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ON THE RELATIVITY PROBLEM

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After two eminent specialists have presented their objections to relativity theory in this journal, it must be not undesirable for the readers if an adherent of this new theoretical direction expounds his view. This shall be done as concisely as possible in the following.

Currently we have to distinguish two theoretical systems, both of which fall under the name “relativity theory.” The first of these, which we will call “relativity theory in the narrower sense,” is based on a considerable body of experience and is accepted by the majority of theoretical physicists to be one of the simplest theoretical expressions of these experiences. The second, which we will call “relativity theory in the broader sense,” is as yet by no means established on the basis of physical experience. The majority of my colleagues regard this second system either sceptically or dismissively. It should be said immediately that one can certainly be an adherent of the relativity theory in the narrow sense without admitting the validity of relativity theory in the wider sense. For that reason we will discuss the two theories separately.

I. RELATIVITY THEORY IN THE NARROWER SENSE

It is well known that the equations of the mechanics established by Galileo and Newton are not valid with respect to an arbitrarily moving coordinate system, if one adheres to the requirement that the description of motion admits only central forces satisfying the | law of equality of action and reaction. But if the motion is referred to [338]
a coordinate system K, such that Newton’s equations are valid in the indicated sense, then that coordinate system is not the *only one* with respect to which those laws of mechanics hold. Rather, all of the coordinate systems K' with arbitrary spatial orientation that have uniform translational motion with respect to K have the property that relative to them the laws of motion hold. We call the assumption of the equal value of all these coordinate systems K, K', etc. for the formulation of the laws of motion, actually for the general laws of physics, the “relativity principle” (in the narrow sense).

Jürgen Renn (ed.). *The Genesis of General Relativity,* Vol. 3
Gravitation in the Twilight of Classical Physics: Between Mechanics, Field Theory, and Astronomy.

As long as one believed that classical mechanics lies at the foundation of the theoretical representation of all processes, one could not doubt the validity of this relativity principle. But even abstaining from that, it is difficult from an empirical standpoint to doubt the validity of that principle. In fact if it did not hold, then the processes of nature referred to a reference system at rest with respect to the Earth would appear to be influenced by the motion (velocity) of the Earth's yearly orbital motion around the Sun; the terrestrial space of observations would have to behave physically in an anisotropic manner due to the existence of this motion. But despite the most arduous searching, physicists have never observed such an apparent anisotropy.

The relativity principle is hence as old as mechanics, and no one could ever have questioned its validity from an empirical standpoint. That it has been nonetheless doubted, and is again doubted today, is due to the fact that it seemed to be incompatible with Maxwell-Lorentz electrodynamics. Whoever is in a position to judge this theory, in light of its inclusiveness, the small number of its fundamental assumptions, and its successful representation of phenomena in the domain of electrodynamics and optics, will find it difficult to dispel the impression that the main features of this theory are as definitively established as are the equations of mechanics. It has also not been accomplished to set another theory against this one that could even tolerably compete with it. |

[339] It is easy to specify wherein lies the apparent contradiction between Maxwell-Lorentz electrodynamics and the relativity principle. Suppose that the equations of that theory hold relative to the coordinate system K. This means that every light ray propagates in vacuo with a definite velocity c, with respect to K, which is independent of direction and of the state of motion of the light source; this proposition will be called the "principle of the constancy of the speed of light" in the following. Now if one such light ray were to be observed by an observer moving relative to K, then the propagation speed of this light ray, as estimated from the standpoint of this observer, in general seems to be different than c. For example, if the light ray propagates in the direction of the positive x- axis of K with speed c, and our observer moves in the same direction with the temporally constant speed v, then one would believe that one can immediately conclude that the light ray's propagation speed must be $c - v$ according to the moving observer. Relative to the observer, that is, relative to a coordinate system K' moving with the same velocity, the principle of the constancy of the speed of light does not appear to hold. Hence, here is an apparent contradiction with the principle of relativity.

