condition in XIA (which is only a necessary condition of adequacy),

(2) the condition of the uniqueness of the designatum \(\infty\): Every
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sentence in $M^{oo}$ of the form 'if $D(..., --)$ and $D(..., -. -)$, then $- - = o_o$. ' follows from the definition or the rules for 'D'.

On the basis of the theorems under (B), $Des^{oo}$ for $L_1$ fulfills also the condition (2) and therefore is an adequate predicate in $M^{oo}$ for designation$^{oo}$ in $L_1$.

D. Theorems.

Henceforth, when we refer to a relation $Des^{oo}$ for a system $L$ it is assumed that the rules for it in $M^{oo}$ are such that the adequacy condition (C) is fulfilled. Then the following holds for $L$.

(1) For any $A_i$ and $p$, if $Des^{oo} (A_i, p)$, then

(a) $A_i$ is true iff $p$;
(b) $A_i$ is false iff not $p$.

(2) For any $A_i$, if $A_i$ is true, $A_i$ is not false. (Indirect proof. The assumption that $A_i$ is both true and false leads to the conclusion that, for some $p$, $p$ and not $p$, which is impossible.)

(3) For any sentence $S_i$, $S_i$ is false iff $S_i$ is not true.

XVI. Truth

A. Rules of truth for $L_1$.

The following rules may take the place of the rules of $Des^{oo}$ for sentences (XB, R3 and XVA) and the definition of truth (XIB (1)); they lead to the same results concerning truth in $L_1$.

R3'.

(a) If $Des^{oo}(pr_i, F)$ and $Des^{oo}(in_j, x)$ then $pr_i in_j$ is true (in $L_1$) iff $F(x)$.
(b) $\sim S_i$ is true iff $S_i$ is not true.
(c) $S_i \lor S_j$ is true iff $S_i$ is true or $S_j$ is true.

The usual truth-tables for sentential connectives are diagram formulations of rules of truth, corresponding to formulations in words like (b) and (c) above.

The rules R3' give a recursive definition for 'true in $L_1$'. (An equivalent explicit definition can be formulated with the help of the concept of closure, V (6).)
B. Sufficient and necessary condition of adequacy for truth in a system \( L \).

A one-place predicate \('T'\) in \( M \) is an adequate predicate for truth in \( L \) iff the following condition is fulfilled. For every sentence \( S_i \) in \( L \), a sentence in \( M \) of the form \('T(...) \iff \cdot \cdot \cdot') with a spelling description of \( S_i \) in the place of \('\cdot \cdot \cdot') and a translation of \( S_i \) into \( M \) in the place of \('\cdot \cdot \cdot') follows from the definition of the rules for \('T'). (This is the so-called Leśniewski requirement; see Tarski, "Wahrheitsbegriff", p. 305, "Konvention W"; Tarski, "The semantic conception of truth", sect. 4; Carnap, *Intr. Sem.*, pp. 26–29.)

Suppose that \('T' fulfills the above condition. Then \('T(...) \iff \cdot \cdot \cdot') is logically true. Therefore, the sentence \('T(...)') in \( M \) is logically equivalent to the translation of \( S_i \) into \( M \) and hence also to \( S_i \).

Thus, to assert that a sentence is true means the same as to assert the sentence itself (see Tarski, "The semantic conception of truth", and Carnap, "Remarks on induction and truth", sect. 3). If 'true in \( L_1 \)' is introduced in either of the two ways mentioned in (A), then the condition of adequacy is fulfilled.

XVII. Denotation

A. Following R.M. Martin (*J.S.L.* 18, 1953, 1–8), we use 'denotes' in such a sense that '\( P_1 \)' in \( L_1 \) is said to denote every single individual which is large (not, as in traditional terminology, the class of large individuals). In \( M^{oo} \), we can define '\( Den^{oo} \)' on the basis of '\( Des^{oo} \)'.

\[
(1) \quad Den^{oo}(A_i, x) = \exists F \left[ Des^{oo}(A_i, F) \text{ and } Fx \right].
\]

On the other hand, if '\( Den^{oo} \)' is introduced by \( Den^{oo} \)-rules (see below), we can define '\( Des^{oo} \)' for predicators:

\[
(2) \quad Des^{oo}(A_i, F) = \exists x \left[ Den^{oo}(A_i, x) \right].
\]

Hence we obtain:

\[
(3) \quad \text{For any predicator } A_i, Des^{oo}(A_i, \lambda x) \left( Den^{oo}(A_i, x) \right).
\]

[The above definitions and the subsequent \( Den^{oo} \)-rules may be
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used in $M^e$ and $M^i$. It is, however, doubtful whether the adherents of a sense-logic would admit them in $M^e$.]

B. Rules of denotation. If Des-rules for predicates are used, predicate variables are needed in the formulation of Des-rules for sentences (e.g., in XB, R3(a)) or of truth-rules (e.g., in XCV, R3' (a)). If, instead, Den-rules for predicates are used, predicate variables are not needed (unless the object language contains predicate variables). [However, predicate variables are needed for an explicit definition of truth with the help of closure.] Furthermore, Den-rules do not contain $\lambda$-expressions. We can then obtain results of the simple form (d) in the examples in (D) and (E) below without the use of $\lambda$-conversion (compare XCV(2)). We may then drop $\lambda$-conversion in the definition of synonymy (XIIA4(c)), which seems preferable.

C. Den-rules for $L_1$. The following rules may take the place of XB, R2 and XVA, R2(a):

R2' Den-rules for predicates in $L_1$.
   (a) $Den^{\circ}(pr_1, x) = ^{\circ}(x$ is large).
   (b) $Den^{\circ}(pr_2, x) = ^{\circ}(x$ is red).
   etc.

Then XCVIA, R3'(x) is replaced by the following rule: R3'' (x). If $Des^{\circ}(in_j, x)$ then $pr_i in_j$ is true (in $L_1$) iff $Den^{\circ}(pr_i, x)$.

D. Example of a proof, with the rules in (C).

XB, R1(x) $Des^{\circ}(in_1, L.A.)$ (a)
R2'(x) $Den^{\circ}(pr_1, L.A.) = ^{\circ}(L.A. is large)$ (b)
(a), R3''(x) $pr_i in_1$ is true in $L_1$ iff $Den^{\circ}(pr_1, L.A.)$ (c)
(b), (c), XCVIA2(2) $pr_i in_1$ is true in $L_1$ iff L.A. is large (d)

E. Des-rule for atomic sentences in $L_1$ based on the Den-rules. On the basis of the rules R2' above, the following rule takes the place of XB, R3(x), while (b) and (c) remain unchanged.

R3'''(x). If $Des^{\circ}(in_p, x)$, then $Des^{\circ} (pr_i in_j Den^{\circ}(pr_i, x))$.

Example of a proof.
(a) and (b) as in (D).

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(a), R3""(a)  \(\text{Des}^{\circ}(\text{pr}_1\text{in}_1, \text{Den}^{\circ}(\text{pr}_1, \text{L.A.}))\)  (c)
(b), (c), XIIIIB(3)  \(\text{Des}^{\circ}(\text{pr}_1\text{in}_1, \text{L.A. is large})\)  (d)

XVIII. Interpretation in the extensional metalanguage \(M^e\)

A. Do the rules for ‘\(\text{Des}^e\) in \(L\)’ convey the intended interpretation of \(L\)?

The rules are supposed to fulfill the condition of adequacy (XVC). Hence, for any designator \(A\), the rules yield a \(\text{Des}^e\)-sentence containing a translation of \(A\). E.g., for \(L_1\), we obtain:

(1) \(\text{Des}^e(\text{pr}_1\text{in}_1, \text{L.A. is large})\).

But the following sentence is likewise true:

(2) \(\text{Des}^e(\text{pr}_1\text{in}_1^1, \text{Paris is in France})\).

This shows again that \(\text{Des}^e\) is the relation between a designator and its extension, not its meaning. The sentence (1) does convey information about the intended meaning of \(\text{pr}_1\text{in}_1\), but (2) does not. (1) follows logically from the rules, while for (2) the factual premise ‘L.A. is large = e Paris is in France’ is needed. In every sentence of the form ‘\(\text{Des}^e(\text{pr}_1\text{in}_1, ---)\)’, which follows logically from the rules, ‘---’ is logically equivalent to ‘L.A. is large’. Therefore every sentence of this kind gives the intended interpretation (logical meaning, content) of \(\text{pr}_1\text{in}_1\). In this sense, the intended interpretation of \(L\) is conveyed by the \(\text{Des}^e\)-rules in \(M^e\).

