CHAPTER
7

The Grand Design—A New Physics

The publication of Isaac Newton's *Principia* in 1687 was one of the most notable events in the whole history of physical science. In it one may find the culmination of thousands of years of striving to comprehend the system of the world, the principles of force and of motion, and the physics of bodies moving in different media. It is no small testimony to the vitality of Newton's scientific genius that although the physics of the *Principia* has been altered, improved, and challenged ever since, we still set about solving most problems of celestial mechanics and the physics of gross bodies by proceeding essentially as Newton did some 300 years ago. Newtonian principles of celestial mechanics guide our artificial satellites, our space shuttles, and every spacecraft we launch to explore the vast reaches of our solar system. And if this is not enough to satisfy the canons of greatness, Newton was equally great as a pure mathematician. He invented the differential and integral calculus (produced simultaneously and independently by the German philosopher Gottfried Wilhelm Leibniz), which is the language of physics; he developed the binomial theorem and various properties of infinite series; and he laid the foundations for the calculus of variations. In optics, Newton began the experimental study of the analysis and composition of light, showing that white light is a mixture of light of many colors, each having a characteristic index of refraction. Upon these researches have risen the science of spectroscopy and the methods of color analysis. Newton invented a reflecting telescope and so showed astronomers how to transcend the limitations of telescopes built of lenses. All in all, his was a fantastic scientific

achievement—of a kind that has never been equaled and may never be equaled again.

In this book we shall deal exclusively with Newton's system of dynamics and gravitation, the central problems for which the preceding chapters have been a preparation. If you have read them carefully, you have in mind all but one of the major ingredients requisite to an understanding of the Newtonian system of the world. But even if that one were to be given—the analysis of uniform circular motion—the guiding hand of Newton would still be required to put the ingredients together. It took genius to supply the new concept of universal gravitation. Let us see what Newton actually did.

First of all, it must be understood that Galileo himself never attempted to display any scheme of forces that would account for the movement of the planets, or of their satellites. As for Copernicus, the *De revolutionibus* contains no important insight into a celestial mechanics. Kepler had tried to supply a celestial mechanism, but the result was never a very happy one. He held that the *anima motrix* emanating from the sun would cause planets to revolve about the sun in circles. He further supposed that magnetic interactions of the sun and a planet would shift the planet during an otherwise circular revolution into an elliptical orbit. Others who contemplated the problems of planetary motion proposed systems of mechanics containing certain features that were later to appear in Newtonian dynamics. One of these was Robert Hooke, who quite understandably thought that Newton should have given him more credit than a mere passing reference for having anticipated parts of the laws of dynamics and gravitation.

NEWTONIAN ANTICIPATIONS

The climactic chapter in the discovery of the mechanics of the universe starts with a pretty story. By the third quarter of the seventeenth century, a group of men had become so eager to advance the new mathematical experimental sciences that they banded together to perform experiments in concert, to present problems for solution to one another, and to report on their own researches and on those of others as revealed by correspondence, books, and pamphlets. Thus it came about that Robert Hooke, Edmond Halley, and Sir Christopher Wren, England's foremost architect, met to discuss the question, Under what law of force would a planet follow an elliptical orbit? From Kepler's laws—especially the third or harmonic law, but also the second or law of areas—it was clear that the sun somehow or other must control or at least affect the motion of a planet in accordance with the relative proximity of the planet to the sun. Even if the particular mechanisms proposed by Kepler (an *anima motrix* and a magnetic force) had to be rejected, there could be no doubt that some kind of planet-sun interaction keeps the planets in their courses. Furthermore, a more acute intuition than Kepler's would sense that any force emanating from the sun must spread out in all directions from that body, presumably diminishing according to the inverse of the square of its distance from the sun—as the intensity of light diminishes in relation to distance. But to say this much is a very different thing from *proving* it mathematically. For to prove it would require a complete physics with mathematical methods for solving all the attendant and consequent problems. When Newton declined to credit authors who tossed off general statements without being able to prove them mathematically or fit them into a valid framework of dynamics, he was quite justified in saying, as he did of Hooke's claims: "Now is not this very fine? Mathematicians that find out, settle, and do all the business must content themselves with being nothing but dry calculators and drudges; and another, that does nothing but pretend and grasp at all things, must carry away all the invention, as well of those that were to follow him as of those that went before." (See, further, Supplement 11).

In any event, by January 1684 Halley had concluded that the force acting on planets to keep them in their orbits "decreased in the proportion of the squares of the distances reciprocally,"

$$F \propto \frac{1}{D^2},$$

but he was not able to deduce from that hypothesis the observed motions of the celestial bodies. When Wren and Hooke met later

in the month, they agreed with Halley's supposition of a solar force. Hooke boasted "that upon that principle all the laws of the celestial motions were to be [i.e., could be] demonstrated, and that he himself had done it." But despite repeated urgings and Wren's offer of a considerable monetary prize, Hooke did not—and presumably could not—produce a solution. Six months later, in August 1684, Halley decided to go to Cambridge to consult Isaac Newton. On his arrival he learned the "good news" that Newton "had brought this demonstration to perfection." Here is DeMoivre's almost contemporaneous account of that visit:

> After they had been some time together, the Dr. [Halley] asked him what he thought the curve would be that would be described by the planets supposing the force of attraction towards the sun to be reciprocal to the square of their distance from it. Sir Isaac replied immediately that it would be an ellipsis. The Doctor, struck with joy and amazement, asked him how he knew it. Why, saith he, I have calculated it. Whereupon Dr. Halley asked him for his calculation without any further delay. Sir Isaac looked among his papers but could not find it, but he promised him to renew it and then to send it him. Sir Isaac, in order to make good his promise, fell to work again, but he could not come to that conclusion which he thought he had before examined with care. However, he attempted a new way which, though longer than the first, brought him again to his former conclusion. Then he examined carefully what might be the reason why the calculation he had undertaken before did not prove right, and he found that, having drawn an ellipsis coarsely with his own hand, he had drawn the two axes of the curve, instead of drawing two diameters somewhat inclined to one another, whereby he might have fixed his imagination to any two conjugate diameters, which was requisite he should do. That being perceived, he made both his calculations agree together.

Spurred on by Halley's visit, Newton resumed work on a subject that had commanded his attention in his twenties when he had laid the foundations of his other great scientific discoveries: the nature of white light and color and the differential and integral calculus. He now put his investigations in order, made great progress, and in the fall term of the year, discussed his research in a series of lectures on dynamics that he gave at Cambridge University, as required by his professorship. Eventually, with Halley's encouragement, a draft of these lectures, *De motu cor-*

porum, grew into one of the greatest and most influential books any man has yet conceived. Many a scientist has echoed the sentiment that Halley expressed in the ode he wrote as a preface to Newton's *Principia* (or, to give Newton's masterpiece its full title, *Philosophiae naturalis principia mathematica, Mathematical Principles of Natural Philosophy,* London, 1687):

Then ye who now on heavenly nectar fare,
Come celebrate with me in song the name
Of Newton, to the Muses dear; for he
Unlocked the hidden treasuries of Truth:
So richly through his mind had Phoebus cast
The radiance of his own divinity.
Nearer the gods no mortal may approach.

THE PRINCIPIA

The *Principia* is divided into three parts or books; we shall concentrate on the first and third. In Book One Newton develops the general principles of the dynamics of moving bodies, and in Book Three he applies the principles to the mechanism of the universe. Book Two deals with a facet of fluid mechanics, the theory of waves, and other aspects of physics.

In Book One, following the preface, a set of definitions, and a discussion of the nature of time and space, Newton presented the "axioms, or laws of motion":

Law I

Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed upon it.

Law II

A change in motion is proportional to the motive force impressed and takes place in the direction of the straight line along which that force is impressed. [See Suppl. Note on p. 184.]

