Einstein for Everyone, Problem Set 01

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1. Suppose you are in a closed, windowless cabin on board the USS Enterprise, traveling at Warp 0.9 (73% the speed of light) velocity with respect to the Earth. Your cabin comes equipped with the most sophisticated available tools to measure distances and times, such as an atomic clock, an interferometer much more precise than Michelson and Morley's, and so on. You also have a meter stick and a watch. An earthbound observer (if they could see inside the cabin) would see that your meter stick is shorter than one meter, and that your watch runs slow. Will you be able to measure these effects? Why or why not?

2. According to Einstein, do observers in relative motion (such as Al and Bob in the examples in Janssen 2013) agree on the time order of all events? Explain your answer (briefly, in a few sentences).

3. Draw a spacetime diagram including the following elements, all of which should be clearly labeled:

   a. An event p.

   b. The lightcone (including past and future "lobes") from the event p.

   c. The wordline for an observer O passing through p.

   d. The hyperplane of events that will be simultaneous with p according to the observer O.

   e. An event q that is spacelike separated from p.

   f. An event r that is timelike separated from p.

4. Draw, in a second spacetime diagram, two inertial observers, O and O', who are moving with respect to each other. Observer O has a measuring rod. (You should draw a diagram with only one spatial dimension in order to make things easier.) Draw the worldlines for the end-points of O's measuring rod. On the diagram, mark the events p and q, corresponding to

"marking the two endpoints of the rod, at the same time according to O". Mark an event r that corresponding to "marking the other endpoint of the rod at the same time as p, according to O'". Are q and r the same point, or not? Briefly comment on how this relates to length contraction.

5. Suppose Al is at rest standing in between two sets of railroad tracks. Bob goes by in one direction at a speed u, as measured by Al, and Carl goes by on the other tracks going in the opposite direction at the same speed. Al, Bob, and Carl all have meter sticks and whatever other instruments needed to compare their length.

   a. Suppose that $u = 1/1000\ c$ (one thousandth of the speed of light). Approximately how fast is Bob going according to Carl?

   b. Suppose that $u = 9/10\ c$. What does Al say about Bob's and Carl's meter sticks? What does Bob say regarding Al's and Carl's meter sticks? How are these two different accounts compatible?

6. Suppose Einstein's theory of relativity is true. Now consider what happens if light moves with infinite speed rather than the speed c. In particular, explain what would then be true regarding the relativistic effects we have considered in class – length contraction, time dilation, and the relativity of simultaneity.

7. Alice is riding in a sled in inertial motion. She (naturally) considers herself to be at rest, and she measures the sled to be exactly 3 meters in length. The sled is on a frozen lake, which she measures to be moving past her at $0.9c$. She is approaching a hole in the frozen lake. Bob, who is moving inertially as well, stands next to the hole. He measures the size of the hole to be exactly 3 meters (in the direction Alice's sled is approaching it). According to Alice, the hole (in her line of travel) is less than 3 meters wide. According to Bob, Alice's sled is less than 3 meters in length. What happens when Alice in her sled crosses the hole? Does the sled fall through the hole or not?

References

Janssen, M. (2013). Appendix on special relativity. In M. Janssen (Ed.), *Cambridge Companion to Einstein* (Second ed.)., Chapter [***], pp. [***]. Cambridge: Cambridge University Press. Forthcoming.