B. Do the rules for ‘true in \(L\)’, formulated in \(M^e\), convey the intended interpretation of \(L\)?

The rules are supposed to fulfill the condition of adequacy (XVIB). The situation is analogous to that in (A). It is often said that to understand the meaning of a sentence is to know under what conditions it would be true. But this should be qualified as follows: a logically true statement of a (necessary and sufficient) truth condition for a sentence \(S\) conveys the meaning of \(S\). For example, (3) and (4), are both true in \(M^e\).

(3) \(\text{pr}_1\text{in}_1\) is true in \(L_1\) iff L.A. is large.

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(4) \( \text{pr}_1 \text{in}_1 \) is true in \( L_1 \) iff Paris is in France.

(3) gives the intended meaning, (4) does not. In every sentence in \( M^e \) of the form \( \text{pr}_1 \text{in}_1 \) is true in \( L_1 \) iff --- which follows logically from the rules, ‘---’ is logically equivalent to ‘L.A. is large’. Thus the rules in \( M^e \) for ‘true in \( L \)’ convey the intended interpretation of \( L \).

XIX. Philosophical issues concerning the semantical concept of truth

A. Some philosophers commit a confusion of ‘true’ with ‘verified’ (or ‘known to be true’, ‘well established’, ‘highly probable’, etc.). The distinction is important. Verification is relative to person and time, truth is not. Read pp. 119–123 of Carnap, “Truth and Confirmation”, in Feigl-Sellars.

For (B) to (F), read Tarski, “The semantic conception of truth”, esp. Part II “Polemical remarks”.

B. Objection: ‘true’ can be eliminated, and thus is useless. (See Tarski, sect. 16, 20–22.)

C. The question of agreement with the classical conception (correspondence theory of truth, e.g. Aristotle). (See Tarski, sect. 17.)

D. The question of agreement with every-day usage. (See Tarski, sect. 17; compare Ness.)

E. Objection: the concept has no philosophical importance. (See Tarski, sect. 18.) Later Black raised this objection (“The semantic definition of truth”); for his main arguments, see (G), (H), (J) and (K) below.

F. Objection: the semantical conception of truth involves metaphysics. (See Tarski, sect. 19, with reference to Nagel; compare Nagel, 1944, p. 67n.)

G. Objection: the concept is defined only for artificial languages. Black (sect. 6) believes that only a definition for colloquial English would be philosophically relevant (sect. 6). He admits
that, in principle, such a definition could be given; but then objection (H) would hold.

H. *Objection:* The definition of truth is not general, but is based on an enumeration of instances; any attempt to generalize the set of sentences of the form '... is true iff ---' referred to in the adequacy condition (XVI B) leads to nonsensical formulations: we seem to understand the general principle underlying the definition, but this principle cannot be formulated (Black, sect. 3, 4, 6, 7). *Answer.* The definition can be stated in a general form in terms of designation (see XI B (1)); this was D17-C1 in *Intr. Sem.* This form of the definition expresses the underlying principle. The definition is based on a general definition for 'Des' with respect to sentences (X B R3). This, in turn, is based on Des-rules for individual constants and predicates (X B, R1, R2). The latter rules proceed indeed by enumeration; this is inevitable because the interpretation of a language must ultimately go back to its dictionary.

J. *Objection:* The semantical definition of truth is neutral with respect to the philosophical controversy; the adherents of the correspondence theory, the coherence theory, and the pragmatist theory of truth would all accept the sentences of the form '... is true iff ---' specified in the adequacy condition (XVI B) (Black, sect. 8, 9). *Answer.* If this were the case, then the three theories would be based on essentially the same concept, because if each of two predicates fulfills the adequacy condition, then they are logically equivalent.

K. *Objection:* The philosophically important concept of truth is not, like the semantical concept, a property of sentences expressed in the metalanguage, but rather a concept used in the object language in the form "it is true that ...". (Black, sect. 8; Strawson, "Truth", *Analysis* 9, 1949, reprinted in MacDonald: "Truth is not a property of symbols; for it is not a property.") *Answer.* This use of 'true' seems indeed more frequent in the everyday language. It is useful for purposes of emphasis, opposition, and the like. But its usefulness for
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theoretical purposes, i.e., for expressing cognitive content, is nil. It can be explicated by the explicit definition:

(1) \( T(p) = \text{df} \ p \) (Intr. Sem. D17-1).
And analogously for "It is false that . . .":

(2) \( F(p) = \text{df} \sim p. \)

Thus 'T' and 'F' are extensional connectives. 'F' is merely a sign of negation. 'T' is the redundant connective; its omission does not change the content. E.g., the following two sentences are logically equivalent:

(3) It is true that L.A. is large.
(4) L.A. in large.

Likewise the following two:

(5) It is false that L.S. is large.
(6) L.A. is not large.

XX. Semantical system for the language \( L_2 \) with individual variables

A. General remarks on a language \( L \).

1. Let \( L \) be an object language with individual variables. Rules of interpretation for \( L \) must contain a rule specifying the domain of individuals of \( L \), i.e., the class of those entities which are to be taken as the values of the individual variables. The domain may be infinite. It is not required that \( L \) contain an individual constant for every individual in \( L \).

2. A value assignment (VA) for the individual variables in \( L \) is a function which assigns to every individual variable in \( L \) one individual. We take in \( M \) 'VA_1', 'VA_2', etc. as constants for VA, and 'VA_k', 'VA_m', etc. as variables. We write, e.g., 'VA_2(inv_4)' as short for 'the individual assigned by VA_2 to inv_4'.

3. An open designator formula, e.g., 'P_1x_1 \lor P_2x_3', does by itself not designate anything. However, we can give an interpretation for it by specifying what the formula
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designates with respect to a given \( VA_k \); \( VA_k(inv_1) \) is then, so to speak, taken as the designatum of \( inv_1 \). Note that a \( VA \) assigns an individual, not an individual constant. This has the advantage that open formulas can be interpreted even if \( L \) does not contain individual constants for all individuals.

4. In the following our metalanguage will always be \( M^e \), unless the contrary is stated. We write briefly ‘\( M \)’ for ‘\( M^e \)’, ‘\( = \)’ for ‘\( =^e \)’, ‘\( \text{Des} \)’ for ‘\( \text{Des}^e \)’, and ‘\( des \)’ for ‘\( des^e \)’. Because of the uniqueness of the designatum (XV), we may use a functor ‘\( des \)’ instead of the predicate ‘\( \text{Des} \)’. We write ‘\( des(A_i) \)’ for ‘the designatum of \( A_i \)’, i.e., ‘the entity (extension) to which \( A_i \) bears the relation \( \text{Des} \)’. We write ‘\( des_k(A_i) \)’ for ‘the designatum (i.e., the extension) of \( A_i \) with respect to the value assignment \( VA_k \).’

B. Signs of \( L_2 \).

(1) – (4) like those of \( L_1 \) (XA).

(5) Individual variables, e.g. \( x_1 \), \( x_2 \), etc. Their names in \( M \): ‘\( inv_1 \)’, ‘\( inv_2 \)’, etc.

C. Rules of formation for \( L_2 \).

An expression \( A_i \) in \( L_2 \) is a sentential formula in \( L_2 \) iff \( A_i \) has one of the following five forms, where \( S_j \) and \( S_k \) are sentential formulas:

(1) \( pr_i \) \( in_j \) (atomic sentence),
(2) \( pr_i \) \( inv_j \) (open atomic sentential formula),
(3) \( \neg S_j \) (negation),
(4) \( (S_j \lor S_k) \) (disjunction),
(5) (\( inv_1 \)) \( (S_j) \) (universal sentential formula).

\( A_i \) is a sentence in \( L_2 \) = def \( A_i \) is a closed sentential formula in \( L_2 \).

