Lecture: Fndns Thermo & SM – Probability in SM 10 Jan 2018

(1)

- read Jaynes' quotes in online doc (course webpage)
  - → distinction between formalism, approach, interpretation, w/r epi, epi
- (cf. Good's "46656 Varieties of Bayesians" – objective, subjective, ontic, def'n of prob'y, def'n of cond'l prob'y, only basis for statistical inference, only basis for decision-making, —)

Broadest Categories of Approach to Prob'y

- epistemic vs ontic (not standard objective vs subjective, coz one can be obj or subj epi, etc.)
  - epistemic: prob'y feature, property, constraint, consequence, etc. of agent's epistemic state; may be subject to some external 'rationality constraints' (e.g., coherence, princ of indifference, Kolmogorov axioms, exchangeability, etc.) or not
  - obj: all rational agents w/ same information (and background beliefs?) assign same prob's
  - subj: a rational agent can assign any probs they want, perhaps subject to constraint

Approaches

- logical → e.g. "rel'ns of objective evidential support/confirmation"
- degree of belief
- Bayes
- frequentist
- propensity
- best-systems
- ...

most can be either epi or ont (degree of belief always epi, propensity always ontic)

Frequency & Sampling

- fundamental distinction between frequency as prob'y (comparison class problem, "long run", anomalous runs, etc.) and frequency as evidence for prob'y, which all interp's must rely on, treated as strongest evidence
- not enough to decide probs by 'symmetry' or 'counting', etc. – sampling procedures must be fair (laying down vs shaking a die; Diaconis flipping a coin and his proof of optimal shuffling of cards)

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Rational Constraints: Peirce's paradox   } Peirce's "Doctrine of Chances", § IV

Kolmogorov Axioms

prob space $(X, \mathcal{A}, p)$

X : space of 'events', 'propositions', ...

$\mathcal{A}$: "algebra" on X (unions/complements, disjunction/negation, ...)

p : func $X \to [0, 1]$

axioms

1) non-negativity : $p(x) \geq 0 \quad \forall x \in X$
2) normalization : $p(X) = 1$ (more precisely: $p(\{\bigcup_X x\}) = 1$)
3) additivity : $p(x \cup y) = p(x) + p(y)$ for $x \cap y = \emptyset$
   (or $p(x \vee y) = p(x) + p(y)$ when $x \perp y$)

consequences

1) $p(\neg x) = 1 - p(x)$
2) $p(x \cap y) = p(x)\,p(y)$ when x & y 'independent'

example: loaded die

$X = \{ \{1\}, \{2\}, \ldots, \{1,2\}, \{1,3\}, \ldots \{1,2,3\}, \ldots \}$

p : $p(1) = 3/12$, $p(6) = 1/12$, $p(2,3,4,5) = 1/6$

so $p(\{1,3\}) = p(\{1\} \cup \{3\}) = 3/12 + 1/6 = 5/12$

$X^2 = X \times X$, all ordered pairs of rolls (rolling die twice in row)

exercise: $p(\{(1,3)\}) = ?$ $\quad p(\{(1,3), (1,6)\}) = ?$

prove: prob'y of 2 simultaneous pairs of rolls turning up resp'y $(1,3)$ and $(1,6)$? $= p(\{1,3\})\,p(\{1,6\})$

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Expectation Value, Weighted Avg, Phase Avg

consider a function $f : X \to \mathbb{R}$
we want to know what it's expected value is when "sampled" over X (long trial runs, say))

$$\langle f \rangle = \sum p(x) f(x)$$

Say in our example of loaded die, we have a procedure that produces a particle of mass m_i when the die is rolled and i pips show; if we roll the die j times (space is X^j), then phase avg/expected val is: $\frac{1}{j} \sum_{i=1}^{j} m_i p(i)$

Gibbs SM

- ensemble: "imaginary" collection of identical copies of macro-sys of interest, one for each possible microstate it can occupy subject to some constraint (fixed engy, fixed volume, etc.)
- distribution: a "weighting" or 'prob'y' or 'prob'y density' assigned to each element of ensemble ~~available macro-regime~~ phase
  $\Rightarrow$ a function ρ on space T and time s.t. $\int_T \rho(x,t)\, dx = 1 \quad \forall t \in \mathbb{R}$
- phase avg of macro-qty

$$\bar{F}(t) = \int_T f(x,t)\, \rho(x,t)\, dx$$

: 'what is measured in observations'? (except abs temp and entropy)

- entropy (or 'fine-grained' or 'ensemble' entropy)

$$S(\rho) = -k_B \int_T \rho \ln(\rho)\, dx$$

sketch 'standard story': measurement time long compared to characteristic scale of micro-dyns; so measurement is essentially time avg weighted by distr $\Rightarrow$ ergodic story

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– stationary dist and equilibrium

since we want measured qs to be constant in equil, by defn of phase avg and it's association of 'what is observed' we define equil to be a stationary dist:

$$\frac{\partial \rho}{\partial t} = 0$$

then: "maximum entropy princ" says 'real equil' should be (unique) stationary dist that maximizes $S(\rho)$ ☆ see discussion of Shannon entropy on next page

problem

Liouville's thm implies that $\frac{d\rho}{dt} = 0$

so how can equil ever be 'approached' or 'achieved' when sys begins out of it ??

how to interpret ρ as prob'y   Jaynes: "what is φ of formalism?"

