Entropy Is Modal—What's Up with That?

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January 17, 2018

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1 Two Kinds of Modality

There are two kinds of modality that may play a role in the nature of entropy as a physical quantity in the different theories and frameworks in which it is variously defined.

1. The definition of the quantity itself derives essentially from a principle that itself is modal in character (*e.g.*, placing restrictions on the possibility of kinds of physical systems, transformations, or processes).

2. The quantity is not an intrinsic property of a single state of a system, but is rather a property of a modally characterize class of states (*e.g.*, those "possible" given certain constraints).

I am not sure about the relation between them, if any.

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2 Thermodynamical Entropy

The Second Law (Kelvin Postulate, Clausius Postulate, Planck Postulate) all place restrictions on which physical processes are possible. Since these form the basis of the definition of entropy, entropy is fundamentally modal in thermodynamics. Also, entropy as defined by the Clausius formula ($\int_A^B \frac{dQ}{T}$) is not a property of a single state, but is a relation between states: how much heat would be transferred (at a given temperature) if the system were to evolve quasi-statically between this state and that one? This forms an equivalence class of states relative to a given state: those with respect to it have the same entropy. Thus, the entropy is not a property of a single state, but is characterized by a modal relation among states. How would we define the thermodynamical entropy of a system that could never, in principle, change its state (*e.g.*, a system with zero free energy in an otherwise empty universe)? We could not. This is not true of its energy, or any other thermodynamical property.

3 Entropy in Boltzmannian Statistical Mechanics

It is not a property of an individual micro-state. If all I know is the micro-state, I do not know the entropy. I have to know the partition of phase space into the macro-states. Thus the entropy is a measure of how many different micro-states the system *could* be in, though it actually is not. For no other quantum physics of interest in Boltzmannian statistical mechanics does one need to know the partition into macro-states in order to determine its value.

The modality is even more complex and multi-faceted for the combinatorial entropy. The same goes for the fine-grained entropy. (See, *e.g.*, **?**, §2.2 and Frigg and Werndl 2011, p. 124 for remarks relevant to, but not directly addressing, this claim.)

4 Entropy in Gibbsian Statistical Mechanics

It is not a property of an individual element of the ensemble. It is a property of the probability distribution over the ensemble. If I know only the state the individual is actually in, but not the probability distribution, then I do not know its entropy. I need to know all other states all other possible members of the ensemble can be in, and how likely they are. Thus, there is a double modality here, the states of the non-actual members of the ensemble *and* how likely each is, roughly speaking. This holds true no matter how one interprets probability in Gibbsian statistical mechanics. (See **?**, §3.1 and Frigg and Werndl 2011, p. 128 for remarks relevant to, but not directly addressing, this claim.)

Note that this is true for other observable physical quantities as well, except absolute temperature and energy, not only entropy. This makes Gibbsian statistical mechanics *weird*.

5 Shannon Entropy

Again, this is not a property of a message or of a channel itself, but of the probability distribution over possible messages (roughly speaking).

See Frigg and Werndl (2011) for a discussion of the differences between the discrete and the continuous Shannon entropy.

6 Von Neumann Entropy

This one I'm not sure about. I have to think more about it. I strongly suspect it is modal.

7 Black-Hole Entropy

Not sure whether this is modal or not. I think that Sorkin (2005) would argue it is not—the "natural" coarse-graining uniquely determines it. Also, it is an intrinsic property of the black hole itself (modulo the issue of picking a spacelike slicing with respect to which one determines the area—or does this make it modal?).

8 No Entropometers

It follows as an immediate consequence of the modal character of entropy that there can be no such thing as an entropometer, a measuring instrument that couples directly with and measures the value of entropy, of any kind. Indeed, it follows that entropy can mediate no physical couplings between systems, in the way, say, that temperature mediates heat flow between thermodynamical systems.

Even though the area of the event horizon of a black hole is not a modal quantity in either sense of the idea I'm using here, still it mediates no physical couplings, so there is no entropometer. (Thus the relation between modality and lack of an entropometer is not a biconditional.)

9 The Peculiarity of Entropy

Although other physical quantities satisfy principles that limit physically possible states and processes (*e.g.*, the principle of the conservation of energy), none is *defined* by such a principle.

No other physical quantity has this sort of intrinsically modal nature (except some of those in Gibbsian statistical mechanics).

It speaks against the Humean mosaic. Lewisians can deny that it is a "fundamental" quantity—but, come on, if the Second Law isn't fundamental, what is?

None of this has anything to do with "entropy as ignorance" or "entropy as disorder". Maybe the coarse-grained entropy in Boltzmannian statistical mechanics has to do with ignorance, if one interprets $G(D)$ as a measure of knowledge, and claims that knowledge of the distribution is less than knowledge of the arrangement—but given that the combinatorial argument works *only* because we assume there is no possible knowledge that would allow us to distinguish among particles, this seems weak. If one accepts Jaynes' account, then ignorance may work for Gibbsian entropy, but there are strong arguments against it.

References

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