D. Rules of interpretation for \( L_2 \).

\( RI \) The individuals in \( L_2 \) are the material bodies at a given time \( t_0 \).
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The subsequent rules RD 1 to 6 constitute a recursive definition which enables us to determine \( \text{desk}(A_i) \) for any designator formula \( A_i \) with respect to any \( VA_k \). Later the function \( \text{des} \) will be defined, but only for designators, i.e., closed designator formulas. For any sentential formula \( S_b \), \( \text{desk}(S_b) \) is one of the truth-values, either \( T \) (for truth) or non-\( T \) (for falsity), ('\( T \)' is a sentential constant in \( M \); it may be regarded as short for, say, '\( a_1 = a_1 \).') If a rule says: "\( \text{desk}(S_i) = T \) iff...", this is meant to imply that, if the condition "..." is not fulfilled, \( \text{desk}(S_i) = \text{non-T} \). "\( \text{desk}(S_i) = T \)" says in effect that \( S_i \) is true with respect to \( VA_k \) or that \( S_i \) is satisfied by \( VA_k \). Thus the rules RD yield, for any sentence \( S_i \) in \( L_2 \), a necessary and sufficient condition for the truth of \( S_i \) with respect to any given \( VA_k \). While RD 1 and 2 are analogous to X BR 1 and 2 for \( L_1 \), RD 4 to 6 are analogous to the truth-rules XVI R3' (a) to (c).

RD. Rules of \( \text{desk} \) for \( L_2 \).

RD1. For predicates.
   (a) \( \text{desk}(pr_1) = (\lambda x)(x \text{ is large}), \) etc.

RD2. For individual constants.
   (a) \( \text{desk}(in_1) = \text{Los Angeles}, \) etc.

RD3. For individual variables. For any \( inv_j \), \( \text{desk}(inv_j) = VA_k(inv_j) \).

RD4. If \( s_j \) is either an individual constant or an individual variable and \( \text{desk}(s_j) = x \) and \( \text{desk}(pr_i) = F \), then \( \text{desk}(pr_i; s_j) = T \) iff \( F(x) \).

RD5. \( \text{desk}(\sim S_i) = T \) iff \( \text{desk}(S_i) = \text{non-T} \).

RD6. \( \text{desk}(S_i \lor S_j) = T \) iff \( \text{desk}(S_i) = T \) or \( \text{desk}(S_j) = T \).

RD7. Let \( S_i \) be \( (inv_i)(S_j) \). Then \( \text{desk}(S_i) = T \) iff, for every value assignment \( VA_m \) that differs from \( VA_k \) at most for \( inv_i \), \( \text{desk}(S_j) = T \).
E. Example for RD7.

(Here and in the examples G, we shall use 'S_1', 'S_2', etc. also as individual constants in M.) Let S_2 be a universal sentential formula of the following form with free variables 'x_2' and 'x_3'; let the operand be S_1.

\[ S_2: \ (x_1)(x_2 \ldots x_n) \]

- Individuals assigned by VA_k: \( a_2 \ a_3 \ a_5 \)
- Individuals assigned by VA_{m_1}: \( a_1 \ a_3 \ a_5 \)
- Individuals assigned by VA_{m_2}: \( a_2 \ a_3 \ a_5 \) (= VA_k)
- Individuals assigned by VA_{m_3}: \( a_3 \ a_3 \ a_5 \)

etc. etc.

VA_{m_1}, VA_{m_2}, etc. are those VA which differ from the given VA_k either for no variable (e.g., VA_{m_2}) or for 'x_1' only. Thus, according to RD7, S_2 is true with respect to the given VA_k iff S_1 is true for those values of 'x_2' and 'x_3' which are assigned by VA_k and for every value of 'x_1'.

F. Definitions of designation, truth, and falsity. According to the rules RD 1 to 7, if \( A_i \) is a designator (and hence closed), then \( des(A_i) \) is independent of VA_k. Therefore we define as follows, using an arbitrarily chosen VA_1:

1. For any designator \( A_i \) in \( L_2 \), \( des(A_i) = \text{df} \ des_1(A_i) \), where VA_1 assigns \( a_1 \) to every individual variable.
2. \( A_i \) is true in \( L_2 = \text{df} \ A_i \) is a sentence and \( des(A_i) = T \).
3. \( A_i \) is false in \( L_2 = \text{df} \ A_i \) is a sentence and \( des(A_i) = \text{non-T} \).

G. Examples.

1. For an atomic sentence. Let \( S_3 = \text{pr}_{1}in_1 \). For any VA_k, hence also for VA_1, we obtain from RD 1, 2, 4 (writing 'a_1' for 'Los Angeles'):

   (a) \( des_1(S_3) = T \) iff \( a_1 \) is large.

By F(1), and with ' = ' for 'iff' (see XIII A(1)):

   (b) \( (des(S_3) = T) = a_1 \) is large.

The following is a tautology (see, e.g., Foundations, T21-5u(1)):
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(c) \( (p = T) = p \).
Hence from (b):
(d) \( \text{des}(S_3) = a_1 \) is large.
This result corresponds to XC(2) for \( L_1 \). From (b), with
F(2):
(e) \( S_3 \) is true iff \( a_1 \) is large.

2. For a universal sentence. Let \( S_1 \) be \( 'P_1x_1' \) and \( S_2 = '(x_1) (P_1x_1)' \). Take the various \( VA \)'s as in the earlier example E, but now with \( 'x_1' \) as the only free variable in \( S_1 \). Then
\( VA_k(inv_1) = a_2 \), and, for every \( i \), \( VA_{m_1}(inv_1) = a_i \). Hence by
RD 3, \( \text{des}_k(inv_1) = a_2 \) and, for every \( i \), \( \text{des}_{m_1}(inv_1) = a_i \) by
RD 1(a) and RD 4:
(a) For every \( i \), \( \text{des}_{m_1}(S_1) = T \) iff \( a_i \) is large.
Hence by RD 7:
(b) \( \text{des}_k(S_2) = T \) iff, for every \( VA_{m_1} \), \( \text{des}_{m_1}(S_1) = T \).
(c) \( \text{des}_k(S_2) = T \) iff, for every \( i \), \( a_i \) is large.
(d) \( \text{des}_k(S_2) = T \) iff every individual is large.
Since \( S_2 \) is closed, (d) holds for every \( VA_k \), hence also for
\( VA_1 \).
Then by F(1):
(e) \( \text{des}(S_2) = T \) iff every individual is large.
Hence with (c) of Example (1):
(f) \( \text{des}(S_2) = T \) iff every individual is large.
From (e) with F(2):
(g) \( S_2 \) is true iff every individual is large.
Since \( 'every individual is large' \) is the translation of \( S_2 \),
the adequacy conditions for designation and for truth are
fulfilled.

XXI. Preliminary explanations of the language \( L_3 \) with a type
system

A. The metalanguage \( M \).

1. We use, as before (XX A4), \( M \) (i.e., \( M^o \)) and we write \( '=' \)
and \( 'des' \).
2. We use in $M$ numerals ‘1’, ‘2’, etc. as individual constants. They refer to enumerated positions in an ordered domain. (Thus $M$ is a coordinate language, see $M \& N$, sect. 18.) ‘3 is blue’ is understood as ‘the position No. 3 is blue’. We use as individual variables in $M$ ‘$j$’, ‘$k$’, . . . , ‘$n$’. Their values are numbers (‘number’ is here meant as ‘positive integer’) and, secondarily, positions (see $M \& N$, p. 86).

3. We sometimes use in $M$ ‘$u$’, ‘$v$’, etc. as variables without fixed type. (Strictly speaking, they should be regarded as variables of a transfinite level, see the remark in X B.)

B. The system of types. This system serves as a classification of designator formulas of the object language $L_3$, and also as a classification of the corresponding extensions. The types are 0, 1, 2, etc. Type 0 comprises the individuals, type 1 the classes of individuals, type 2 the classes of classes of individuals, etc. The $m$-the constant of type $n$ ($n = 0, 1, 2, \ldots$) in $L_3$ consists of the letter ‘$a$’ followed by $n$ (0 or more) superscript primes, followed by $m$ (one or more) subscript primes. As a convenient unofficial notation, we write numerals instead of the strings of primes, e.g., ‘$a_3^2$’ and ‘$a_0^0$’. The $m$-th variable of type $n$ consists of ‘$x$’ with $n$ superscript primes and $m$ subscript primes. The names in $M$ of constants in $L_3$ are formed with ‘$c$’, and the names of variables with ‘$v$’; e.g., ‘$c_3^2$’ is the name of ‘$a_3^2$’, and ‘$v_4^2$’ is the name of ‘$x_2^3$’. We use ‘$n$’ and ‘$m$’ as numerical variables in $M$.