Observe that if a body is in uniform motion in a straight line, a force at right angles to the direction of motion of the body will not affect the forward motion. This follows from the fact that the acceleration is always in the same direction as the force producing it, so that the acceleration in this case is at right angles to the direction of motion. Thus in the toy train experiment of Chapter 5, the chief force acting is the downward force of gravity, producing a vertical acceleration. The ball, whether moving forward or at rest, is thus caused to slow down in its upward motion until it comes to rest, and then be speeded up or accelerated on the way down.

A comparison of the two sets of photographs (p. 83) shows that the upward and downward motions are exactly the same whether the train is at rest or in uniform motion. In the forward direction there is no effect of weight or gravity, since this acts only in a downward direction. The only force in the forward or horizontal direction is the small amount of air friction, which is almost negligible; so one may say that in the horizontal direction there is no force acting. According to Newton's first law of motion, the ball will continue to move in the forward direction with uniform motion in a straight line just as the train does—a fact you can check by inspecting the photograph. The ball remains above the locomotive whether the train is at rest or in uniform motion in a straight line. This law of motion is sometimes called the *principle of inertia,* and the property that material bodies have of continuing in a state of rest or of uniform motion in a straight line is sometimes known as the bodies' *inertia.* *

Newton illustrated Law I by reference to projectiles that continue in their forward motions "so far as they are not retarded by the resistance of the air, or impelled downward by the force of gravity," and he referred also to "the greater bodies of planets and comets." (On the inertial aspect of the motion of "greater bodies" such as "planets and comets," see Supplement 12.) At

*The earliest known statement of this law was made by René Descartes in a book that he did not publish. It appeared in print for the first time in a work by Pierre Gassendi. But prior to Newton's *Principia* there was no completely developed inertial physics. It is not without significance that this early book of Descartes was based on the Copernican point of view; Descartes suppressed it on learning of the condemnation of Galileo. Gassendi likewise was a Copernican. He actually made experiments with objects let fall from moving ships and moving carriages to test Galileo's conclusions about inertial motion. Descartes first published his version of the law of inertia in his *Principles of Philosophy* (1644); the earlier statement, in Descartes's *The World,* was published after Descartes's death in 1650. See Suppl. 8.

this one stroke Newton postulated the opposite view of Aristotelian physics. In the latter, no celestial body could move uniformly in a straight line in the absence of a force, because this would be a "violent" motion and so contrary to its nature. Nor could a terrestrial object, as we have seen, move along its "natural" straight line without an external mover or an internal motive force. Newton, presenting a physics that applies simultaneously to both terrestrial and celestial objects, stated that in the absence of a force bodies do not necessarily stand still or come to rest as Aristotle supposed, but they may move at constant rectilinear speed. This "indifference" of all sorts of bodies to rest or uniform straight-line motion in the absence of a force clearly is an advanced form of Galileo's statement in his book on sunspots (p. 88), the difference being that in that work Galileo was writing about uniform motion along a great spherical surface concentric with the earth.

Newton said of the laws of motion that they were "such principles as have been received by mathematicians, and . . . confirmed by [an] abundance of experiments. By the first two Laws and the first two Corollaries, Galileo discovered that the descent of bodies varies as the square of the time and that the motion of projectiles is in the curve of a parabola, experience agreeing with both, unless so far as these motions are a little retarded by the resistance of the air." The "two Corollaries" deal with methods used by Galileo and many of his predecessors to combine two different forces or two independent motions. Fifty years after the publication of Galileo's *Two New Sciences* it was difficult for Newton, who had already established an inertial physics, to conceive that Galileo could have come as close as he had to the concept of inertia without having taken full leave of circularity and having known the true principle of linear inertia.

Newton was being very generous to Galileo because, however it may be argued that Galileo "really did" have the law of inertia or Newton's Law I, a great stretch of the imagination is required to assign any credit to Galileo for Law II. This law has two parts. In the second half of Newton's statement of Law II, the "change in motion" produced by an "impressed" or "motive" force—whether that is a change in the speed with which a body moves

or a change in the direction in which it is moving—is said to be "in the direction of the straight line along which that force is impressed." This much is certainly implied in Galileo's analysis of projectile motion because Galileo assumed that in the forward direction there is no acceleration because there is no horizontal force, except the negligible action of air friction; but in the vertical direction there is an acceleration or continual increase of downward speed, because of the downward-acting weight force. But the first part of Law II—that the change in the magnitude of the motion is related to the motive force—is something else again; only a Newton could have seen it in Galileo's studies of falling bodies. This part of the law says that if an object were to be acted on first by one force F_1 and then by some other force F_2, the accelerations or changes in speed produced, A_1 and A_2, would be proportional to the forces, or that

$$\frac{F_1}{F_2} = \frac{A_1}{A_2}, \text{ or}$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

But in analyzing falling, Galileo was dealing with a situation in which only one force acted on each body, its weight W, and the acceleration it produced was g the acceleration of a freely falling body. (For the two forms of Newton's Law II, see p. 184.)

Where Aristotle had said that a given force gives an object a certain characteristic speed, Newton now said that a given force always produces in that body a definite acceleration A. To find the speed V, we must know how long a time T the force has acted, or how long the object has been accelerated, so that Galileo's law

$$V = AT$$

may be applied.

At this point let us try a thought-experiment, in which we assume we have two cubes of aluminum, one just twice the volume of the other. (Incidentally, to "duplicate" a cube—or make a cube having exactly twice the volume as some given cube—is as impossible within the framework of Euclidean geometry as to trisect an angle or to square a circle.) We now subject the smaller cube to a series of forces F_1, F_2, F_3, . . . and determine the corresponding accelerations A_1, A_2, A_3, . . . In accordance with Law II, we would find that there is a certain constant value of the ratio of force to acceleration

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} = \frac{F_3}{A_3} = \ldots\ldots = m_s$$

which for this object we may call m_s. We now repeat the operations with the larger cube and find that the same set of forces F_1, F_2, F_3, . . . respectively produces *another* set of accelerations a_1, a_2, a_3, . . . In accordance with Newton's second law, the force-acceleration ratio is again a constant which for this object we may call m_l

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \ldots\ldots = m_l$$

For the larger object the constant proves to be just twice as large as the constant obtained for the smaller one and, in general, so long as we deal with a single variety of matter like pure aluminum, *this constant* is proportional to the volume and so *is a measure of the amount of aluminum in any sample.* This particular constant is a measure of an object's resistance to acceleration, or a measure of the tendency of that object to stay as it is—either at rest, or in motion in a straight line. For observe that m_l was twice m_s; to give both objects the same acceleration or change in motion the force required for the larger object is just twice what it must be for the smaller. The tendency of any object to continue in its state of motion (at constant speed in a straight line) or its state of rest is called its *inertia;* hence, Newton's Law I is also called the principle of inertia. The constant determined by finding the constant force-acceleration ratio for any given body may thus be called the body's inertia. But for our aluminum blocks this same constant is also a measure of the "quantity of matter" in the object, which is called its *mass.* We now make precise the condition that two objects of different material—say one of brass and the other of wood—shall have the same "quantity of matter": it is that they have the *same mass* as determined by the force-acceleration ratio, or the *same inertia.*

In ordinary life, we do not compare the "quantity of matter" in objects in terms of their inertias, but in terms of their weight. Newtonian physics makes it clear why we can, and through its clarification we are able to understand why at any place on the earth two unequal weights in a vacuum fall at the same rate. But we may observe that in at least one common situation we always compare the inertias of objects rather than their weights. This happens when a person hefts two objects to find which is heavier, or has the greater mass. He does not hold them out to see which pulls down more on his arm; instead, he moves them up and down to find which is easier to move. In this way he determines which has the greater resistance to a change in its state of motion in a straight line or of rest—that is, which has the greater inertia. (On Newton's concept of inertia, see Supplement 15.)

