Philosophy of Space, Time, and Spacetime
Lecture: Introduction to Special Relativity

Dr. Erik Curiel

Munich Center For Mathematical Philosophy
Ludwig-Maximilians-Universität

Chasing A Beam of Light

If I pursue a beam of light with the velocity c (velocity of light in a vacuum), I should observe such a beam of light as an electromagnetic field at rest though spatially oscillating.

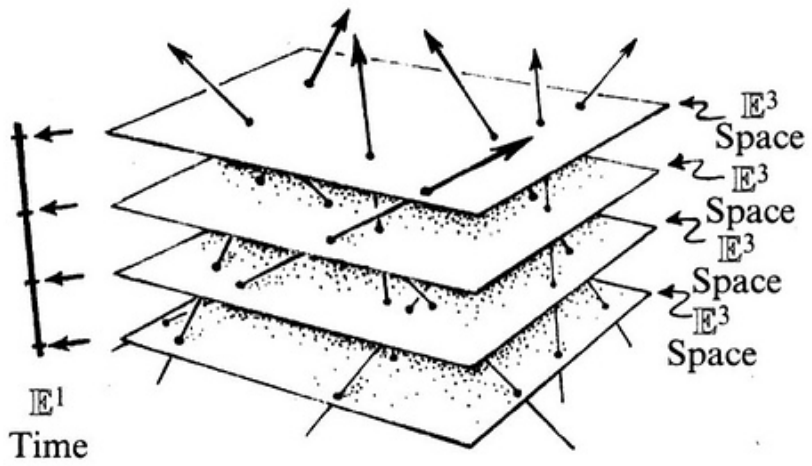
There seems to be no such thing, however, neither on the basis of experience nor according to Maxwell's equations.

From the very beginning it appeared to me intuitively clear that, judged from the standpoint of such an observer, everything would have to happen according to the same laws as for an observer who, relative to the earth, was at rest. For how should the first observer know or be able to determine, that he is in a state of fast uniform motion? (Einstein 1946)

Einstein's Breakthrough: Revising Time

But a friend of mine living in living in Bern (Switzerland) [Michele Besso] helped me by chance. One beautiful day, I visited him and said to him: 'I presently have a problem that I have been totally unable to solve. Today I have brought this "struggle" with me.' We then had extensive discussions, and suddenly I realized the solution. The very next day, I visited him again and immediately said to him: 'Thanks to you, I have completely solved my problem.' My solution actually concerned the concept of time. Namely, time cannot be absolutely defined by itself, and there is an unbreakable connection between time and signal velocity.
Using this idea, I could now resolve the great difficulty that I previously felt. After I had this inspiration, it took only five weeks to complete what is now known as the special theory of relativity. (Einstein, 1922 lecture)

Absolute Time (Classical, Newtonian Picture)



From Roger Penrose, *The Road to Reality*

Absolute Time

- Spacetime composed of "events"
- Global: well-defined time interval between *any* two events
- Absolute: time interval *does not depend* on state of motion

Question

- How to establish simultaneity of distant events?
  - Synchronized Clocks, . . . but *how are they synchronized?. . .* ". . . there is an unbreakable connection between time and signal velocity."
- Signal Speed
  - Instantaneous Signal
    Assumed in earlier theories
  - Signal with Finite Speed (e.g. *light*)