<table>
<thead>
<tr>
<th>Class</th>
<th>Designator Formulas</th>
<th>Extensions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Open or closed</td>
<td>Closed</td>
</tr>
<tr>
<td>$S$</td>
<td>sentential formulas</td>
<td></td>
</tr>
<tr>
<td>type 0</td>
<td>individual formulas</td>
<td></td>
</tr>
<tr>
<td>type 1</td>
<td>predicate formulas</td>
<td></td>
</tr>
<tr>
<td>type 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td>sentences</td>
</tr>
<tr>
<td></td>
<td></td>
<td>individuators</td>
</tr>
<tr>
<td></td>
<td></td>
<td>predicates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Classes</td>
</tr>
</tbody>
</table>
C. Other signs and expressions in $L_3$.

1. $L_3$ contains connectives for negation and disjunction. ‘$\equiv$’ is used as sign of identity of extensions (like ‘$\equiv$’ in $M^c$, XII A), hence also as a biconditional connective ($M$ & $N$, pp. 13 f).

2. Variables of all types are used in $L_3$ in three kinds of operators:
   (a) universal quantifiers: ($v^n_m$);
   (b) lambda-operators for class expressions ($M$ & $N$, p. 3): ($\lambda v^n_m$);
   (c) iota operators for descriptions: ($\nu^n_m$).

3. In order to assure that a closed iota-description $D_j$ of the form ($\nu^n_m$(Sk)) has a unique designatum even if $S_k$ does not fulfill the uniqueness condition (i.e., if either no extension of type $n$ or more than one satisfy $S_k$), we make the following convention (see $M$ & $N$, sect. 8, method III b):

<table>
<thead>
<tr>
<th>Type $n$</th>
<th>des($D_j$) in the case of non-uniqueness</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 0</td>
<td>an arbitrarily chosen individual, say, the position No. 1</td>
</tr>
<tr>
<td>(b) $n$ (&gt; 0)</td>
<td>the null class of type $n$</td>
</tr>
</tbody>
</table>

The rules for des$_p$ and des for $L_3$ (in XXIV and XXVI) will be made so as to yield these results.

XXII. Rules of formation for $L_3$

A. $L_3$ contains the following eleven signs:

<table>
<thead>
<tr>
<th>Sign in $L_3$</th>
<th>Explicit name of sign in $M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $\sim$</td>
<td>sign of negation</td>
</tr>
<tr>
<td>(2) $\lor$</td>
<td>sign of disjunction</td>
</tr>
<tr>
<td>(3) $\equiv$</td>
<td>sign of identity (and biconditional)</td>
</tr>
<tr>
<td>(4) $a$</td>
<td>sign of constants</td>
</tr>
<tr>
<td>(5) $x$</td>
<td>sign of variables</td>
</tr>
</tbody>
</table>
In spelling descriptions in $M$, we use ‘c’ as the name of ‘a’, ‘v’ as the name of ‘x’, and each of the other signs as a name of itself. For convenience, we take ‘$c^2_a$’ as the name of ‘$a^2_a$’, and ‘$v^0_1$’ as the name of ‘$x^0_1$’ etc.

B. Designator formulas in $L_3$.

An expression is a designator formula in $L_3$ iff it has one of the following ten forms, where $D^n_j$ and $D^n_k$ are any designator formulas of type $n$, and $S_j$ and $S_k$ are any sentential formulas, i.e., formulas of class $S$. (For ‘designator’ and other terms, see VIII C.)

<table>
<thead>
<tr>
<th>Form of Expression</th>
<th>Class</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c^n_m$</td>
<td>type $n$</td>
</tr>
<tr>
<td>$v^n_m$</td>
<td>type $n$</td>
</tr>
<tr>
<td>$D^{n+1}_j$</td>
<td>$S$</td>
</tr>
<tr>
<td>$(D^n_j = D^n_k)$</td>
<td>$S$</td>
</tr>
<tr>
<td>$(S_j = S_k)$</td>
<td>$S$</td>
</tr>
<tr>
<td>$\sim S_j$</td>
<td>$S$</td>
</tr>
<tr>
<td>$(S_j \lor S_k)$</td>
<td>$S$</td>
</tr>
<tr>
<td>$(v^n_m)S_k$</td>
<td>$S$</td>
</tr>
<tr>
<td>$(v^n_m)S_k$</td>
<td>type $n$</td>
</tr>
<tr>
<td>$(\lambda n^n_m)S_k$</td>
<td>type $n + 1$</td>
</tr>
</tbody>
</table>

(In the actual writing of formulas, we omit parentheses in accordance with the usual conventions.)
C. Unofficial abbreviations in $L_3$, introduced by definition schemata.

Conjunction:
(1) $(S_i \cdot S_j) \equiv \sim (\sim S_i \lor S_j)$.

Conditional:
(2) $(S_i \Rightarrow S_j) \equiv (\sim S_i \lor S_j)$.

Existential quantifier:
(3) $(\exists v^n_m S_i) \equiv \sim (v^n_m) \sim S_i$.

(4) For $k \geq 1$, $\{v^n_1, v^n_2, \ldots, v^n_k\} \equiv (\lambda v^n_{k+1}) [v^n_{k+1} \equiv v^n_1 \lor \ldots \lor (v^n_{k+1} \equiv v^n_k)]$.

This is the customary notation for finite classes defined by enumeration. (Note that in $L_3$ only homogeneous classes occur, i.e. those whose elements belong to the same type.)

Recursive definition for iterated unit class formation:
(5) (a) $0(v^n_m) \equiv v^n_m$;
    (b) $p+1(v^n_m) \equiv \{p(v^n_m)\}$.

Thus, e.g., $3(a^0_1)$ is the class $\{\{a^0_1\}\}$ of type 3.

This notation can be used for the formation of a homogeneous class expression out of terms of different types. An ordered $k$-tuple can be defined as a certain class expression of type $n + 3$, where $n$ is the highest type of the members of this tuple. A $k$-adic relation $R$ can now be construed as a class of ordered $k$-tuples, thus as a class of type $n + 4$, if $n$ is the highest type of the members of $R$.

XXIII. Preliminary explanations of models for $L_3$

A. The use of models. We shall later define 'L-truth' as an explicatum for logical truth, i.e., truth in all possible states (of the universe of discourse). In a simple language (e.g., $L_2$), even if the domain of individuals is denumerably infinite, it is possible to represent every possible state by a state-description, i.e., an infinite class of sentences which contains, for every atomic sentence $S_j$, either $S_j$ or $\sim S_j$, but not both, and no
other sentences. (For the method of defining the L-concepts with the help of state-descriptions, see M & N, sect. 2, and, in greater detail, Foundations, sections 18 A, D and 20). For a language like $L_3$, which contains constants and variables of higher levels, it is not possible to represent all possible states by classes of sentences. Therefore the rules must here refer, not to the state-descriptions, but to the possible states themselves. The possible states will be construed here, not as propositions (which would require a non-extensional metalanguage, see Intr. Sem., sect. 18), but as models. A model for $L_3$ is a function which assigns to every descriptive constant in $L_3$ an extension of the corresponding type. [Concerning the use of models, see: Tarski, 1956, ch. VIII (concept of truth) and ch. XVI (logical consequence); Carnap, Syntax, ch. III C; Kemeny, "Models" (J.S.L 13, 1948, 16–30), 1953, and 1956.

B. The individual constants in $L_3$ are regarded as logical signs, since numbers are taken as individuals. Therefore the rules will assign to each $c_m^0$ one fixed extension (viz., the number $m$) in all models. Any identity sentence with two different $c^0$, e.g. $a_1^0 = a_2^0$, will then turn out to be logically false.

