In Search of Lost Thermodynamics

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How Thermodynamical Are Black Holes?

A LOT

Outline

PROBLEMS:

1. zeroth law
2. Hawking radiation not from internal degrees of freedom, but external scattering field
3. only reason for entropy is to save Second Law, but we don't think that's fundamental anyway

I'll consider only first problem

Kerr Black Holes

- region of "no escape" (event horizon)
- asymptotically flat: "isolated system"
- stationary (asymptotic time-translation symmetry): "in equilibrium"
- rotational symmetry: "has angular momentum"
- zero electric charge
- "no hair": characterized by mass, angular momentum, electric charge—like a thermodynamical system (and a fundamental particle!)

(Hayward's dynamical trapping horizons are fascinating, but beyond scope of this talk)

Thermodynamical Objects?

late 1960s Wheeler poses fundamental puzzles about black holes and the Second Law of Thermodynamics

1970 Penrose shows how to extract energy; Hawking proves Area Theorem; Christodoulou characterizes irreducible mass, defines reversible and irreversible processes; Geroch's infamous *Gedankenexperiment*

1971–1973 Bekenstein proposes entropy proportional to surface area, formulates Generalized Second Law

1973 Bardeen, Carter and Hawking prove four Laws of Black-Hole Mechanics; most think analogy with thermodynamics purely formal

1974–1975 using quantum effects, Hawking shows black holes radiate like perfect black bodies: after brief resistance, most now think black holes are "true thermodynamical objects"

The Status of the Analogy?

A purely formal analogy? Or are black holes truly thermodynamical objects? Are the black-hole laws the laws of thermodynamics extended to cover black holes?

Orthodoxy: Yes!

- Hawking radiation justifies the claim that κ is a physical temperature
- the postulated validity of the Generalized Second Law justifies the claim that A is a physical entropy

(*N.b.*: black-hole entropy is NOT entropy of Hawking radiation, and is not based on it!)

Philosophical Memory and the Conceptual Identity of Thermodynamics

concepts, principles, and relations among them get modified in philosophically rich ways in extension to new framework $\Rightarrow$ new avenues to attack old problems

Scientific Memory and the Physical Identity of Thermodynamics

delineate what of standard thermodynamics black holes do and do not satisfy $\Rightarrow$ constraints on form of underlying statistical theory (quantum gravity?)

Fundamental Principles and Features of Standard Thermodynamics

not just four laws, but also:

- state space of equilibrium states
- temperature as mediator/measure of physical couplings
- classification of processes as adiabatic, reversible, and quasi-static
- important conjugacy relations between intensive and extensive variables
- additivity of entropy
- equilibrium states maximize entropy and minimize free energy
- the Clausius and Kelvin Postulates
- ...

Extending Thermodynamics into New Fields

How to Decide Whether Blackbody Radiation Is Thermodynamical?

- need Gibbsian "heat" term to save First Law?
- analogue of temperature physically couples in right way to ordinary thermodynamical systems?
- violations of Second Law without "entropy" of blackbody radiation (*e.g.*, *perpetuum mobile* of 2nd kind: Clausius and Kelvin Postulates)?
- analogues of as much of the rest as possible. . .

THEN...

blackbody radiation is real thermodynamical system

BUT...

differences point to need to modify statistical grounding, and place constraints on how to do it

can we do same for black holes?

Zeroth Law

Thermodynamics The temperature T is constant throughout a body in thermal equilibrium.

Black Holes The surface gravity κ is constant over the event horizon of a stationary black hole.

REAL Zeroth Law

Two bodies in equilibrium with a third are in equilibrium with each other.

Fundamental Role of Transitivity

- construction of state space of equilibrium states
- state-function of intensive quantity physically identified as temperature
- equilibrium implies constancy of temperature
- temperature as mediator and measure of thermal coupling
- definition of entropy, its additivity
- characterization of processes as adiabatic, reversible, and quasi-static
- conjugacy of intensive and extensive quantities
- entropy maximization, free-energy minimization

CONSTANCY OF TEMPERATURE ALONE DOES NOT SUFFICE!

A Strengthened Zeroth Law for Black Holes?

Yes!

Claim: Constancy + Kelvin Postulate $\Rightarrow$ Transitivity

Recall:

The Kelvin Postulate

A transformation whose only final result is to transform into work heat extracted from a source that is at the same temperature throughout is impossible.

What Is "Mutual Equilibrium"?

claim: all we need is a sufficient condition

Condition

there will be no spontaneous transfer of heat from one system to another other, and neither will spontaneously perform work on the other, when two bodies in mutual equilibrium are brought into contact

Deriving the Zeroth Law (Crude Sketch)

three ordinary thermodynamical systems A, B and C

1. assume A and C in mutual equilibrium, and B and C, but not A and B
2. bring A and C into contact: no heat spontaneously transferred, so they are same temperature (similarly for B and C) $\Rightarrow$ A and B same temperature
3. bring A and B in contact
4. *ex hypothesi*, one must spontaneously perform work on the other
5. $\Rightarrow$ violation of the Kelvin Postulate
6. $\Rightarrow$ they are in mutual equilibrium

"Mutual Equilibrium" for Black Holes

so: what are work and heat for black holes?

Irreducible Mass

M_{irr} a convex, positive-definite function of total mass and angular momentum; total mass cannot be reduced below initial value of M_{irr} by any physical process

$$M_{\text{irr}}^2 := \frac{1}{2}[M^2 + (M^4 - J^2)^{\frac{1}{2}}]$$

so

$$M^2 = M_{\text{irr}}^2 + \frac{J^2}{4M_{\text{irr}}^2} \geq M_{\text{irr}}^2$$

Irreducibility of Irreducible Mass

- $A = 16\pi M_{\mathsf{irr}}^2$
- thus Second Law implies M_{irr}^2 cannot decrease through any physical process

Free Energy, "Heat", and Work for Black Holes

- since M cannot be reduced below M_{irr}:

$$\text{“free energy”} := M - M_{\text{irr}}$$

- so "quantity of heat" transferred in any process: change in total energy minus change in free energy:

$$\Delta Q_{\text{BH}} := \Delta M_{\text{irr}} \, (> 0)$$

- so "work": change in total energy not due to change in free energy (rotational and radiative processes)

The Kelvin Postulate for Black Holes

A transformation whose only final result is that a quantity of heat is extracted from a stationary black hole and transformed entirely into work is impossible.

Proof (Crude Sketch):

1. assume such a transformation possible
2. if heat extracted is purely gravitational, then change in irreducible mass must be strictly greater than change in total mass (*i.e.*, irreducible mass must increase)
3. so area (entropy) must also increase, violating the Postulate

(rigorous proof *à la* Wald's "physical process" version of the GSL; straightforward emendation to accommodate heat transfer by Hawking radiation)

Promissory Note

construction of equilibrium state-space, *etc*.