FINAL FORMULATION OF THE LAW OF INERTIA

At one point in his *Discourses and Demonstrations Concerning Two New Sciences,* Galileo imagined a ball to be rolling along a plane and noted that "equable motion on this plane would be perpetual if the plane were of infinite extent." A plane without limit is all right for a pure mathematician, who is a Platonist in any case. But Galileo was a man who combined just such a Platonism with a concern for applications to the real world of sensory experience. In the *Two New Sciences,* Galileo was not interested only in abstractions as such, but in the analysis of real motions on or near the earth. So we understand that having talked about a plane without limit, he does not continue with such a fancy, but asks what would happen on such a plane if it were a real earthly plane, which for him means that it is "ended, and [situated] on high." The ball, in the real world of physics, falls off the plane and begins to fall to the ground. In this case,

> the movable (which I conceive of as being endowed with heaviness), driven to the end of this plane and going on further, adds on to its

previous equable and indelible motion that downward tendency which it has from its own heaviness. Thus there emerges a certain motion, compounded from equable horizontal and from naturally accelerated downward [motion], which I call "projection."

Unlike Galileo, Newton made a clear separation between the world of abstract mathematics and the world of physics, which he still called philosophy. Thus the *Principia* included both "mathematical principles" as such and those that could be applied in "natural philosophy," but Galileo's *Two New Sciences* included only those mathematical conditions exemplified in nature. For instance, Newton plainly knew that the attractive force exerted by the sun on a planet varies as the inverse-square of the distance

$$F \propto \frac{1}{D^2}$$

but in Book One of the *Principia* he explored the consequences not only of this particular force but of others with quite different dependence on the distance, including

$$F \propto D$$

"THE SYSTEM OF THE WORLD"

At the beginning of Book Three, which was devoted to "The System of the World," Newton explained how it differed from the preceding two, which had been dealing with "The Motion of Bodies":

> In the preceding Books I have laid down . . . principles not philosophical [pertaining to physics] but mathematical: such, namely, as we may build our reasonings upon in philosophical inquiries. These principles are laws and conditions of certain motions, and powers or forces, which chiefly have respect to philosophy; but, lest they should have appeared of themselves dry and barren, I have illustrated them here and there with some philosophical scholiums, giving an account of such things as are of a more general nature, and which philosophy seems chiefly to be founded on: such as the density and the resistance of bodies, spaces void of all bodies, and the motion of light and sounds. It remains that, from the same principles, I now demonstrate the structure of the System of the World.

I believe it fair to say that it was the freedom to consider problems either in a purely mathematical way or in a "philosophical" (or physical) way that enabled Newton to express the first law and to develop a complete inertial physics. After all, physics as a science may be developed in a mathematical way but it always must rest on experience—and experience never shows us pure inertial motion. Even in the limited examples of linear inertia discussed by Galileo, there was always some air friction and the motion ceased almost at once, as when a projectile strikes the ground. In the whole range of physics explored by Galileo there is no example of a physical object that has even a component of pure inertial motion for more than a very short time. It was perhaps for this reason that Galileo never framed a general law of inertia. He was too much a physicist.

But as a mathematician Newton could easily conceive of a body's moving along a straight line at constant speed forever. The concept "forever," which implies an infinite universe, held no terror for him. Observe that his statement of the law of inertia, that it is the natural condition for bodies to move in straight lines at a constant speed, occurs in Book One of the *Principia,* the portion said by him to be mathematical rather than physical. Now, if it is the natural condition of motion for bodies to move uniformly in straight lines, then this kind of inertial motion must characterize the planets. The planets, however, do not move in straight lines, but rather along ellipses. Using a kind of Galilean approach to this single problem, Newton could say that the planets must therefore be subject to two motions: one inertial (along a straight line at constant speed) and one always at right angles to that straight line drawing each planet toward its orbit. (See, further, Supplements 11 and 12.)

Though not moving in a straight line, each planet nevertheless represents the best example of inertial motion observable in the universe. Were it not for that component of inertial motion, the force that continually draws the planet away from the straight line would draw the planet in toward the sun until the two bodies collided. Newton once used this argument to prove the existence of God. If the planets had not received a push to give them an inertial (or tangential) component of motion, he said, the solar attractive force would not draw them into an orbit but instead would move each planet in a straight line toward the sun itself. Hence the universe could not be explained in terms of matter alone.

For Galileo pure circular motion could still be inertial, as in the example of an object on or near the surface of the earth. But for Newton pure circular motion was not inertial; it was accelerated and required a force for its continuance. Thus it was Newton who finally shattered the bonds of "circularity" which still had held Galileo in thrall. And so we may understand that it was Newton who showed how to build a celestial mechanics based on the laws of motion, since the elliptical (or almost circular) orbital motion of planets is not purely inertial, but requires additionally the constant action of a force, which turns out to be the force of universal gravitation.

Thus Newton, again unlike Galileo, set out to "demonstrate the structure of the System of the World," or—as we would say today—to show how the general laws of terrestrial motion may be applied to the planets and to their satellites.

In the first theorem of the *Principia* Newton showed that if a body were to move with a purely inertial motion, then with respect to any point not on the line of motion, the law of equal areas must apply. In other words, a line drawn from any such body to such a point will sweep out equal areas in equal times. Conceive a body moving with purely inertial motion along the straight line of which PQ is a segment. Then in a set of equal time intervals (Fig. 30) the body will move through equal distances AB, BC, CD, . . . because, as Galileo showed, in uniform motion a body moves through equal distances in equal times. But observe that a line from the point O sweeps out equal areas in these equal times, or that the areas of triangles OAB, OBC, OCD, . . . are equal. The reason is that the area of a triangle is one-half the product of its altitude and its base; and all these triangles have the same altitude OH and equal bases. Since

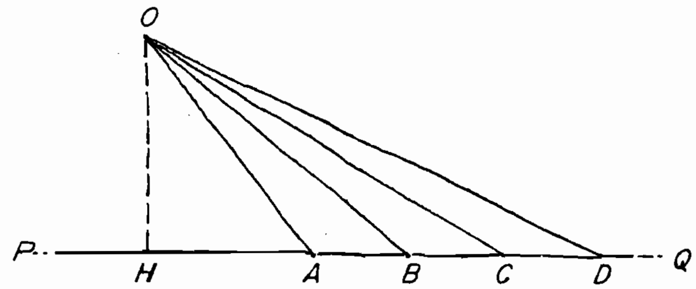


FIG. 30.

$$AB=BC=CD=. . .$$

it is true that

$$\tfrac{1}{2}AB\times OH=\tfrac{1}{2}BC\times OH=\tfrac{1}{2}CD\times OH=. . . . .$$

or

$$\text{area of } \Delta OAB=\text{area of } \Delta OBC=\text{area of } \Delta OCD= \ . . .$$

Thus the very first theorem proved in the *Principia* showed that purely inertial motion leads to a law of equal areas, and so is related to Kepler's second law. Newton then proved that if at regular intervals of time, a body moving with purely inertial motion were to receive a momentary impulse (a force acting for an instant only), all these impulses being directed toward the same point S, then the body would move in each of the equal time-intervals between impulses so that a line from it to S would sweep out equal areas. This situation is shown in Fig. 31. When the body reaches the point B it receives an impulse toward S. The new motion is a combination of the original motion along AB and a motion toward S, which produces a uniform rectilinear motion toward C, etc.: The triangles SAB, SBC, and SCD . . . have the same area. The next step, according to Newton, is as follows:

> . . . Now let the number of those triangles be augmented, and their breadth diminished *in infinitum;* and (by Cor. iv, Lem. iii) their ultimate perimeter ADF will be a curved line: and therefore the centripetal