C. Logical and descriptive definitions of extensions and models.

1. An extension may be specified in $M$ either in logical or in descriptive terms. For example, for a number (type 0): '4' is logical, 'the one number (position) which is blue and cold' is descriptive; for a class of numbers (type 1): '{3, 5}' (i.e., '(n) (n = 3 or n = 5)') is logical, '(n) (n is blue)' is descriptive. '(\lambda n) (n is blue) = \{3, 5\}' is a factual sentence; it says that the (descriptively specified) property $(\lambda n) (n is blue)$ has the (logically specified) extension $\{3, 5\}$.

2. A model for $L_3$ may likewise be specified in $M$ either in logical or in descriptive terms. For the definitions of the L-concepts and A-concepts (which will be given in XXIV) the models may be regarded as logical models (sometimes
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called "mathematical models"). Likewise, the value assignments for variables (VA, in XXIV) may be regarded as logically specified. Only later, in the rules of interpretation and the definition of truth, shall we refer to descriptively defined extensions and models.

D. On A-postulates.

Suppose that, by virtue of the intended meanings of the descriptive constants in $L_3$, logical dependencies hold between two predicates (e.g., logical implication or incompatibility) or a structural property (e.g., transitivity) holds for a relational predicate. Then some models do not represent possible states. In this case the rules for A-truth (logical truth in the wider sense, analyticity) and other A-concepts must give A-postulates (meaning postulates) for $L_3$, which express the dependencies and structural properties (XXIV B). The admissible models or A-models will then be defined as those models in which the A-postulates hold. The A-models represent the possible states. Therefore we can then define a sentence as A-true iff it holds in all A-models. Read: Carnap, "Meaning postulates".

XXIV. The A-concepts for $L_3$

A. Value assignments and models.

1. A value assignment (VA) for the variables in $L_3$ is a function which assigns to every variable $v^n_m$ in $L_3$ an extension of type $n$. We use ‘$VA_r$’ and ‘$VA_s$’ as variables for $VA$ in $M$. We write ‘$VA_r(v^n_m)$’ for ‘the extension assigned to the variable $v^n_m$ by $VA_r$’.

2. The value assignment $VA_0$ is defined as follows (it assigns the same extension to all variables of the same type). For any $v^0_m$ of type 0, $VA_0(v^0_m) = 1$; for any variable $v^n_m$ of any type $n > 0$, $VA_0(v^n_m)$ is the null class of type $n$.

3. A model for $L_3$ is a function which assigns to every con-
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stant of every type \( n > 0 \) an extension of type \( n \). We use in \( M \) \( \text{\textit{Mod}}_p \) and \( \text{\textit{Mod}}_q \) as variables for models. We write \( \text{\textit{Mod}}_p(c^n_m) \) for 'the extension assigned to \( c^n_m \) by \( \text{\textit{Mod}}_p \).

B. The A-postulates of \( L_3 \).

1. The following sentences are A-postulates of \( L_3 \):

   (1) \( (v^0_1 \sim (c^1_2(v^0_1) \cdot c^1_3(c^0_1))). \)

   (b) \( c^2_1(c^1_2). \)

   (c) \( c^2_1(c^1_3). \)

   (d) \( (v^0_1)(v^0_2)(v^0_3) (c^4_1 <v^0_1, v^0_2> \cdot c^4_1 <v^0_2, v^0_3> \subseteq c^4_1 <v^0_1, v^0_3>). \)

   (e) \( (v^0_1(v^0_2))(c^4_1 <v^0_1, v^0_2> \supseteq \sim c^4_1 <v^0_1, v^0_2>). \)

2. Explanations. In what follows we shall assume that the whole class of A-postulates of \( L_3 \) has been specified, either by enumeration (if the class is finite) or else by schemata or rules in \( M \). The five examples (a) to (e) given above state (a) the incompatibility of \( c^1_2 \) and \( c^1_3 \); (b) and (c) the membership of the classes designated by \( c^1_2 \) and \( c^1_3 \), respectively, in the class designated by \( c^2_1 \); (d) the transitivity of \( c^4_1 \); (e) the asymmetry of \( c^4_1 \). [The rules of direct designation to be given later (XXVI A) stipulate that the designata of the constants \( c^1_1, c^1_2, c^1_3, c^2_1, \) and \( c^4_1 \) are, respectively, the classes Cold, Blue, Red, Color, and the relation Warmer. Therefore the five A-postulates given above are in agreement with the interpretation of the constants stated by the rules of designation, since (a) Blue and Red are incompatible, (b) and (c) Blue and Red are colors, (d) and (e) the relation Warmer is transitive and asymmetric; and this holds, not as a matter of contingent fact, but in virtue of the meanings of the terms. Note, however, that in our present context, i.e., the definition of the A-concepts on the basis of the A-postulates, the rules of designation are not used.]

C. Rules of relative designation for \( L_3 \).

For every designator formula \( D_m \) in \( L_3 \), the following rules (1a) to (10) (for the forms listed in the rules of formation,
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XXII B) determine \( \text{des}_{pr}(D_m) \), i.e., the designators of \( D_m \) with respect to \( Mod_p \) and \( VA_r \). If \( D_m \) is of type \( n \), \( \text{des}_{pr}(D_m) \) is an extension of type \( n \). '\( D^m \)' refers to designator formulas of type \( n \). (For explanations of the rules, see D below.)

<table>
<thead>
<tr>
<th>( D_m )</th>
<th>( \text{des}_{pr}(D_m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1a) ( c_m^0 )</td>
<td>( m )</td>
</tr>
<tr>
<td>1b ( c_m^n ) (for any ( n &gt; 0 ))</td>
<td>( Mod_p(c_m^n) )</td>
</tr>
<tr>
<td>(2) ( v_m^n )</td>
<td>( VA_r(v_m^n) )</td>
</tr>
<tr>
<td>(3) ( D_j^{n+1}(D_k^n) )</td>
<td>( \text{des}<em>{pr}(D_j^n) ) is an element of the class ( \text{des}</em>{pr}(D_j^{n+1}) ).</td>
</tr>
<tr>
<td>(4) ( D_j^n \equiv D_k^n )</td>
<td>( \text{des}<em>{pr}(D_j^n) = \text{des}</em>{pr}(D_k^n) ).</td>
</tr>
<tr>
<td>(5) ( S_j \equiv S_k )</td>
<td>( \text{des}<em>{pr}(S_j) = \text{des}</em>{pr}(S_k) ).</td>
</tr>
<tr>
<td>(6) ( \sim S_j )</td>
<td>Not ( \text{des}_{pr}(S_j) ).</td>
</tr>
<tr>
<td>(7) ( S_j \lor S_k )</td>
<td>( \text{des}<em>{pr}(S_j) ) or ( \text{des}</em>{pr}(S_k) ).</td>
</tr>
<tr>
<td>(8) ( (\lambda v_m^n)S_j )</td>
<td>The class ( (\lambda u^n) (\text{des}_{ps}(S_j)) ), where ( VA_s ) is that ( VA ) which assigns to ( v_m^n ) the extension ( u^n ) and otherwise is like ( VA_r ).</td>
</tr>
<tr>
<td>(9) ( (w_m^n)S_j )</td>
<td>The one extension ( u^n ) (of type ( n )) such that either (a) ( u^n ) is the only element of the class ( \text{des}_{pr}((\lambda v_m^n)S_j) ), or (b) this class does not have exactly one element and ( u^n = VA_0(v_m^n) ).</td>
</tr>
<tr>
<td>(10) ( (v_m^n)S_j )</td>
<td>Every extension of type ( n ) is an element of ( \text{des}_{pr}((\lambda v_m^n)S_j) ).</td>
</tr>
<tr>
<td>(11) For a class ( K_m ) of sentential formulas, ( \text{des}<em>{pr}(K_m) = \text{Dr} ) for every element ( S_j ) of ( K_m ), ( \text{des}</em>{pr}(S_j) ).</td>
<td></td>
</tr>
<tr>
<td>(12) For a designator ( D_m ), ( \text{des}<em>{pr}(D_m) = \text{Dr} \text{des}</em>{ps_0}(D_m) ) (referring to ( VA_0 ), see A2 above).</td>
<td></td>
</tr>
<tr>
<td>(13) For a class ( K_m ) of sentences, ( \text{des}<em>{pr}(K_m) = \text{Dr} \text{des}</em>{ps_0}(D_m) ).</td>
<td></td>
</tr>
</tbody>
</table>

