Lecture: Struc/Form Theories: Halvorson

"What Theories Could Not Be"

22 Dec 2016

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- Identity of theories based on equivalence classes
  - why important: [stat. structural realism]
    - ontology: diff't formns may suggest radically diff't — essential for struct realism
    - epistemology: diff't formns may lead to diff't warrant for diff't props
    - methodology: how phy'ts know they're collaborating, not competing
  - why equivalence
    - not always clear what is 'same' (Matrix vs Schrödinger)
  - consequence: sem view really says "theory is class of models up to equivalence rel'n – equiv class of models" [define equivalence rel'n, equiv classes]
  - — pick up here

Def'ns (context: 1st order langs, classical logic)

"F an interpretation of T in T'": $F : L(T) \to L(T')$ [" $L(T)$ " langs w/ struc s.t. it is a model of T]

takes vars to vars, preds to wffs. Then F can be extended to map terms of $L(T)$ to terms of $L(T')$ and formulas to formulas. F is interp of T in T' iff for each axiom $\varphi \vdash \psi$, $F(\varphi) \vdash F(\psi)$ is thm of T'.

n.b.: interps induce naturally model maps $F^*: M(T') \to M(T)$ ('contravariant functor')

n.b.: "preds mapped to wffs" ⇒ def'ns

"T and T' def'y equiv": F, G interps; G is weak inverse of F iff each wff φ of $L(T)$, $GF(\varphi)$ T-provably equiv to φ, vice-versa. T' and T are def'y equiv iff ∃ weakly invertible interp F

Equinumerosity is criterion equiv: $L(T)$ has countably ∞ 0-place preds $\{p_i\}$. T empty theory (consequences are tautologies), $L(T')$ has new propositional constant (0-place predicate) q and T' axioms $\{q \vdash p_i : i \in \mathbb{N}\}$

Then $M(T)$ and $M^q(T')$ equinumerous, but T and T' not def'y equiv

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Model-wise isomorph as standard of equiv

$L(T)$ w/ countably ∞ 1-place preds $P_1, \ldots P_n, \ldots$, T single axiom $\exists x (x=x)$

$L(T')$ 1-place preds $\{Q_i\}_{i \in \mathbb{N}}$, T' axioms $\exists x (x=x)$ and $Q_0 x \vdash_x Q_i x \ \forall i \in \mathbb{N}$

$\Rightarrow$ same # of models

every model is isomorphic to one of other and vice versa.

(a model is domain w/ single object w/ countable ∞ ~~...~~ monadic properties)

but ~~...~~ "intuitively" not equivalent, since T' has "more info" (rel'ns among preds)

$\Rightarrow$ make precise: not def'y equiv

identity of models as standard of equiv - clearly too strong

- distinguishes obv manifestly equiv theos just because their models are formulated in diff't languages (and so have diff't L-struc) : automorphs of groups

modelwise : also too strict, distinguishing manifestly equiv theos

~~...~~ $\Rightarrow$ def'n equiv does not entail that model-maps yield isomorphic models

$\Rightarrow$ "interpretive corollary" : using def'l equiv as standard of equiv, "theor'l equiv" is global : does not necessarily induce ~~...~~ model-wise isomorphs

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Consequences

1) showing models not isomorphic not suff't for theory inequiv
2) trouble for structuralists (structural realists)

[~~discuss/state structural realism~~]

Problem w/ Halvorson's arg?

- why we don't require to ground his args if semantic view theorist wants to do away w/ langs as fundamental?
  → because we must use languages in practice — and we use them to define and pick out 'family of models' in the first place; otherwise, we have no way of knowing what set-struc to count as a viable 'family of models constituting a theory' in the 1st place

Upshot

A theory cannot be just "a collection of models"
⇒ more formalization/maths is needed?
(e.g., relations among models)