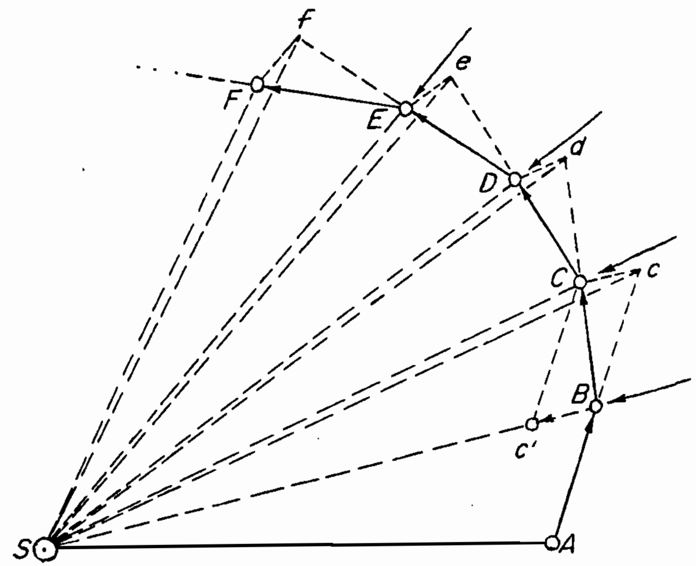


FIG. 31. If at B the body had received no impulse, it would, during time T, have moved along the continuation of AB to c. The impulse at B, however, gives the body a component of motion toward S. During T if the body's only motion came from that impulse, it would have moved from B to c'. The combination of these two movements, Bc and Bc', results during time T in a movement from B to C. Newton proved that the area of the triangle SBC is equal to the area of the triangle SBc. Hence, even when there is an impulsive force directed toward S, the law of equal areas holds.

force, by which the body is continually drawn back from the tangent of this curve, will act continually; and any described areas *SADS*, *SAFS*, which are always proportional to the times of description, will, in this case also, be proportional to those times. Q.E.D.

In this way Newton proceeded to prove:

Proposition 1. Theorem 1.

The areas which revolving bodies describe by radii drawn to an immovable centre of force do lie in the same immovable planes, and are proportional to the times in which they are described.

In simple language, Newton proved in the first theorem of Book One of the *Principia* that if a body is continually drawn toward some center of force, its otherwise inertial motion will be transformed into motion along a curve, and that a line from the center of force to the body will sweep out equal areas in equal times. In proposition 2 (theorem 2) he proved that if a body moves along a curve so that the areas described by a line from the body to any point are proportional to the times, there must be a "central" (centripetal) force continuously urging the body toward that point. The significance of Kepler's Law I does not appear until proposition 11 when Newton sets out to find "the law of the centripetal force tending to a focus of the ellipse." This force varies "inversely as the square of the distance." Then Newton proves that if a body moving in an hyperbola or in a parabola is acted on by a centripetal force tending to a focus, the force still varies inversely as the square of the distance. Several theorems later, in proposition 17, Newton proves the converse, that if a body moves subject to a centripetal force varying inversely as the square of the distance, the path of the body must be a conic section: an ellipse, a parabola, or a hyperbola. (See Supplement 13).

We may note that Newton has treated Kepler's laws exactly in the same order as Kepler himself: first the law of areas as a general theorem, and only later the particular shape of planetary orbits as ellipses. What seemed at first to be a rather odd way of proceeding has been shown to represent a fundamental logical progression of a kind that is the opposite of the sequence that would have been followed in an empirical or observational approach.

In Newton's reasoning about the action of a centripetal force on a body moving with purely inertial motion, mathematical analysis, for the first time, disclosed the true meaning of Kepler's second law, of equal areas! Newton's reasoning showed that this law implies a center of force for the motion of each planet. Since the equal areas in planetary motion are reckoned with respect to the sun, Kepler's second law becomes in Newton's treatment the basis for proving rigorously that a central force emanating from the sun attracts all the planets.

So much for the problem raised by Halley. Had Newton stopped his work at this point, we would still admire his achievement enormously. But Newton went on, and the results were even more outstanding.

THE MASTERSTROKE: UNIVERSAL GRAVITATION

In Book Three of the *Principia,* Newton showed that as Jupiter's satellites move in orbits around their planet, a line from Jupiter to each satellite will "describe areas proportional to the times of description," and that the ratio of the squares of their times to the cubes of their mean distances from the center of Jupiter is a constant, although a constant having a different value from the constant for the motion of the planets. Thus if T_1, T_2, T_3, T_4 be the periodic times of the satellites, and a_1, a_2, a_3, a_4 be their respective mean distances from Jupiter,

$$\frac{(a_1)^3}{(T_1)^2} = \frac{(a_2)^3}{(T_3)^2} = \frac{(a_3)^3}{(T_3)^2} = \frac{(a_4)^3}{(T_4)^2}$$

Not only do these laws of Kepler apply to the Jovian system, but they also apply to the five satellites of Saturn known to Newton —a result wholly unknown to Kepler. The third law of Kepler could not be applied to the earth's moon because there is only one moon, but Newton did state that its motion agrees with the law of equal areas. Hence, one may see that there is a central force, varying as the inverse-square of the distance, that holds each planet to an orbit around the sun and each planetary satellite to an orbit around its planet.

Now Newton makes the masterstroke. He shows that a single universal force (a) keeps the planets in their orbits around the sun, (b) holds the satellites in their orbits, (c) causes falling objects to descend as observed, (d) holds objects on the earth, and (e) causes the tides. It is the force called *universal gravity,* and its fundamental law may be written

$$F = G\frac{mm'}{D^2}$$

This law says that between any two bodies whatsoever, of masses m and m', wherever they may be in the universe, separated by a distance D, there is a force of attraction that is *mutual*, and each body attracts the other with a force of identical magnitude, which is *directly proportional to the product of the two masses* and *inversely proportional to the square of the distance between them.* G is a constant of proportionality, and it has the same value in all circumstances —whether in the mutual attraction of a stone and the earth, of the earth and the moon, of the sun and Jupiter, of one star and another, or of two pebbles on a beach. This constant G is called the *constant of universal gravitation* and may be compared to other "universal" constants—of which there are not very many in the whole of science—such as c, the speed of light, which figures so prominently in relativity, or h, Planck's constant, which is so basic in quantum theory.

How did Newton find his law? It is difficult to tell in detail, but we can reconstruct some of the basic aspects of the discovery.

From a later memorandum (about 1714), we learn that Newton as a young man "began to think of gravity extending to the orb of the moon, and having found out how to estimate the force with which [a] globe revolving within a sphere presses the surface of the sphere, from Kepler's rule of the periodical times of the planets being in a sesquialterate proportion [i.e., as the 3/2 power] of their distances from the centers of their orbs, I deduced that the forces which keep the planets in their orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them answer [i.e., agree] pretty nearly."

With this statement as guide, let us consider first a globe of mass m and speed v moving along a circle of radius r. Then, as Newton found out, and as the great Dutch physicist Christiaan Huygens (1629–1695) also discovered (and to Newton's chagrin, published first; see Supplement 14), there must be a central acceleration, of magnitude v^2/r. That is, an acceleration follows from the fact that the globe is not at rest nor moving at constant speed in a straight line; from Law I and Law II, there must be a force and hence an acceleration. We shall not prove that this acceleration has a magnitude v^2/r, but that it is directed toward the center you can see if you whirl a ball in a circle at the end of a string. A force is needed to pull the ball constantly toward the center, and from Law II the acceleration must always have the same direction as the accelerating force. Thus for a planet of mass m, moving approximately in a circle of radius r at speed v, there must be a central force F of magnitude

$$F = mA = m\frac{v^2}{r}.$$

If T is the period, or time for the planet to move through 360°, then in time T the planet moves once around a circle of radius r, or through a circumference of $2\pi r$. Hence the speed v is $2\pi r/T$, and

$$F = mA = mv^2 \times \frac{1}{r} = m\left[\frac{2\pi r}{T}\right]^2 \times \frac{1}{r}$$

$$= m \times \frac{4\pi^2 r^2}{T^2} \times \frac{1}{r}$$

$$= m \times \frac{4\pi^2 r^2}{T^2} \times \frac{1}{r} \times \frac{r}{r}$$

$$= \frac{4\pi^2 m \times r^3}{T^2 \times r^2} = \frac{4\pi^2 m}{r^2} \times \frac{r^3}{T^2}.$$

Since for every planet in the solar system, r^3/T^2 has the same value K (by Kepler's rule or third law),

$$F = \frac{4\pi^2 m}{r^2} \times K = 4\pi^2 K\frac{m}{r^2}.$$

The radius r of the circular orbit corresponds in reality to D the average distance of a planet from the sun. Hence, for any planet the law of force keeping it in its orbit must be

$$F = 4\pi^2 K\frac{m}{D^2}$$

where m is the mass of the planet, D is the average distance of the planet from the sun, K is "Kepler's constant" for the solar system (equal to the cube of the mean distance of any planet from the sun divided by the square of its period of revolution), and F is the force with which the sun attracts the planet and draws it continually off its purely inertial path into an ellipse. Thus far mathematics and logic may lead a man of superior wit who knows the Newtonian laws of motion and the principles of circular motion.