D. Remarks on the rules in C.

1. For any \( D_m \) in \( L_3 \), there is one of the rules C(1a) to (10) which yields a sentence in \( M \) of the form ‘\( \text{des}_{pr}(D_m) = \ldots \)’,

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where a designator formula $A_i$ of $M$ stands in the place of the three dots; if $D_m$ is a designator formula of type $n$ in $L_3$, $A_i$ is a formula of type $n$ in $M$; and if $D_m$ is a sentential formula in $L_3$, $A_i$ is a sentential formula in $M$. [For example, for the individual constant $c^0_3$, we obtain by the rule C(1a): 'des$_p$($c^0_3$) = 3'; for $v^0_3 = v^0_5$, we obtain from C(4): 'des$_p$($v^0_3 = v^0_5 = (des$_p$($v^0_3$) = des$_p$($v^0_5$))'.] If the formula $A_i$ contains 'des$_p$', $A_i$ is transformed by further rules (in the second example, C(2) is applied twice), until finally a formula $A_i$ is obtained which does no longer contain 'des$_p$'. If a particular model, say $Mod_3$, and a particular $VA$, say $VA_5$, are defined in $M$ in logical terms (i.e., without the use of descriptive terms of the translation vocabulary, like 'blue'), then for any $D_m$ we obtain finally a result of the form 'des$_3,5(D_m) = ...$', where a logical designator in $M$ stands in the place of the three dots.

2. Note that 'des$_p$($S_j$)' is a sentential formula in $M$. We have (see XX Gl (c))
   (a) 'des$_p$($S_j$)' and 'des$_p$($S_j$) = $T$' are logically equivalent in $M$.
   (b) 'not des$_p$($S_j$)' and 'des$_p$($S_j$) = non-$T$' are logically equivalent in $M$.

Therefore an expression in $M$ of the form 'des$_p$($S_j$) = ...', where a sentential formula in $M$ stands in the place of the three dots, may be transformed in 'des$_p$($S_j$) = $T$) = ...'; thus it may be read, not only as 'the designatum of $S_j$ with respect to $Mod_p$ and $VA_r$ is ...', but also as '$S_j$ is true with respect to $Mod_p$ and $VA_r$, iff ...' or as '$S_j$ is satisfied by $VA_r$ and $Mod_p$ iff ...'.

3. Remarks on particular rules.
   (a) The rules C(2), (6), (7), and (10) (the latter in combination with (8)) are analogous to some rules for $L_2$ (viz., XX D, RD 3, 5, 6, 7 respectively). C(3) is a generalized analogue to RD4.
   (b) C(9) is in agreement with the convention on iota-descriptions stated in XXI D (for $VA_o$, see A2).
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(c) The rules C(11) and (13) are based on the conjunctive interpretation of a class of sentences; a class is regarded as true iff every sentence belonging to it is true (Intr. Sem., p. 34).

(d) It is easily seen from the rules that, if $D_m$ is a designator and thus without free variables, then $\text{des}_p(D_m)$ depends only on $\text{Mod}_p$ but is independent of $VA_r$. Therefore we may use here a function $\text{des}_p$ instead of $\text{des}_{pr}$. In its definition an arbitrarily chosen $VA$ may be used; in C(12) we have taken $VA_0$.

E. L-concepts and A-concepts.

The term 'L-truth' is here used for logical truth in the narrower sense, i.e., truth based on the meanings of the logical constants (e.g., a sentence of the form $S_1 \lor \sim S_1$ is L-true). I use the term 'A-truth' for logical truth in the wider sense, or analytic truth, i.e., truth based on the meanings of the logical constants and on the relations between the meanings of descriptive constants expressed by A-postulates (e.g., a sentence of $L_3$ saying that, if the position 3 is blue, it is not red, is A-true but not L-true). Since $L_3$ contains A-postulates, the A-concepts are here more important than the L-concepts. We shall give only definitions of A-concepts. The definition of an L-concept is analogous to that of the corresponding A-concept; it is obtained from the later by replacing the prefix 'A-' throughout by 'L-', and 'A-model' by 'model'. Thus the L-concepts are based on the totality of all models, while the A-concepts are based on the narrower class of A-models, i.e., those models which satisfy all A-postulates.

1. $\text{Mod}_p$ is an A-model (admissible model) for $L_3 = \text{Df } \text{des}_p(K_A)$, where $K_A$ is the class of the A-postulates in $L_3$.

2. A designator $D_m$ is an A-determinate designator $= \text{Df }$ for any two A-models $\text{Mod}_p$ and $\text{Mod}_q$, $\text{des}_p(D_m) = \text{des}_q(D_m)$.

3. $c_m^a$ is a logical designator constant $= \text{Df } c_m^a$ is A-determinate. Otherwise $c_m^a$ is called a non-logical or descriptive designator constant.
4. \( D_m \) is a *descriptive designator formula* = \( \text{Df } D_m \) contains at least one descriptive designator constant. Otherwise, \( D_m \) is a *logical designator formula*.

The following definitions (5) through (8) refer to *sentential formulas*.

5. \( S_j \) is *A-valid* (in \( L_3 \)) = \( \text{Df } S_j \) is A-valid.

6. \( S_j \) is *A-contravalid* = \( \text{Df } \sim S_j \) is A-valid.

7. \( S_j \) *A-implies* \( S_k \) = \( \text{Df } \sim S_j \lor S_k \) is A-valid.

8. \( S_j \) *A-equivalent* to \( S_k \) = \( \text{Df } S_j \equiv S_k \) is A-valid.

The following definitions refer to sentences.

9. A sentence \( S_j \) is *A-true* = \( \text{Df } S_j \) is A-valid.

10. A sentence \( S_j \) is *A-false* = \( \text{Df } \sim S_j \) is A-true.

11. A sentence \( S_j \) is *A-indeterminate* = \( \text{Df } \sim S_j \) is neither A-true nor A-false.

F. Theorems.

On the basis of the rules and definitions stated, theorems for A-concepts hold in analogy to the usual theorems for L-concepts (e.g., in *Intr. Sem.*, sect. 14, the postulates P14–5 through 9, and 11 through 15.)

We have furthermore:

1. A sentence \( S_i \) is A-determinate iff \( S_i \) is either A-true or A-false.

2. If the designator \( D_m \) contains no descriptive designator constant, \( D_m \) is A-determinate (and, moreover, L-determinate).

The definition E3 is adequate also for systems in which the models include the individual constants. Since this is not the case for \( L_3 \) (see A3), here the theorems (3) and (4) hold:

3. Any \( c_m^0 \) in \( L_3 \) is a logical designator constant. (From C(1a).)

4. If the designator \( D_m \) in \( L_3 \) contains no designator constant of any type \( n > 0 \), \( D_m \) is A-determinate.

5. A sentence \( S_j \) is A-true iff, for every A-model \( Mod_p \), \( \text{des}_p(S_j) \).
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G. Example.
Let $S_1$ be $c_1(v_1^0)$, $S_2$ be $S_1 \lor \neg S_1$, and $S_3$ be $(v_1^0)(S_2)$. By C(6) and (7), the following holds for any $\text{Mod}_p$, and $VA_r$, and any $VA_s$:

(a) $\text{des}_{pr}(S_2) = \text{des}_{pr}(S_1)$ or not $\text{des}_{pr}(S_1)$.
(b) $T$.
(c) $\text{des}_{pr}(S_2) = T$.

From C(8):
(d) $\text{des}_{pr}((\lambda v_1^0)S_2)$ = the class of all numbers $u^0$ such that $\text{des}_{ps}(S_2)$, where $VA_s$ is that $VA$ which assigns to $v_1^0$ the number $u^0$ and otherwise is like $VA_r$.

Hence with (c):
(e) $\text{des}_{pr}((\lambda v_1^0)S_2)$ = the class of all numbers $u^0$ such that $T$,
(f) $\text{des}_{ps}(S_2)$ = the class of all numbers.

Hence with C(10):
(g) $\text{des}_{pr}(S_3)$ = every number is an element of the class of all numbers,
(h) $T$.

From (b) and (h) by D2 (a):
(i) $\text{des}_{pr}(S_2)$,
(j) $\text{des}_{pr}(S_3)$.