However, an exact analysis of the physical content of our spatial and temporal determinations leads to the well-known result that the implied contradiction is only apparent, since it depends on both of the following arbitrary assumptions:

1. The assertion that whether two events occurring in different places occur simultaneously has content independently of the choice of a reference system.
2. The spatial distance between the places in which two simultaneous events occur is independent of the choice of a reference system.

Given that the Maxwell-Lorentz theory as well as the relativity principle are empirically supported to such a large degree, one must therefore decide to drop both the aforementioned arbitrary assumptions, the apparent evidence for which rests solely on the facts that light gives us information about distant events *apparently instantaneously*, and that the objects | we deal with in daily life have velocities that [340]
are small compared to the velocity of light c.

By abandoning these arbitrary assumptions, one achieves compatibility between the principle of the constancy of the speed of light, which results from Maxwell-Lorentz electrodynamics, and the relativity principle. One can retain the assumption that one and the same light ray propagates with velocity c relative to all reference systems K' in uniform translational motion with respect to a system K, rather than only relative to K. One only has to choose the transformation equations, which exist between the spacetime coordinates (x, y, z, t) with respect to K and those (x', y', z', t') with respect to K', in an appropriate way; one will thereby be led to the system of transformation equations called the "Lorentz transformation." This Lorentz transformation supercedes the corresponding transformations that until the development of relativity theory were regarded as the only conceivable ones, which, however, were based on the assumptions (1) and (2) given above.

The heuristic value of relativity theory consists in the fact that it provides a constraint that all of the systems of equations that express general laws of nature must satisfy. All such systems of equations must be constructed such that with the application of a Lorentz transformation they go into a system of equations of the same form (covariance with respect to the Lorentz transformations). Minkowski presented a simple mathematical schema to which equation systems must be reducible if they are to behave covariantly with respect to Lorentz transformations; thereby he achieved the advantage, that for the accommodation of the system of equations with the constraint mentioned above it is certainly not necessary to in fact carry out a Lorentz transformation on those systems.

From what has been said it clearly follows that relativity theory by no means gives us a tool for deducing previously unknown laws of nature from nothing. It only provides an always applicable criterion that constrains the possibilities; in this respect
| it is comparable to the law of energy conservation or the second law of thermody- [341]
namics.

It follows from a close examination of the most general laws of theoretical physics that Newtonian mechanics must be modified to satisfy the criterion of relativity theory. These altered mechanical equations have proven to be applicable to cathode rays and β- rays (motion of free electrical particles). Moreover, the implementation of relativity theory has lead to neither a logical contradiction nor a conflict with empirical results.

Only one result of relativity theory will be given here in particular, because it is of importance for the following analyses. According to Newtonian mechanics the inertia of a system constituted by a collection of material points (that is, the inertial resistance against acceleration of the system's center of gravity) is independent of the state of the system. By contrast, according to relativity theory the inertia of an isolated system (floating in a vacuum) depends upon the state of the system, such that the inertia increases with the energy content of the system. Thus according to relativity theory, it is ultimately *energy* that inertia can be attributed to. The energy, rather than the inertial mass of a material point, is what we have ascribed indestructibility to; hence the theorem of the conservation of mass is incorporated into the theorem of the conservation of energy.

It was remarked above that it would be a great mistake to regard relativity theory as a universal method that allows one to develop an unequivocally appropriate theory for a domain of phenomena regardless of how little this has been explored empirically. Relativity theory only *reduces* by a significant amount the empirical conclusions necessary for the development of a theory. There is only one domain of fundamental importance where we have such poor empirical knowledge that this knowledge, in combination with relativity theory, is not sufficient, by a wide margin, for a clear determination of the general theory. This is the domain of gravitational phenomena. Here we can only reach our goal by complementing what is empirically
[342] known with physical hypotheses in order to complete the basis | of the theory. The following considerations are firstly to show how one arrives at what are, in my opinion, the most natural such hypotheses.

