Lecture: Fndns Thermo & SM – Nature of Entropy

17 Jun 2018

Outline

1) overview: how ~~has~~ entropy been conceived; diff't fmn's in diff't theors, and in same theory; modality; appears everywhere

2) entropy in thermo, BSM, GSM, cosmo, BHs

3) modality

Everywhere: phys, logic, stats, biology, neuro-cognitive sci/neuropsychology, economics, communication/network theory, traffic, geometry, —

Overview: Conceptions

- ent as randomness: not even a clear form'n (what's 'random'?)
- entropy as disorder: plausibly only in BSM, and not really when one looks close; although if Penrose Conformal Curvature Hypothesis is right, maybe something like this in cosmo
- entropy as ignorance: plausibly only in GSM, but hard to justify when looking closely
- entropy as measure of available engy: greater S, smaller available engy: true in thermo, plausibly true in SM, not for bhs, difficult even to formulate in cosmo
- entropy as measure of dissipation of engy: free engy drops as total engy remains constant: not equiv to entropy increase, but intimately related (see Maxwell Theory of Heat, ch. XII, p. 192)
- entropy as determining 'arrow of time' – we'll get to that

Overview: diff't formulations (we ignore zillions, e.g. Shannon, von Neumann, etc.)

- Thermo: seems like just one
- BSM: 4
- GSM: 2
- cosmo: hard to say ('ordinary grav + ordinary matter' vs measures of curvature)
- BHs: 1 (excluding conjectural attempts to give statistical underpinning – there dozens) will get to

(to think later in course)

⇒ arguably, all fundamental diff in character, hard to see physical and conceptual relations – but they all seem to enter into 2nd Law!

Overview: modality

- w/ possible exception of cosmo & bhs, they all seem fundamentally modal in character, only phys gy with this nature – wtf?

Therm Entropy

- Kelvin, Clausius, Planck Postulates serve as basis for defn, viz. they imply

$$\oint \frac{dQ}{T} \leq 0 \quad \text{(cycle, Q = heat \underline{received} by sys)}$$

⇒ if reversible, trivially $= 0$

⇒ define $\Delta S = \int_A^B \frac{dQ}{T}$, fix value for A, then   S(A) ← "standard/reference state"

$$S(B) := S(A) + \int_A^B \frac{dQ}{T} \quad \text{for } A \to B \text{ any \underline{reversible} path}$$

and then follows that (no proof given)

$S(C) \leq S(B)$ for all transformations, w/ equality for reversible, from any states along path $B \to C$

⇒ fixed only up to additive constant, depending on initial assignment of $S(A)$

- can't change unless heat is exchanged
- association w/ decrease in available energy:

Helmholtz free energy = amount of total energy available to do work at constant T & V

$$A = U - TS \quad \text{(log of partition func for canonical ensemble)}$$

⇒ increase in entropy → decrease in capacity to do work

Gibbs free energy = available for work at constant temp & pressure

$$G = U + pV - TS \quad (U + pV = \text{"enthalpy"})$$

⇒ minimum of free energy = max entropy

Lecture: Fndns Thermo & SM – Nature of Entropy   17 Jun 2018

BSM

(1) let $F(x,t)$ be dist func on T (single particle space)

then $\underline{H(F,t)} := \int_T F(x,t) \log F(x,t)\, dx$

⇒ H-thm says H decreases "monotonically", reaching equil at Maxwell distribution – to prove must make "ad hoc" dyn'l assumptions, like Stosszahlansatz (introduces asymmetry)

(2) fine-grained entropy

$S_{fg}(F,t) := -kNH(F,t)$, N = # particles, k = Boltzmann constant

⇒ if H-thm right, then 2nd Law
– this increases with homogeneity of particles in cells – not "disorder"

Probs for both: How to interpret dist? relative frequency or prob'y? If latter, what kind of prob'y (epistemic-objective, epistemic-subjective, ontic)? what diffs will make?

(3) coarse-grained/combinatorial entropy

see "IntroSM, BSM" notes, 15 Nov 2017, pp. 5-6

then $S_c(D) = k \log G(D)$

– standard interp: S_c measures "# of arrangements for given dist D"
→ larger S_c, "less info about actual arrangement"
but this assumes "amount of knowledge in knowing arrangement always greater than that for dist", to interpret S_c as "measure of ignorance",
but combinatorial arg works only for we assume ∃ knowledge that allows us to distinguish particles, viz., it seems ~~to~~ content of knowledge in knowing arrangement same as for dist!
– to increase, must make "ad hoc" dyn'l assumptions (e.g. ergodicity)

Lecture: Frodas Thermo & SM – Nature of Entropy 17 Jan 2017

④

④ neo-Boltzmannian: measure-theoretic

$$S_M = k \log(\mu(M))$$

for macrostate M

How all related?

- Must make non-trivial, sometimes ad hoc assumptions, plausibly (only approx) true only in limited circumstances
- their conceptual rel'ns not at all clear

→ e.g., for $S_{F_G} \approx S_M$, we need:

i) F not vary much from cell to cell (more homogeneity ⇒ higher ent ⇒ closer to equil)

ii) large # particles

iii) no inter-particle forces

rel'n to thermo

≈ thermo ent only under limited circumstances, must be proven separately on case-by-case basis

GSM

$\rho(x,t)$ = "prob dist" on T^N (ensemble state space)

prob of finding state of system in $R \subseteq T^N$ at time t

$$p(R) = \int_R \rho(x,t)\, dx$$

① fine-grained entropy

$$S_F(\rho) := -k \int_{T^N} \rho(x,t) \log \rho(x,t)\, dx$$

Lecture: Fndns Thermo of SM – Nature of Entropy

17 Jan 2018

probs

- how to interpret ρ? (see lecture notes on prob'y in SM)
- doesn't increase over time (indeed constant), following from fundamental Liouville Thm

⇓

solve increase prob by introducing

② coarse-grained entropy

i) partition Γ^N into cells ω_i of equal volume $\delta\omega$ (measure)

ii) coarse-grained density $\bar{\rho}_\omega(x,t) := \frac{1}{\delta\omega}\int_{\omega(x)} \rho(x',t)\, dx'$

iii) $S_{c,\omega}(\rho) := S_F(\bar{\rho}_\omega) = -k\int_{\Gamma^N} \bar{\rho}_\omega \log \bar{\rho}_\omega \, dx$

Thm provably $S_c \geq S_F$, equality iff ρ is uniform over all cells; not constant over time (not necess'ly)

⇒ but does S_c increase over time?

- one sol'n: Jaynes' "Princ of Max Ent" seems to guarantee this – but ∃ probs (how to apply GSM framework out of equil)

rel'n to other entropies?

≈ thermo only under limited circumstances, proven on case-by-case basis
- conceptual rel'n not at all clear

≈ BSM, same

Modality: see PDF "Entropy Is Modal – What's Up with That?"