But now we rewrite the equation as

$$F = \left[\frac{4\pi^2 K}{M_s}\right]\frac{M_s m}{D^2}$$

where M_s is the mass of the sun and say that the quantity

$$\frac{4\pi^2 K}{M_s} = G$$

is a *universal constant*, that the law

$$F = G\frac{M_s m}{D^2}$$

is not limited to the force between the sun and a planet. It applies also to every pair of objects in the universe, M_s and m becoming the masses m and m' of those two objects and D becoming the distance between them:

$$F = G\frac{mm'}{D^2}$$

There is no mathematics—whether algebra, geometry, or the calculus—to justify this bold step. One can say of it only that it is one of those triumphs that humble ordinary men in the presence of genius. And just think what this law implies. For instance, this book that you hold in your hands attracts the sun in a calculable degree; the same force makes the moon follow its orbit and an apple fall from the tree. Late in life Newton said it was this last comparison that inspired his great discovery. (See, further, Supplement 14.)

The moon (see Fig. 32) if not attracted by the earth would have a purely inertial motion and in a small time t would move uniformly along a straight line (a tangent) from A to B. It does not, said Newton, because while its inertial motion would have carried it from A to B, the gravitational attraction of the earth will have made it fall toward the earth from the line AB to C. Thus the moon's departure from a purely inertial rectilinear path is caused by its continual "falling" toward the earth—and its falling is just like the falling of an apple. Is this true? Well, Newton put the proposition to a test, as follows:

Why does an apple of mass m fall to the earth? It does so, we may now say, because there is a force of universal gravitation between it and the earth, whose mass is M_e. But what is the distance between the earth and the apple? Is it the few feet from the apple to the ground? The answer to this question is far from obvious. Newton eventually was able to prove that the attraction

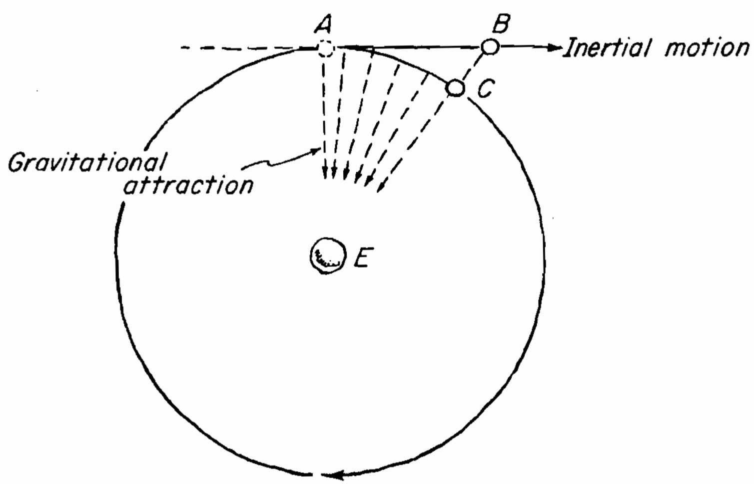


FIG. 32.

between a small object and a more or less homogeneous and more or less spherical body is exactly the same as if all the large mass of the body were concentrated at its geometric center. This theorem means that in considering the mutual attraction of earth and apple, the distance D in the law of universal gravitation may be taken to be the earth's radius, R_e. Hence the law states that the attraction between the earth and an apple is:

$$F = G\frac{mM_e}{R_e{}^2},$$

where m is the mass of the apple, M_e the mass of the earth, and R_e the earth's radius. But this is an expression for the *weight* W of the apple, because the weight of any terrestrial object is merely the magnitude of the force with which it is gravitationally attracted by the earth. Thus,

$$W = G\frac{mM_e}{R_e{}^2}.$$

There is a second way of writing an equation for the weight of an apple or of any other terrestrial object of mass m. We use Newton's Law II, which says that the mass m of any object is the ratio of the force acting on the object to the acceleration produced by that force,

$$m = \frac{F}{A}$$

or

$$F = mA.$$

Note that when an apple falls from the tree, the force pulling it down is its weight W, so that

$$W = mA.$$

Since we now have two different mathematical statements of the same force or weight W, they must be equal to each other, or

$$mA = G\frac{mM_e}{R_e^{\,2}}$$

and we can divide both sides by m to get

$$A = G\frac{M_e}{R_e^{\,2}}.$$

So, by Newtonian principles, we have at once explained why at any spot on this earth all objects—whatever their mass m or weight W may be—will have the same acceleration A when they fall freely, as in a vacuum. The last equation shows that this acceleration of free fall is determined by the mass M_e and radius R_e of the earth and a universal constant G, none of which *depends in any way* on the particular mass m or weight W of the falling body.

Now let us write the last equation in a slightly different way,

$$A = G\frac{M_e}{D_e^{\,2}}$$

where D_e stands for the distance from the center of the earth. At or near the earth's surface D_e is merely the earth's radius R_e. Now consider a body placed at a distance D_e of 60 earth-radii from the earth's center. With what acceleration A' will it fall toward the center of the earth? The acceleration A' will be

$$A' = G\frac{M_e}{(60\ R_e)^2} = G\frac{M_e}{3600\ R_e^{\,2}} = \frac{1}{3600}\,G\frac{M_e}{R_e^{\,2}}.$$

We just saw that at the surface of the earth an apple or any other object will have a downward acceleration equal to $G\frac{M_e}{R_e^{\,2}}$, and now we have proved that an object at 60 earth-radii will have an acceleration just 1/3600th of that value. On the average, a body at the earth's surface falls in one second toward the earth through a distance of 16.08 feet, so that out at a distance of 60 earth-radii from the earth's center a body should fall

$$1/3600 \times 16.08 \text{ feet} = 1/3600 \times 16.08 \times 12 \text{ inches} = 0.0536 \text{ inches.}$$

It happens that there is a body, our moon, out in space at a distance of 60 earth-radii and so Newton had an object for testing his theory of universal gravitation. If the same gravitational force makes both the apple and the moon fall, then in one second the moon should fall through 0.0536 inches from its inertial path to stay on its orbit. A rough computation, based on the simplifying assumptions that the moon's orbit is a perfect circle and that the moon moves uniformly without being affected by the gravitational attraction of the sun, yields a distance fallen in one second of 0.0539 inches—or a remarkable agreement to within 0.0003 inches! Another way of seeing how closely observation agrees with theory is to observe that the two values differ by 3 parts in about 500, which is the same as 6 parts in 1000 or 0.6 parts per hundred (0.6 per cent). Another way of seeing how this calculation can be made (perhaps following the lead Newton himself gave in the quotation on page 165) is as follows:

1) For a body on earth (the apple), the acceleration (g) of free fall is

$$g = G\frac{M_e}{R_e^{\,2}}.$$

2) For the moon, the form of Kepler's third law is

$$k = \frac{R_m^{\,3}}{T_m^{\,2}}$$

where R_m and T_m are respectively the radius of the moon's orbit and the moon's period of revolution. If the gravitational force *is* universal, then the relation derived earlier for planets moving around the sun

$$G = \frac{4\pi^2 K}{M_s}$$

can be rewritten for the moon moving around the earth, in the form

$$G = \frac{4\pi^2 k}{M_e}.$$

Hence, we may compute g from Equation (1), as follows:

$$g = \left[\frac{4\pi^2 k}{M_e}\right]\frac{M_e}{R_e^{\,2}} = 4\pi^2 k\left[\frac{1}{R_e^{\,2}}\right]$$

$$= 4\pi^2\left[\frac{R_m^{\,3}}{\mathrm{T_m}^{2}}\right]\left[\frac{1}{R_e^{\,2}}\right] = 4\pi^2\left[\frac{R_m^{\,3}}{T_m^{\,2}}\right]\left[\frac{1}{R_e^{\,2}}\right]\left[\frac{R_e}{R_e}\right]$$

$$= 4\pi^2\left[\frac{R_m}{R_e}\right]^3\left[\frac{R_e}{T_m^{\,2}}\right].$$

Since

$$\frac{R_m}{R_e} = 60, \text{ and } R_e = 4{,}000 \times 5{,}280 \text{ feet}$$

$$T_m = 28d = 28 \times 24 \times 3600 \text{ sec}$$

we may compute that

$$g \approx 32 \text{ ft/sec}^2$$

or

$$g \approx 1000 \text{ cm/sec}^2.$$

Newton said, in the autobiographical memorandum I have quoted, that he "compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth."