From these with E 5:
(k) $S_2$ is A-valid (and, moreover, L-valid).
(l) $S_3$ is A-valid.

Hence with E 9:
(m) $S_3$ is A-true (and, moreover, L-true).

XXV. Preliminary remarks on interpretation and truth for $L_3$

A. On the rules of interpretation. A complete statement of these rules (which is not intended here) would have to include the following specifications:

(1) a specification of the domain of individuals, here positions,
(2) a specification of the enumeration of the positions, i.e., for every $n$, an explanation as to which position is taken as the position No. $n$,
(3) a specification of the meanings of the predicate constants \( c^n(n > 0) \).

We shall omit here the points (1) and (2), assuming that they have been specified. We shall give (in XXVI A) a few examples of rules of the kind (3), merely to illustrate the form of these rules.

We assume that \( L_3 \) contains only a finite number of predicate constants. For each of them its direct designatum is to be stated by a rule of the form ‘\( \text{ddes}(c^n_m) = \ldots \)’, where ‘\( \ldots \)’ is a predicator in \( M \) of the type \( n \). In order to fulfill the requirement of adequacy (see D below), this predicator ‘\( \ldots \)’ in \( M \) must be a translation of \( c^n_m \) according to the intended interpretation. The designata stated in the rules must furthermore be in agreement with the class of A-postulates for \( L_3 \) (of which we have given some examples in XXIV B1); more specifically, all logical relations and properties which hold for the designata, and no others, must be expressed by the A-postulates. According to the rule XXVI A3(a) (below), \( c^n_1 \) designates the relation Warmer, which is logically transitive; thus this rule is in agreement with the A-postulate XXIV B1(d). We assume that all the designata assigned by the \( \text{ddes} \)-rules for \( L_3 \) are descriptive; this is the case for the rules stated in XXVI A.

B. 1. The \( \text{ddes} \)-rules constitute a definition of the function \( \text{ddes} \) for \( L_3 \). This function assigns to every primitive constant \( c^n \) in \( L_3 \) (for \( n > 0 \)) a class of type \( n \) as its extension, specified in descriptive terms. Thus \( \text{ddes} \) is a model for \( L_3 \) (XXIV A3). However, it is not a logical model, like those previously considered, but a descriptive model (i.e., the constant ‘\( \text{ddes} \)’ in \( M \) is not logical, but descriptive). Since we assume that the \( \text{ddes} \)-rules are in agreement with the A-postulates, \( \text{ddes} \) is an A-model.

2. There is exactly one logical model \( \text{Mod}_p \) such that \( \text{Mod}_p = \text{ddes} \); that is to say, such that, for any primitive constant \( c^n_m(n > 0) \), the logically specified class assigned by \( \text{Mod}_p \) to \( c^n_m \) happens to be identical with the descriptively specified class \( \text{ddes}(c^n_m) \). The identity between this logical
model and $ddes$ can only be determined on the basis of factual knowledge (see the example XXVI CI below).

C. On the basis of $ddes$, the general designation function $des$, applicable to all designators, will be defined (XXVI B1). The function $des$ is the special case of $des_p$ (XXIV C(12)) for that $Mod_p$ which is identical with $ddes$. Thus the function $des$ assigns to any designator that designatum which is based on the direct designata assigned to the primitive predicate constants by $ddes$.

D. If designation is expressed by a functor, the uniquesences condition is necessarily fulfilled (because, from ‘$des(D_m) = \ldots$’ and ‘$des(D_m) = \ldots$’ follows). Therefore, the following sufficient and necessary condition of adequacy for designation in $L$ takes now the place of the earlier one (XV C), again for all three metalanguages $M^{\infty}$:
A functor ‘$d$’ in $M^{\infty}$ is an adequate functor for designation $^{\infty}$ in $L$ iff, for every designator $D_m$ in $L$, a sentence in $M^{\infty}$ of the form ‘$d(\ldots) = \ldots$’, with a spelling description of $D_m$ in the place of ‘$\ldots$’ and a translation of $D_m$ into $M^{\infty}$ in the place of ‘$\ldots$’, follows from the definition or the rules for ‘$d$’.

E. A sentence will be said to be true iff its designatum holds (XXVI B3). This definition is essentially the same as XI B(1). [The earlier definiens would be reformulated with the functor ‘$des$’ for sentences as ‘there is a $p$ such that $p = des(S_j)$ and $p$’; and this is logically equivalent in $M$ to ‘$des(S_j)$’.]

XXVI. Rules of interpretation and truth for $L_3$

A. 1. Rules of direct designation ($ddes$) for $L_3$.
(a) $ddes(c_1^1) = (\lambda x) (x \text{ is cold}),$
(b) $ddes(c_2^1) = (\lambda x) (x \text{ is blue}),$
(c) $ddes(c_3^1) = (\lambda x) (x \text{ is red}),$

etc.

2. For predicates of type 2:
(a) \(ddes(c_1^2) = (\lambda x_1^2) \ (x_1^2 \text{ is a color})\), etc.

3. For predicates of type 4:
(a) \(ddes(c_4^2) = (\lambda x_1^2) \ (\lambda x_2^0) \ (x_1^3 = \langle x_1^0, x_2^0 \rangle \text{ and } x_1^0 \text{ is warmer than } x_2^0)\), etc.

Similar rules may be stated for constants of other types.

B. Definitions of designation and truth for \(L_3\).
1. For any designator \(D_m\), \(des(D_m) = desp(D_m)\) for \(Mod_p = ddes\).
2. Theorems.
(a) For any primitive descriptive constant \(c_m^n (n > 0)\), \(des(c_m^n) = ddes(c_m^n)\). (From (1) and XXIV C(lb).)
(b) For any \(c_m^0\), \(des(c_m^0) = m\). (From XXIV C(1a).)
3. For any sentence \(S_j\), \(S_j\) is true in \(L_3 = \text{des}(S_j)\).

C. Examples. We assume for these examples that \(L_3\) contains only two primitive descriptive constants, viz., \(c_1^1\) and \(c_2^1\), with the \(ddes\)-rules A 1(a) and (b). (In this case, there are no A-postulates.)

1. Example for \(ddes\) and models. Let us suppose that, as a matter of fact, the positions 1, 2, and 3 are the only cold ones and that the positions 3 and 5 are the only blue ones. Thus the following two class identities hold factually (see XXIII c1):
(a) \((\lambda x) \ (x \text{ is cold}) = \{1, 2, 3\}\),
(b) \((\lambda x) \ (x \text{ is blue}) = \{3, 5\}\).

From these we derive with the rules A for \(ddes\) two factual sentences:
(c) \(ddes(c_1^1) = \{1, 2, 3\}\),
(d) \(ddes(c_2^1) = \{3, 5\}\).

These sentences give the actual extensions of the two constants. Now there is in \(M\) a logical model constant, say
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`Mod₈`, which is defined by the following sentences (e) and (f):

(e) \( Mod₈(c₁) = \{1, 2, 3\} \),
(f) \( Mod₈(c₂) = \{3, 5\} \).

Thus the constant \( c₁ \) has in \( Mod₈ \) the same extension as in the model \( ddes \); and \( c₂ \) has also in both models the same extension. Hence the descriptive model \( ddes \) is factually identical with the logical model \( Mod₈ \):

\[ ddes = Mod₈. \]

We may say that the true model (or the actual model), i.e., the model which ascribes to the primitive constants those extensions which they (in their intended interpretations) actually have, is descriptively specified in \( ddes \), and logically specified in \( Mod₈ \).

Note that the interpretation of the constants is given only by the \( ddes \)-rules in A above, but not by the factual sentences (c) and (d) without those rules, let alone by the definition of the logical model constant `Mod₈` (in (e) and (f)).

2. Example for \( des \) and truth. Let \( S₅ \) be the sentence \( c₂(c₃) \) in \( L₃ \).

From A 1(b) and B2(a) and (b):

(a) \( des(c₂) = (\lambda x) \, (x \text{ is blue}) \),
(b) \( des(c₃) = 3 \).

Hence with XXIV C(e):

(c) \( des(S₅) = (3 \text{ is an element of } (\lambda x) \, (x \text{ is blue}) \).\)

From the definition of truth (B3):

(d) \( S₅ \text{ is true } = des(S₅) \),
(e) \( S₅ \text{ is true iff } 3 \text{ is an element of } (\lambda x) \, (x \text{ is blue}) \).

