

① S, T, ST: Riemann, Helmholtz, Poincaré - Structure of  
Lecture

16 Dec 2015  
①

① Riemann's mathematical achievements

- a) topology, measure/magnitude concepts
- b) metric - "Pythagorean" form postulated as simplest
- c) ...

② Helmholtz's analysis

- a) mathematical results
- b) physical analysis of possibility of geometry, necessary relation to possibility of 'free mobility of rigid bodies'

③ Poincaré's Conventionalism

a) you not synth a priori, not explicit facts, only possibility left as ~~con~~ conventions [the masses Riemann's insight - they are not purely experimental, but we determined confirmed by the structure of physical theory in which they participate as substructure, in resp. all physical phenomena that depend to any degree on spatial relations for their description]

→ it is not just description of motion of rigid bodies, but let physical interaction - and that it is at just saying int. that we have basis of what to make our choice of geom. (correlative to 1st point about all geom. characterized all phys. interaction)

④ Conclude w/ Riemann's methodological and metaphysical

sophistication:

- 1) no need for  $\exists$  of perfect free mobility of perfectly rigid bodies - only approx, and that gives us only geometry for sake of those bodies
- 2) in realm of 'immeasurably small', we must

contra Helmholtz

- i) be open-minded about possible sources of information about spatial struc at diff't levels
- ii) our spatial notions, in so far as they are brought to bear in physics, have significance only as structural aspects of a more embracing structure: that of the possibility of physical interaction itself

contra Poincaré

Q 5, T, ST: Riemann, Helm, Poincaré - Structure of Lect

16 Dec  
2015  
②

3) if spatial manifold is continuous, then material relations must be found outside the bare notion of space itself, in binding forces that act upon it

⇓

physical bodies as equilibrium configurations, means we must be ready to engage in analysis of what 'rigidity of body' means, wrt spatial relations, and not just analyze space upon unarticulated, primitive notion of rigidity -

Riemann

- <sup>study of metric relativity</sup> begins w/ assumption of quadratic form (explicitly notes it is simplest possible assumption)
- but really begins w/ characterization of non-metric structure - topology, diff'x space
- need to distinguish this from metric space (concepts like 'unbounded' versus 'infinite'; # of dims)
- deriving from this condition needed for free mobility

Helmholtz

- phil analysis of 'conceive/imagine' - profound advance from Kant for judgment of nature & geometrical of spatial concepts of judgment
- free mobility assumed as pre-condition of consistency of measurement (rigidity of bodies, distant congruence)
- shows form of Riemann metric follows from this (led to hyperbolic spaces of constant curvature)

Starting Point: all our knowledge of space comes from observation of the properties of (approx) rigid bodies

[Compare Poincaré's analysis, his observation that in a world w/ only fluids,  $\rightarrow$  geometry]



general properties of space deducible from conditions necessarily satisfied by bodies to be 'rigid'  
(in motion)

$\rightarrow$  conditions expressed in terms of the motions themselves, viz., mappings of space to itself that take rigid figures to congruent figures



Riemann's metric form, postulated by Riemann as simply the 'simplest form' is derived now by Helmholtz

and much more strongly derived w/ severe restrictions, that the spaces be constant curvature



error: read from Howard, "Phi-1 Postscript", p. 22

1) goes out 'experimental' sci for Poincaré

2) def'n of convention: 'stipulated def'n'

- w/ clear, unambiguous application as ground of a form of reasoning

- not a priori fact

- not result of experimental reasoning

⇒ only possibility left: convention

} we must have possibility that method you dynamically affect each other: GR

3) convention is chosen - a projective act, "free"

- guided by, not determined by, experiment, experience

- no "true" geom (metaphys? epist?), only

"more convenient, simpler": saving notes in p. 71 of "space of Geom."

4) geom vs perceptual space

- diff perceptual spaces ⇒ no a priori geom space of objects, but synthesis of diff perceptual spaces

- no rep of geom space

- diff from rep of objects in perceptual space vs reasoning about obs (constructions, c.), in geom space

5) genesis of geom space

- change of state vs change of position

⇒ assumption of rigid bodies, necessity of possibility of our mobility, not just that of studied obj's

⇒ isolated sensations cannot lead us to <sup>concept</sup> idea of geom space, but only study of laws by which sensations succeed each other

6) return of geom space

- complete reversal from Helmholtz:

possibility of displacement is no longer a  
precondition for (certain kinds of geom), though  
they are used in the development of the  
concept of geom space

⇒ when developed, geom becomes <sup>study of</sup> the laws  
of displacement of class of bodies  
↑  
rigid

7) choice of geom

- example of "theory in homogeneous space"

properties to show that one can always make trade-off  
between dists and geom, to choose whatever  
geom one likes