In Book Three of the *Principia,* Newton shows that the moon, in order to keep along its observed orbit, falls away from its straight line inertial path through a distance of 15 1/12 Paris feet (an old measure) in every minute. Imagine the moon, he says, "deprived of all motion to be let go, so as to descend toward the earth with the impulse of all that force by which . . . it is retained in its orb." In one minute of time it will descend through the same distance that it does when this descent occurs together with the normal inertial motion. Next, assume that this motion toward the earth is due to gravity, a force that varies inversely as the square of the distance. Then, at the surface of the earth this force would be greater by a factor 60 × 60 than at the moon's orbit. Since the acceleration is, by Newton's second law, proportional to the accelerating force, a body brought from the moon's orbit to the earth's surface would have an increase in its acceleration of 60 × 60. Thus, Newton argues, if gravity is a force varying inversely as the square of the distance, a body at the earth's surface should fall, starting from rest, through a distance of nearly 60 × 60 × 15 1/12 Paris feet in one minute, or 15 1/12 Paris feet in one second.

From Huygens's pendulum experiment Newton obtained the result that on earth (at the latitude of Paris) a body falls just about that far. Thus he proved that it is the force of the earth's gravity that retains the moon in its orbit. In making the computation, Newton predicted from observations of the moon's motion and from gravitation theory that the distance fallen by a body on earth in one second would be 15 Paris feet, 1 inch and 1 4/9 lines (1 line = 1/12 inch). Huygens's result for free fall at Paris was 15 Paris feet, 1 inch, 1 7/9 lines. The difference was 3/9 or 1/3 of a line and hence 1/36 of an inch—a very small number indeed. By the time Newton wrote the *Principia,* he had found a far better agreement between theory and observation than in that rough test he had made twenty years earlier.

Newton said that in this test observation agreed with prediction "pretty nearly." Two factors were involved in that phrase. First, he chose a poor value of the earth's radius and so obtained bad numerical results, agreeing only roughly or "pretty nearly." Second, since he had not then been able to prove rigorously that a homogeneous sphere attracts gravitationally as if all its mass were concentrated at its center, the proof was at best rough and approximate.

But this test proved to Newton that his concept of universal gravitation was valid. You can appreciate how remarkable it was when you consider the nature of the constant G. We saw earlier that $G = \frac{4\pi^2 K}{M_s}$ and we may well ask what either K (the cube of any planet's distance from the sun divided by the square of the periodic time of that planet's revolution about the sun) or M_s (the mass of the sun) has to do with either the earth's pull on a stone or the earth's pull on the moon. If the fact that the earth happens to be within the solar system lessens the wonder that G should apply to the stone and the moon, consider a system of double stars located millions of light-years away from the solar system. Such a pair of stars may form an eclipsing binary, in which one of the stars encircles the other as the moon encircles the earth. Way out there, beyond any possible influence of the sun, the same constant $G = \frac{4\pi^2 K}{M_s}$ applies to the attraction of each of the stars by the other. This is a universal constant *in spite of the fact* that in the form in which Newton discovered it, it was based on elements in *our solar system.* Evidently, the act of dividing the Kepler constant by the mass of the central body about which the others revolve eliminates any special aspects of that particular system—whether of planets revolving about the sun, or satellites revolving about Jupiter or Saturn. (See, further, Supplement 15.)

THE DIMENSIONS OF THE ACHIEVEMENT

A few further achievements of Newtonian dynamics, or gravitation theory, will enable us to comprehend its heroic dimensions. Suppose the earth were not quite a perfect sphere, but were oblate—flattened at the poles and bulging at the equator. Consider now the acceleration A of a freely falling body at a pole, at the equator, and at two intermediate points a and b. Clearly the "radius" R of the earth, or distance from the center, would increase from the pole to the equator, so that

$$R_p < R_b < R_a < R_e.$$

As a result the acceleration A of free fall at those places would have different values:

$$A_p = G\frac{M_e}{R_p{}^2};\ A_b = G\frac{M_e}{R_b{}^2};\ A_a = \frac{M_e}{R_a{}^2};\ A_e = \frac{M_e}{R_e{}^2},$$

so that

$$A_p > A_b > A_a > A_e.$$

The following data, obtained from actual experiment, show the acceleration varies with latitude:

Latitude	*Acceleration of free fall*	
0° (equator)	978.039 cm/sec^2	32.0878 ft/sec^2
20°	978.641	32.1076
40°	980.171	32.1578
60°	981.918	32.2151
90°	983.217	32.2577

In Newton's day, the acceleration of free fall was found by determining the length of a seconds pendulum—one that has a period of 2 seconds. The equation for the period T of a simple pendulum swinging through a short arc is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the length of the pendulum (computed from the point of support to the center of the bob) and g is the acceleration of free fall. Halley found that when he went from London to St. Helena it was necessary to shorten the length of his pendulum in order to have it continue to beat seconds. Newton's mechanics not only explains this variation but leads to a prediction of the shape of the earth, an oblate spheroid, flattened at the poles and bulging at the equator.

The variations in g, the acceleration of free fall, imply concomitant variations in the weight of any physical object transported from one latitude to another. A complete analysis of this variation in weight requires the consideration of a second factor, the force arising from the rotation of the object along with the earth. The factor that enters here is v^2/r where v is the linear speed along a circle and r the circle's radius. At different latitudes, there will be different values of both v and r. Furthermore, to relate the rotational effect to weight, a component must be taken along a line from the center of the earth to the position in question, since the rotational effect occurs in the plane of circular motion, or along a parallel of latitude. It is because of these rotational forces that the earth, according to Newtonian physics, acquired its shape.

A second consequence of the equatorial bulge is the precession of the equinoxes. In actual fact, the difference between the polar and equatorial radii of the earth may not seem very great:

equatorial radius = 6378.388 km = 3963.44 miles
polar radius = 6356.909 km = 3949.99 miles

But if we represent the earth with an 18-inch globe, the difference between the smallest and greatest diameters would be about 1/16th of an inch. Newton showed that precession occurs because the earth is spinning on an axis inclined to the plane of its orbit, the plane of the ecliptic. In addition to the gravitational attraction that keeps the earth in its orbit, the sun exerts a pull on the bulge, thus tending to straighten the axis. This force of the sun tends to make the earth's axis perpendicular to the plane of the ecliptic (Fig. 33A) or make the plane of the bulge (or of the earth's equator) coincide with the plane of the ecliptic. At the same time the moon's pull tends to make the plane of the bulge coincide with the plane of its orbit (inclined at about 5 degrees to the plane of the ecliptic). The moon's force is somewhat greater in this regard than the sun's. If the earth were a perfect sphere, the pull on it by the sun or moon would be symmetrical and there would be no tendency for the axis of rotation to "straighten out"; the lines of action of the gravitational pulls of sun and moon would pass through the earth's center. But if the earth should be oblate, or flattened at the poles, as Newton supposed, then there would be a net force tending to shift the

