Ratios of Spatial Lengths Don't Fix Absolute Lengths

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Fix 4 (simultaneous) spatial points, p, q, r, s. (We will see in the lecture next Wednesday that the restriction to distances among simultaneous points is essential; for now, just take it as required by fiat.) Write the straight line segment connecting p and q as '$\overline{pq}$', and similarly for r and s. Write the ratio of the distance $\overline{pq}$ to the distance $\overline{rs}$ as $R^{p,q}_{r,s}$. Then the question that was asked can be formulated as follows.

> If we know for all quadruplets of simultaneous spatial points $(p,\ q,\ r,\ s)$ the ratio $R^{p,q}_{r,s}$, why does this not fix the absolute distances between the points?

Let us write '$d(p, q)$' for a potential absolute distance function for any two spatial points p, q. In other words, d is a rule that assigns a real number to every pair of simultaneous spatial points at a given instant of time t, the assigned real number being the absolute distance between those two points. Then the ratio $R^{p,q}_{r,s}$ can be written as $\dfrac{d(p, q)}{d(r, s)}$. Clearly, this ratio will not change if we multiply the real numbers $d(p, q)$ and $d(r, s)$ by the same constant k_t: $\dfrac{k_t d(p, q)}{k_t d(r, s)} = R^{p,q}_{r,s}$. Thus, the new rule "take the assignment of distances given by d and multiply every assignment by the same constant k_t"—which we will represent as '$k_t d$'—gives the same ratios of lengths. (Here, I must stop to apologize to the student who sat in the first row yesterday and tried to point out to me that multiplying the distance function by a constant amounts to nothing more than choosing a unit of measurement. He was right, and my response to him was incoherent and incorrect.)

Now, consider knowing the same thing at a different instant of time t'. Again, $R^{p,q}_{r,s}$ will not change if we multiply the distance function by a constant $k_{t'}$. But now—and this is the crucial point, which I was too muddle-headed to recall and explain clearly yesterday—nothing dictates that we use the same constant at time t' as we did at time t. It is not necessary that $k_t = k_{t'}$, for *any* times t, t'. The ratio $R^{p,q}_{r,s}$ will remain the same whether the constants are the same at different times or not. Since, however, we stipulated that it is only the ratio that we know, it follows that there is no way to say whether the distance between p and q is "really" $d(p, q)$, or $k_t d(p, q)$, or $k_{t'} d(p, q)$, or $d(p, q)$ multiplied by any other strictly positive number.

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