When we speak of a body's *mass*, we associate with this word two definitions that are logically completely independent. By *mass* we understand, first, a constant inherent to a body, which measures its resistance to acceleration ("inertial mass"), and second, the constant of a body that determines the magnitude of the force that it experiences in a gravitational field ("gravitational mass").

It is in no way self-evident *a priori* that the inertial and gravitational mass of a body must agree; we are simply *accustomed* to assume their agreement. The belief in this agreement stems from the empirical fact that the acceleration that various bodies experience in a gravitational field is independent of their material constitution. Eötvös has shown that, in any case, inertial and gravitational mass agree with very great precision, in that, through his experiments with a torsion balance, he ruled out the existence a relative deviation of the two masses from each other on the order of magnitude of 10^{-8}.[1]

Enormous quantities of energy in the form of heat are discharged into the environment by radioactive processes. According to the result regarding the inertia of energy presented above, the decay products generated by the reaction taken together must have a smaller inertial mass than that of the material existing before the radioactive decay. This change of inertial mass is, for the kind of reaction with known heat effect,

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1 Eötvös' experimental method is based on the following. The Earth's gravity and the centrifugal force influence a body found on the Earth's surface. For the first the body's gravitational mass, and for the second the inertial mass, is the determining factor. If the two were not identical, then the direction of the resultant of the two (the apparent weight) would depend on the material of the body. With his torsion balance experiments, Eötvös proved with great precision the non-existence of such a dependence.

of the order 10^{-4}. If the gravitational mass did not change simultaneously with the inertial mass of the system, then the inertial mass would have to differ from the gravitational mass for various elements far more than Eötvös's | experiments would allow. [343] Langevin was the first to call attention to this important point.

From what has been said the identity of the inertial and gravitational mass of closed systems (at rest) follows with great probability; I think that based on the present state of empirical knowledge we should adhere to the assumption of this identity unconditionally. We have thereby attained one of the most important physical demands that, from my point of view, must be imposed on any gravitational theory.

This demand involves a far-reaching constraint on gravitational theories, which one recognizes especially in conjunction with the theorem of the inertia of energy. All energy corresponds to inertial mass, and all inertial mass corresponds to gravitational mass; the gravitational mass of a closed system must therefore be determined by its energy. The energy of its gravitational field also belongs to the energy of a closed system; hence the gravitational field energy itself contributes to the system's gravitational mass rather than only its inertial mass.

Abraham and Mie have proposed gravitational theories. Abraham's theory contradicts the relativity principle, and Mie's theory contradicts the demand of the equality of the inertial and gravitational mass of a closed system. According to the latter theory, were a body to be heated the *inertial* mass *grows* in proportion to the energy gain, but not the gravitational mass; the latter would actually decrease for a gas with rising temperature.[2]

By way of contrast, a gravitational theory recently proposed by Nordström complies with both the relativity principle and the requirement of the gravity of energy of closed systems, with one restriction to be indicated in the following. Abraham's claim to the contrary made in a paper appearing in this journal is not | correct. In fact, I [344] believe that a cogent argument against Nordström's theory cannot be drawn from experience.

According to Nordström's theory the principle of the gravity of energy of closed systems at rest holds as a statistical principle. The gravitational mass of a closed system (with the whole system at rest) is in general an oscillating quantity, whose temporal average is determined by the total energy of the system. As a consequence of the oscillatory character of mass, such a system must emit standing longitudinal gravitational waves. Yet the energy loss expected according to the theory is so small that it must escape our notice.