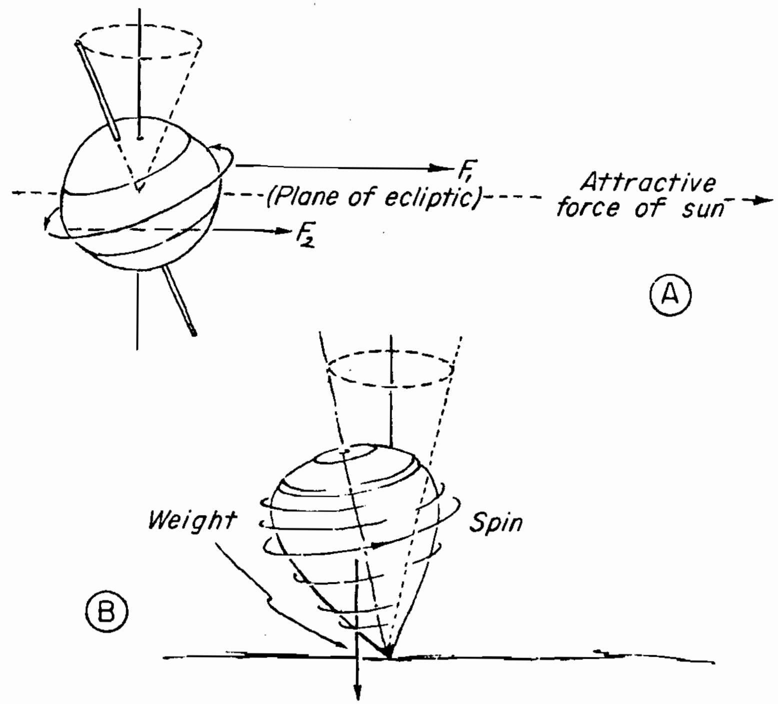


FIG. 33.

axis of the earth. And accordingly there would be a predictable effect.

Now it is a result in Newtonian physics that if a force is exerted so as to change the orientation of the axis of a spinning body, the effect will be that the axis itself, rather than changing its orientation, will undergo a conical motion. This effect may be seen in a spinning top. The axis of rotation is usually not absolutely vertical. The weight of the top acts therefore to turn the axis about the spinning point so as to make the axis horizontal. The weight tends to produce a rotation whose axis is at right angles to that of the top's spin, and the result is the conical motion of the axis shown in Fig. 33B. The phenomenon of precession had been known since its discovery in the second century B.C. by

Hipparchus, but its cause had been wholly unknown before Newton. Newton's explanation not only resolved an ancient mystery, but was an example of how one could predict the precise shape of the earth by applying theory to astronomical observations. Newton's predictions were verified when the French mathematician Pierre L. M. de Maupertuis measured the length of a degree of arc along a meridian in Lapland and compared the result with the length of a degree along the meridian nearer the equator. The result was an impressive victory for the new science.

Yet another achievement of the Newtonian theory was a general explanation of the tides relating them to gravitational action of the sun and moon on the waters of the oceans. We may well understand the spirit of admiration that inspired Alexander Pope's famous couplet:

> *Nature, and Nature's Laws lay hid in Night.*
> *God said,* Let Newton be! *and All was* Light.

In seeing how the Newtonian mechanics enabled man to explain the motions of planets, moons, falling stones, tides, trains, automobiles, and anything else that is accelerated—speeded up, slowed down, started in its motion or stopped—we have solved our original problem. But there remain one or two items that require a word or so more. It is true, as Galileo observed, that for ordinary bodies on the earth (which may be considered as revolving in a large elliptic orbit at an average distance from the sun of about 93 million miles), the situation is very much like being on something that is moving in a straight line, and there *is* an indifference to uniform rectilinear motion and to rest so far as all the dynamical problems are concerned. On the rotating earth, where the arc during any time interval, such as the flight of a bullet, is a part of a "circle" smaller than the annual orbit, another Newtonian kind of principle can be invoked, the principle of the conservation of angular momentum.

The angular momentum of a small object rotating in a circle (as a stone held on the top of a tower on a rotating earth) is given by the expression mvr where r is the radius of rotation, m the mass, and v the speed along the circle. The principle says that

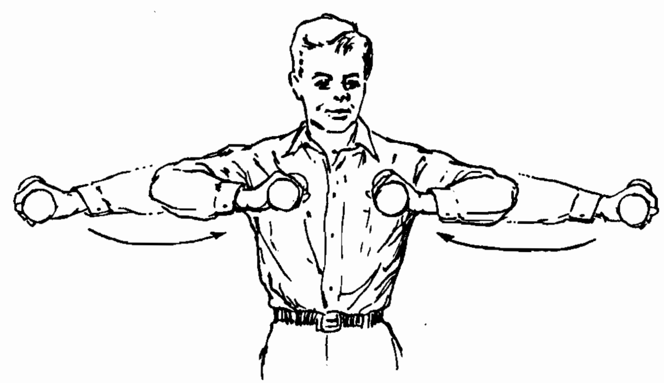
under a large variety of conditions (specifically, in all circumstances in which there is no external force of a special kind), the angular momentum remains constant.

An example may be given. A man stands on a whirling platform, with his arms outstretched and clutching a 10-pound weight in each hand. He is set whirling slowly on the turntable and then is told to bring his hands in toward his body along a horizontal plane so that he looks like Fig. 34. He finds that he spins faster and faster. Stretching his arms out once again will slow him down. For anyone who has never seen such a demonstration before (it is a standard figure in ice skating) the first encounter can be quite startling. Now let us see why these changes occurred. The speed v with which the masses m held in his hands move around is

$$v = \frac{2\pi r}{t}$$

where t is the time for a complete rotation, during which each mass m moves through a circumference of a circle of radius r. At first the angular momentum is

$$mvr = m \times \frac{2\pi r}{t} \times r = \frac{2\pi m r^2}{t}.$$



But as the man brings his arms in to his chest he makes r very much smaller. If $\frac{2\pi mr^2}{t}$ is to keep the same value, as the law of conservation demands, then t must get smaller too, which means that the time for a revolution becomes smaller as r diminishes.

What has this to do with a stone falling from a tower? At the top of the tower the radius of rotation is $R + r$ where R is the radius of the earth and r the height of the tower. When the stone strikes the ground, the radius of rotation is R. Therefore, like the masses drawn inward by the whirling mass, the stone must be moving around in a smaller circle when at the base of the tower than at the top, and so will whirl more quickly. Far from being left behind, the stone, according to our theory, should get a little ahead of the tower. How great an effect is this? Since the problem depends on t, the time for a rotation through 360 degrees, we can get a much better idea of the magnitude of the problem if we study the angular speed than if we consider some linear speed (as we did in Chapter 1). Look at the moving hands of a clock, paying particular attention to the hour hand. By how much does it appear to shift in, say, five minutes, which corresponds to dropping a ball from a much greater height than the Empire State Building? Not by any discernible degree. Now the rotation of the earth through 360 degrees takes just twice as much time as a complete rotation of the hour hand (12 hours). Since in five minutes the angular motion or rotation of the hour hand is not discernible to the unaided eye, a motion that is twice as slow produces practically no effect. Except in problems of long-range artillery firing, analysis of the movements of the trade winds, and other phenomena on a vastly larger scale than the fall of a stone, we may neglect the earth's rotation.

Such was the great Newtonian revolution, which altered the whole structure of science and, indeed, turned the course of Western civilization. How has it fared in the last 300 years? Is the Newtonian mechanics still true?