- ergodic program / frequency
- read probs from Jaynes article pp. 92 ff.

- Jaynes' maximum ent princ.
- probs from Uffink's 'Subj Probys & Stat Phys' pp. 42 ff.

Prob'y in Boltz SM

notes of those lects 15 Nov 2017 pp. 4–5, 13 Dec 2017 p. 8

Cheap criticism of epistemic probs in SM (beliefs don't cause thermo'l behavior) can be dismissed as in discussion in Uffink 'Subj Probys & Stat Phys' p. 45, à la Frigg

mention Malament & Zabell as way to try to justify $F = F^*$ (fundamental problem) using ergodic theory, but in way that avoids problematic limit $\to \infty$

Max Ent Princ of Shannon Ent

why Shannon ent is relevant: the phys sys we're studying is an emitter of info (its coupling to environment based on its state); we're receiver of that info, when we observe and measure by way of the noisy (error-producing, lossy, low-resolution, not discriminating) channel of our instruments; because the channel is noisy, and we can't isolate, control or even determine sources of error, much less determine what actual errors in transmission are, we ~~must~~ know we're in state of uncertainty about what actual info sys emitted; we must try to construct a principled 'proby dist' over all possible info-emissions that could have yielded the info we actually received, subject to constraints on the sys and our knowledge of the noisiness of the channel; Socratic maxim γνῶθι σεαυτόν and princ of epistemic modesty demand jointly that we choose one that represents ~~a state of~~ maximum uncertainty ~~about~~ in our epistemic state; so Shannon ent is measure of that uncertainty; so maximize that, subject to all constraints

Jaynes on the Role and Value of Philosophy in Science, with a Comparison to Maxwell

Jaynes (1967, pp. 91–92):

> [T]he injection of philosophical considerations into science has usually proved fruitless, in the sense that it does not, of itself, lead to any advances in the science. But there is one extremely important exception to this. . . . At the stage in development of a theory where we already have a formalism successful in one domain, and we are trying to extend it to a wider one, some kind of philosophy about what the formalism "means" is absolutely essential to provide us with a sense of direction. And it need not even be a "true" philosophy—whatever that may mean—for its real justification will not lie in whether it is "true", but in whether it does point the way to a successful extension of the theory.
>
> In the construction of theories, a philosophy plays somewhat the same role as scaffolding does in the construction of buildings; you need it desperately at a certain phase of the operation, but when the construction is completed you can remove it if you wish; and the structure will still stand of its own accord. This analogy is imperfect, however, because in the case of theories, the scaffolding is rarely ugly, and many will wish to retain it as an integral part of the final structure. At the opposite extreme to this conservative attitude stands the radical positivist, who in his zeal to remove every trace of the scaffolding, also tears down part of the building. Almost always, the wisest course will lie somewhere between these extremes.

And Jaynes (1967, p. 100):

> Once a philosophy has led to a definite, unambiguous mathematical formalism by which practical calculations may be carried out, then the issue is no longer one of philosophy; but of fact. The formalism either will or will not prove adequate in practice; and it will be judged, quite properly, not by the philosophy which led to it, but by the results which it gives. If you do not like my philosophy, but you find that the formalism, nevertheless, does give useful results, then I am quite sure that you will be able to invent some *other* philosophy by which the formalism can be justified! And, perhaps, that other philosophy will lead to still further generalizations and extensions, to which my own philosophy makes me blind. That is, after all, just the process by which all progress in theoretical physics has been made.

Compare Maxwell's remarks on the standing and value of philosophical questions in science. Maxwell (1870):

> [W]e are met as cultivators of mathematics and physics. In our daily work we are led up to questions the same in kind with those of metaphysics; and we approach them, not trusting to the native penetrating power of our own minds, but trained by a long-continued adjustment of our modes of thought to the facts of external nature.

Maxwell (1875):

> [W]e must bear in mind that the scientific or science-producing value of the efforts made to answer these old standing questions is not to be measured by the prospect they afford us of ultimately obtaining a solution, but by their effect in stimulating men to a thorough investigation of nature. To propose a scientific question presupposes scientific knowledge, and the questions which exercise men's minds in the present state of science may very likely be such that a little more knowledge would shew us that no answer is possible. The scientific value of the question, How do bodies act on one another at a distance? is to be found in the stimulus it has given to investigations into the properties of the intervening medium.

References

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