After a more detailed study of Nordström's theory, everyone will have to admit that this theory, when regarded from an empirical standpoint, is an unobjectionable integration of gravitation into the framework of relativity theory (in the narrower

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2 Due to their smallness, these effects are certainly not accessible to experiments. But it seems to me that there is much to be said for taking the connection between inertial and gravitational mass to be warranted in principle, regardless of what forms of energy are taken into account. According to Mie, one can account for the fact that the equality of inertial and gravitational mass holds for radioactive transformations only with assumptions regarding the special nature of energy in the interior of atoms.

sense). Even though I am of the opinion that we cannot be satisfied with this solution, my reasons for this have an epistemological character that will be described in the following.

II. RELATIVITY THEORY IN THE BROADER SENSE

Classical mechanics, as well as relativity theory in the narrow sense briefly described above, suffer from a fundamental defect, which no one can deny, that is accessible to epistemological arguments. The weaknesses of our physical world picture to be discussed below were already uncovered with full clarity by E. Mach in his deeply penetrating investigations of the foundations of Newtonian mechanics, so that what I will assert in this respect can have no claim to novelty. I will explain the essential point with an example, which is chosen to be quite elementary, in order to allow what is essential to stand out.

Two masses float in space at a great distance from all celestial bodies. Suppose that these two are close enough together that they can exert an influence on each other. Now an observer watches the motion of both bodies, such that he continually
[345] looks along the direction of the line | connecting the two masses toward the sphere of fixed stars. He will observe that the line of sight traces out a closed line on the visible sphere of fixed stars, which does not change its position with respect to the visible sphere of fixed stars. If the observer has any natural intelligence, but has learned neither geometry nor mechanics, he would conclude: "My masses carry out a motion, which is at least in part causally determined by the fixed stars. The law by which masses in my surroundings move is co-determined by the fixed stars." A man who has been schooled in the sciences would smile at the simple-mindedness of our observer and say to him: "The motion of your masses has nothing to do with the heaven of fixed stars; it is rather fully determined by the laws of mechanics entirely independently of the remaining masses. There is a space R in which these laws hold. These laws are such that the masses remain continually in a plane in this space. However, the system of fixed stars cannot rotate in this space, because otherwise it would be disrupted by powerful centrifugal forces. Thus it necessarily must be at rest (at least almost!), if it is to exist permanently; this is the reason that the plane in which your masses move always goes through the same fixed stars." But our intrepid observer would say: "You may be incomparably learned. But just as I could never be brought to believe in ghosts, so I cannot believe in this gigantic thing that you speak of and call space. I can neither see something like that nor conceive of it. Or should I think of your space R as a subtle net of bodies that the remaining things are all referred to? Then I can imagine a second such net R' in addition to R, that is moving in an arbitrary manner relative to R (for example, rotating). Do your equations also hold at the same time with respect to R'?" The learned man denies this with certainty. In reply to which the ignoramus: "But how do the masses know which "space" R, R', etc. with respect to which they should move according to your equations, how do they recognize the space or spaces they orient themselves with respect to?" Now our

learned man is in a quite embarrassing position. He certainly insists that there must be such privileged spaces, but he knows no reason to give for why such spaces could be distinguished from | other spaces. The ignoramus's reply: "Then I will take, for the [346] time being, your privileged spaces as an idle fabrication, and stay with my conception, that the sphere of fixed stars co-determines the mechanical behavior of my test masses."

I will explain the violation of the most elementary postulate of epistemology of which our physics is guilty in yet another way. One would try in vain to explain what one understands by the simple acceleration of a body. One would only succeed in defining *relative* acceleration of bodies with respect to each other. However, having said that, we base our mechanics on the premise that a force (cause) is necessary for the generation of a body's acceleration, ignoring the fact that we cannot explain what it is that we understand by "acceleration," exactly because only *relative* accelerations can be an object of perception.