The right-hand side of (c) and of (e) is a translation of \( S₅ \); thus the conditions of adequacy both for designation (XXV D) and for truth (XVI B) are here fulfilled.

Now we take (b) in Ex. 1 as a factual premise. This yields:

(f) \( 3 \text{ is an element of } (yx) \, (x \text{ is blue}) \).

Hence with (c):

(g) \( S₅ \text{ is true} \).
Semantics may be divided into two parts:

(1) *the theory of extension*, dealing with concepts like extension (designation, XIII), denotation (in the sense of 'Des', XVII), satisfaction, truth, naming, and related ones.

(2) *the theory of intension* (or meaning) dealing with concepts like meaning (or intension or sense as possible explicata, and the relations Des\(^1\) and Des\(^a\)), logical truth and analyticity (L-truth as explicatum), synonymy, and related ones.

Some philosophers, while accepting (1), reject all concepts of the kind (2). See Quine (1953), esp. chs. II, III, VII and White (1960).

Replies defending the meaning concepts: Mates (1951), Martin (1952). (Quine (1953), pp. 35 and 138 makes brief comments on Martin.)

It seems advisable to distinguish two problems:

(a) the problem of meaning concepts for *artificial language systems* defined by their rules,

(b) the problem of meaning concepts for *natural languages*.

The first problem and especially that of explicating logical truth in the wider sense (analyticity) can be solved by special semantical rules, e.g., *meaning postulates*. (See XXIII D, XXIV B and Carnap (1952).

Quine (1953, ch. III) admits the possibility of laying down special semantical rules for meaning concepts. But he doubts whether they explicate meaning, unless there are, as explicanda, meaning concepts which can be applied to *natural languages* on the basis of behavioristic criteria like other concepts of empirical linguistics. I have discussed a concept of this kind, viz., the pragmatical concept of intension, in Carnap (1955).

**XXVIII. Intensions and quasi-intensions**

A. Intensions in \(M^i\).

A few brief remarks about the intensional metalanguage \(M^i\) (XIII) will be made in this section. In \(M^i\), in distinction to
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$M^e$, statements about intensions can be formulated, e.g., a statement saying that the intension (designatum') of a given designator is such and such. Since the variables in $M^i$ have intensions as values, there are also general statements such as "For every proposition, . . ." or "There is a property of the type $[t_1]$ such that . . ." and the like. For each type, these general statements refer to a range of intensions which is much more comprehensive than the class of these intensions for which there are designators in the object language $L$ (say, $L_3$).

In $M^i$, a system of modal logic is used. We may take 'N' as the primitive modal sign, a logical constant. "N(...)" stands for "it is logically necessary that . . ." ($M & N$, ch. V).

B. Quasi-intensions in $M^e$.

The sentences of $M^e$ refer directly to extensions only, not to intensions. But there is a one-one correspondence between the intensions and a special kind of extensions, which we shall call quasi-intensions, such that the logical and semantical properties of any intension are mirrored by those of the corresponding quasi-intension. The following definitions refer to $L_3$: for other language forms, analogous definitions can be formulated.

(1) The **quasi-intensions** corresponding to the type $t$ (for $L_3$) $=_{df}$ the functions from admissible models (for $L_3$) to extensions of the type $t$ (XXI B).

Note that the quasi-intensions corresponding to the type $t$ are themselves extensions, not of the type $t$, but of another type of higher level.

(2) The **quasi-intension of the designator** $D_k$ (in $L_3$) $=_{df}$ that function which, for any admissible model $Mod_p$, has the value $des_p(D_k)$. (For $des_p$, see XXIV C(12).) [It is here assumed, for the sake of simplicity, that in $M^i$ 'N' does not occur in the operand of a $\lambda$-operator or an $\iota$-operator. If such occurrences are admitted, the ranges of intensions in $M^i$ and the corresponding ranges of quasi-intensions in $M^e$ must be still more comprehensive, and the definitions (1) and

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C. Translation from a modal language $L^i$ into an extensional language $L^e$.

If a language $L^i$ with logical modalities and intension variables (similar to $M^i$) is given, it is possible to specify an effective method by which any sentence of $L^i$ is translated into an $L$-equivalent sentence of an extensional language $L^e$. We shall not specify the method here but give only rough indications of the translation for the two most important forms of sentential formulae. A universal formula of the form “for every intension of the type $t, \ldots$” in $L^i$ is translated into one of the form “for every quasi-intension corresponding to the type $t, \ldots$” in $L^e$. And a modal formula “$N(\ldots)$” is translated into one of the form “for every admissible model $Mod_p, \ldots$”. This translation is plausible since a proposition is logically necessary iff it holds in every possible case, that is, in every admissible model ($M \& N$, p. 186).

Even for those who accept only an extensional language, logical modalities and intension variables are shown by this translation to be unobjectionable, provided that the variables in the extensional language $L^e$ have sufficient ranges of extension values to accommodate the quasi-intensions corresponding to the intensions which are values of variables in $L^i$. In particular, the method indicated can be used for translating any sentence of $M^i$ into one of $M^e$. This legitimizes the semantics of intensions for an extensionalist point of view.

XXIX. The controversy about abstract entities in semantics

A. A nominalistic language is one in which all values of all variables are concrete (say, observable objects or events). There is a controversy among analytic philosophers today about the legitimacy of non-nominalistic languages. Some, e.g. Tarski, Quine and Goodman, deny or doubt that a language which
is non-nominalistic and not translatable into a nominalistic one can be accepted as meaningful. (See Quine (1953), esp. ch. VI.) In this contemporary controversy sometimes the old terms "nominalism" and "universalism" (or "realism" or "Platonism") are used. This seems to me inadvisable. The earlier controversies were formulated in a very unclear way, and it seems even doubtful whether the philosophical ("ontological") assertions which were discussed under these labels had any cognitive content. Since sentences about intensions are translatable into sentences about extensions (XXVIII), the controversy concerns essentially the admissibility of variables for abstract (i.e., non-concrete) extensions of various kinds, e.g., classes (of objects), classes of classes (of objects), relations, numbers, functions, etc.

B. In my view, the introduction of variables of a new kind is a matter of practical decision. Certain theoretical investigations are certainly relevant for the decision, e.g., investigations of the logical and semantical features, both the desirable and the undesirable ones, of the enlarged language. Among the features to be considered may be, e.g., the simplicity of the logical structure, the strength in means of expression and means of deduction, the danger of inconsistency, and the like. (There is never any absolute certainty of consistency; and the degree of confidence in consistency often decreases by the introduction of a new kind of abstract variables.) But the legitimacy of the introduction is not dependent upon an alleged prior metaphysical insight into the "existence" or "ontological reality" of the new entities.

Read: Carnap (1950).

C. To the arguments in the paper mentioned I would today add the suggestion that it might be advisable to regard the metalanguage M for syntax and semantics as part of the theoretical language, not of the observation language. (I would prefer to do this even if the object language is part of the observation language.) A given language community may well decide to
admit in their common observation language $L_0$ only sentences which are completely understood by all members of the community and, therefore, to lay down some or all of the following requirements for $L_0$:

1. **Requirement of observability.** All primitive descriptive predicates in $M_0$ designate properties or relations which are directly observable for all members of the community.

2. **Nominalistic requirement.** The values of the variables are observable objects (or object-moments or events).

3. **Wide requirement of finitism.** The rules of $L_0$ do not state or imply that the domain of individuals is infinite.

On the other hand, for the theoretical language $L_T$ we can never have more than an incomplete interpretation. There is no reason to restrict this language by requirements similar to those for $L_0$. On the contrary, we should admit in $L_T$ all means of expression and of deduction which are found to be useful for the purpose of this language, which is, to supply a theoretical superstructure for $L_0$.

This applies also to the semantical metalanguage $M$, now regarded as part of $L_T$. For example, even if the object language is the nominalistic observation language, we should feel free to admit in $M$ variables for classes of objects (and, if it seems useful, also variables for classes of classes, for functions, for intensions or quasi-intensions) and to use these variables in the definitions of semantical concepts, e.g., the concepts of designation, extension, intension, truth, model, L-truth, etc.
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