All too often the misleading statement is made that relativity theory has shown classical dynamics to be false. Nothing could be further from the truth! Relativistic corrections apply to objects moving at speeds v for which the ratio v/c is a significant quantity, c being the speed of light, or 186,000 miles per second. At the speeds attained in linear accelerators, cyclotrons, and other devices for studying atomic and subatomic particles, it is no longer true that the mass m of a physical object remains constant. Rather, it is found that the mass in motion is given by the equation

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

where m is the mass of an object moving at a speed v relative to the observer and m_0 is the mass of that same object observed at rest. Along with this revision goes Albert Einstein's now familiar equation relating mass and energy, $E = mc^2$, and the denial of the validity of Newton's belief in an "absolute" space and an "absolute" time. Well, then, might we agree with the new couplet added by J. C. Squire to the one of Pope's we have quoted?

It did not last: the Devil howling 'Ho,
Let Einstein be,' restored the status quo.

But for the whole range of problems discussed by Newton—exemplified today in the motion of stars, planets, moons, airplanes, spaceships, artificial satellites, automobiles, baseballs, rockets, and every other type of gross body—the speeds v attainable are such that v/c has to all intents and purposes the value zero and we can still apply Newtonian dynamics without correction. (There is, however, one very conspicuous example of a failure of Newtonian physics: a very small error in predicting the advance of the perihelion of Mercury—40″ per century!—for which we need to invoke relativity theory.) Hence for engineering and all physics except a portion of atomic and subatomic physics, it is still the Newtonian physics that explains occurrences in the external world.

While it is true that the Newtonian mechanics is still applicable in the range of phenomena for which it was intended, the student should not make the mistake of thinking that the framework in which the system originally was set is equally valid. Newton believed that there *was* a sense in which space and time were "absolute" physical entities. Any deep analysis of his writings shows how in his mind his discoveries depended on these "absolutes." To be sure, Newton was aware that clocks do not measure absolute time, but only local time, and that we deal in our experiments with local space rather than absolute space. He actually developed not merely a law of gravitational force and a system of rules for computing the answers to problems in mechanics, but constructed a complete system based on a world view, encompassing ideas of space, time, and order. Today, following the Michelson-Morley experiment and relativity, that world view can no longer be considered a valid basis for physical science. The Newtonian principles are considered to be only a special, though extremely important, case of a more general system.

Some scientists hold that one of the greatest validations of Newtonian physics has been the set of predictions concerning satellite motions; they have enabled us to launch into orbit a series of space vehicles and to predict what will happen to them out in space. This may be so, but to the historian the greatest achievement of Newtonian science must ever be the first full explanation of the universe on mechanical principles—one set of axioms and a law of universal gravitation that apply to all matter everywhere: on earth as in the heavens. Newton recognized that the one example in nature in which there is pure inertial motion going on and on and on, without frictional or other interference to bring it to a halt, is the orbital motion of moons and planets. And yet this is not a uniform or unchanging motion along a single straight line, but rather along a constantly changing straight line, because planetary motions are a compounding of inertial motion with a continuing falling away from it. To see that moons and planets exemplify pure inertial motion required the same genius necessary to realize that the planetary law could be generalized into a law of universal attraction for all matter and that the motion of the moon partakes of the motion of the falling apple.

Isaac Newton's system of mechanics came to symbolize the rational order of the world, functioning under the "rule of nature." Not only could Newtonian science account for present and

past phenomena; the principles could be applied to the prediction of future events. In the *Principia* Newton proved that comets are like the planets, moving in great orbits that must (according to Newtonian rules) be conic sections. Some comets move in ellipses and these must return periodically from far out in space to the visible regions of our solar system, whereas others will visit our solar system and never return. Edmond Halley applied these Newtonian results to an analysis of cometary records of the past and found—among others—a comet with a period of some seventy-five and a half years. He made a bold Newtonian prediction that this comet would reappear in 1758. When it did so, right on schedule, though long after Halley and Newton were dead, men and women everywhere experienced a new feeling of awe for the powers of human reason abetted by mathematics. This new respect for science was expressed by such adjectives as "amazing," "phenomenal," or "extraordinary." This successful prediction of a future event symbolized the force of the new science: the perfection of the mathematical understanding of nature, realized in the ability to make reliable predictions of the future. Not surprisingly, men and women everywhere saw a promise that all of human knowledge and the regulation of human affairs would yield to a similar rational system of deduction and mathematical inference coupled with experiment and critical observation. The eighteenth century not only was the Enlightenment, but became "preeminently the age of faith in science." Newton became the symbol of successful science, the ideal for all thought—in philosophy, psychology, government, and the science of society.

Newton's genius enables us to see the full significance of both Galilean mechanics and Kepler's laws of planetary motion as manifested in the development of the inertial principles required for the Copernican-Keplerian universe. A great French mathematician, Joseph Louis Lagrange (1736–1813), best defined Newton's achievement. There is only one law of the universe, he said, and Newton discovered it. Newton did not develop modern dynamics all by himself but depended heavily on certain of his predecessors; the debt in no way lessens the magnitude of his achievement. It only emphasizes the importance of such men as Galileo and Kepler, and Descartes, Hooke, and Huygens, who were great enough to make significant contributions to the Newtonian enterprise. Above all, we may see in Newton's work the degree to which science is a collective and a cumulattive activity and we may find in it the measure of the influence of an individual genius on the future of a cooperative scientific effort. In Newton's achievement, we see how science advances by heroic exercises of the imagination, rather than by patient collecting and sorting of myriads of individual facts. Who, after studying Newton's magnificent contribution to thought, could deny that pure science exemplifies the creative accomplishment of the human spirit at its pinnacle?

SUPPLEMENT NOTE ON THE TWO FORMS OF NEWTON'S SECOND LAW

Newton's *Principia* contains two forms of the second law. Since Newton's day we usually consider only the case of a continuously acting force F acting on a body of mass m to produce an acceleration A, that is $F = mA$. But Newton gave primacy to another case, that of an instantaneous force—an impact or blow—as when a tennis racquet strikes a ball or one billiard ball strikes another. In such cases, the force does not produce a continuous acceleration, but rather an instantaneous change in the body's quantity of motion (or momentum). This is the "change in motion" which is said to be proportional to "the motive force impressed" in Newton's statement of Law II on page 152. Newton conceived that $F = mA$ is a limiting case of the impact law, the situation when the time between successive impacts decreases indefinitely, so that the force ultimately achieves the limiting condition of acting continously. The law $F = mA$ was thus considered by Newton as derived from the impact law, as stated on page 152.

SUPPLEMENT 1

Galileo and the Telescope*

Galileo certainly did not invent the telescope and never claimed to have done so. Nor was he the first observer to point such an instrument toward the heavens. A newsletter of October 1608, about a year before Galileo made his first instrument, carried the news that the spyglass not only could make distant terrestrial objects seem nearer, but enabled one to see "even the stars which ordinarily are invisible to our eyes." There is very good evidence that Thomas Harriot had been observing the moon before Galileo began his telescopic observations; Simon Marius's claims (e.g., that he had discovered the satellites of Jupiter) are less well founded.

Galileo's report (see pp. 56–57) is taken from his *Sidereus nuncius* (1610). He wrote other versions of his first encounter with the telescope, which differ somewhat in detail, for instance, with

*This supplement is based on a report on this topic, by Albert Van Helden, at an international congress on Galileo, held in Pisa, Padua, Venice, and Florence in April 1983, published in the proceedings of this congress, edited by Paolo Galluzzi: *Novità celesti e crisi del sapere* (Suppl. to *Annali dell'Istituto e Museo di Storia della Scienza*, Florence, 1983). See also the monograph by Van Helden in the Guide to Further Reading on p. 243 below.

In *The Sidereal Messenger*, Galileo states that he had only heard of the new device, but had not actually seen one, when he applied his knowledge of the theory of refraction to produce a spyglass. But, by this time, the new instruments were not uncommon in Italy, and one had already arrived in Padua and was being discussed. Perhaps he was in Venice when the spyglass was being shown in Padua. In *The Assayer (Il Saggiatore)* of 1623, he recounted the role he played in the creation of the astronomical telescope and discussed in full the stages that led him to reinvent this instrument. Here, however, we are less concerned with the invention of the telescope than with the use Galileo made of it.