The dubious aspect of proceeding in this vein is very nicely illustrated by a comparison, which I owe to my friend Besso. Suppose we think back to an earlier time, when it was assumed that the surface of the Earth must be approximately *flat*. Imagine that the following conception exists among the learned. In the world there is a physically distinguished direction, the vertical. All objects fall in this direction if they are not supported. Because of this the surface of the Earth is essentially perpendicular to this direction, and this is why it tends towards the form of a plane. While in this case, the mistake lies in privileging one direction over all others without good reason (fictitious cause), rather than simply regarding the Earth as the cause of falling, the mistake in our physics lies in the introduction without good reason of privileged reference systems as fictitious causes; both cases are characterized by forgoing the establishment of a sufficient reason.

Since relativity theory in the narrower sense, rather than only classical mechanics, exhibits the fundamental deficiency explained above, I set myself the goal of generalizing relativity theory in such a way that this imperfection will be avoided. First of all, I recognized that gravitation in general | must be assigned a fundamental role in [347] any such theory. Then from what was explained earlier it already follows that every physical process must also produce a gravitational field, because of the quantity of energy corresponding to it. On the other hand, the empirical fact that all bodies fall with the same speed in a gravitational field suggests the idea that physical processes happen in a gravitational field exactly as they do relative to an accelerated reference system (equivalence hypothesis). In taking this idea as a foundation, I came to the conclusion that the velocity of light is not to be regarded as independent of the gravitational potential. Thus the principle of the constancy of the speed of light is incompatible with the equivalence hypothesis; that is why relativity theory in the narrower sense cannot be made consistent with the equivalence hypothesis. This led me to take relativity theory in the narrow sense to be applicable only in regions within which no noticeable differences in the gravitational potential occur. Relativity theory (in the narrower sense) has to be replaced by a general theory, which contains the former as a limiting case.

The path leading to this theory can only be very incompletely described in words.[3] The equations of motion for material points in a gravitational field that follow from the equivalence hypothesis can be easily written in a form such that these laws are completely independent of the choice of variables determining place and time. By leaving the choice of these variables as *a priori* completely arbitrary, and thus not privileging any spacetime system, one averts the epistemological objection discussed above. A quantity

$$ds^2 = \sum_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu,$$

appears in that law of motion, and it is invariant, i.e., it is a quantity that is independent of the choice of reference system (i.e., of the choice of four spacetime coordinates). The quantities $g_{\mu\nu}$ are functions of $x_1 \ldots x_4$ and represent the gravitational
[348] field. |

With the help of the absolute differential calculus, which has been developed by Ricci and Levi-Civita based on Christoffel's mathematical investigations, one can succeed, based on the existence of the invariant above, in replacing the well-known systems of equations of physics with equivalent systems (when all $g_{\mu\nu}$ are constant), which are valid independent of a choice of the spacetime coordinate system x_ν. All such systems of equations include the quantities $g_{\mu\nu}$, i.e., the quantities that determine the gravitational field. Thus the latter influence all physical processes.

Conversely, physical processes must also determine the gravitational field, i.e., the quantities $g_{\mu\nu}$. One arrives at the differential equations that determine these quantities by means of the hypothesis that the conservation of momentum and energy must hold for material events and the gravitational field taken together. This hypothesis subsequently constrains the choice of spacetime variables x, without thereby evoking again the epistemological doubts analyzed above. Because according to this generalized relativity theory there are no longer privileged spaces with peculiar physical qualities. The quantities $g_{\mu\nu}$ control the course of all processes, which for their part are determined by the physical events in all the rest of the universe.

The principle of the inertia and gravitation of energy is completely satisfied in this theory. Furthermore, the equations of motion for gravitational masses are such that it is, as one must demand based on the considerations above, acceleration with respect to other bodies rather than absolute acceleration (acceleration with respect to "space") that appears as that which is decisive for the appearance of inertial resistance.

Relativity theory in the broader sense signifies a further development of the earlier relativity theory, rather than an abandonment of it, that seems necessary to me for the epistemological reasons I cited.

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3 Cf. A. Einstein and M. Grossmann, *Zeitschrift f. Math. & Physik* 62 (1914): p. 225.