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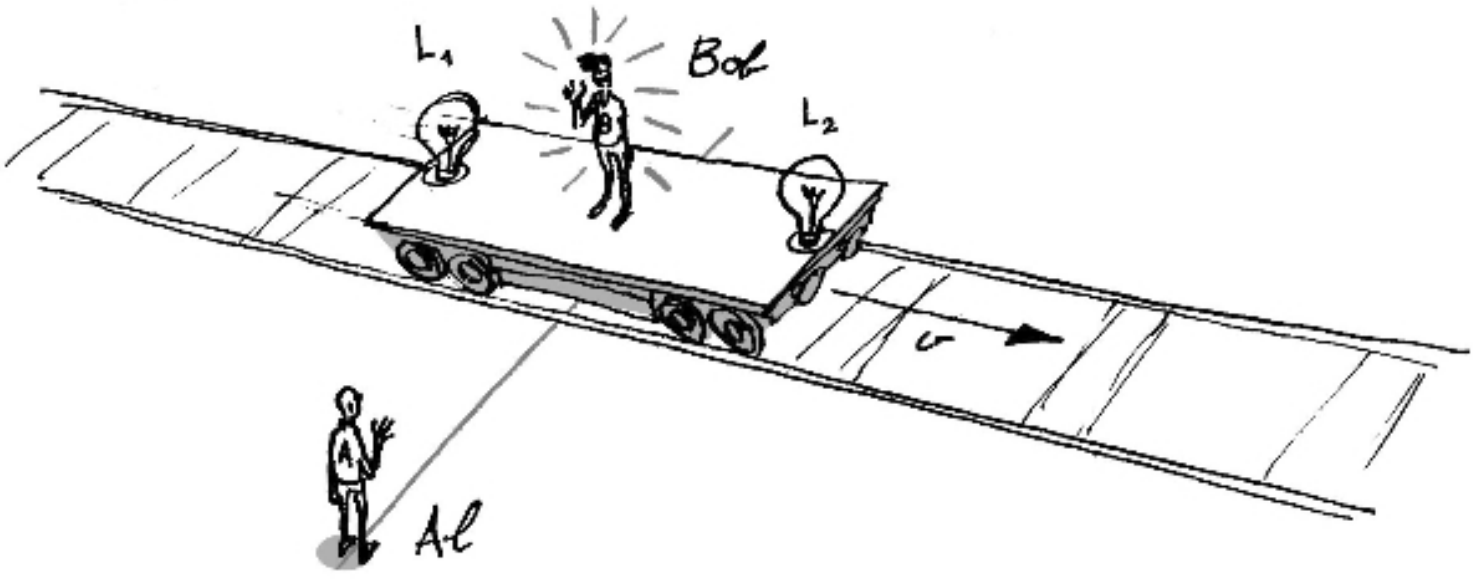
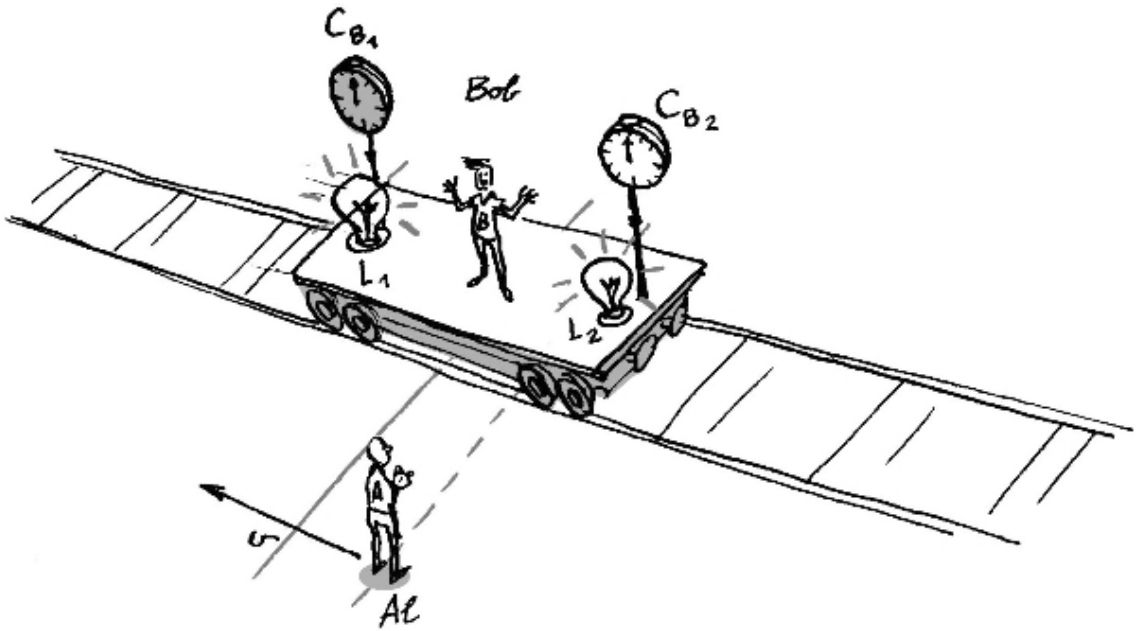


- Setup (see Janssen, "Appendix on SR" §1.2)
  - Bob: in the middle of rail car, lights flash once at each end (L_1 and L_2)
  - Flashes reach Bob at the moment he passes Al
  - Train moving at velocity v past Al

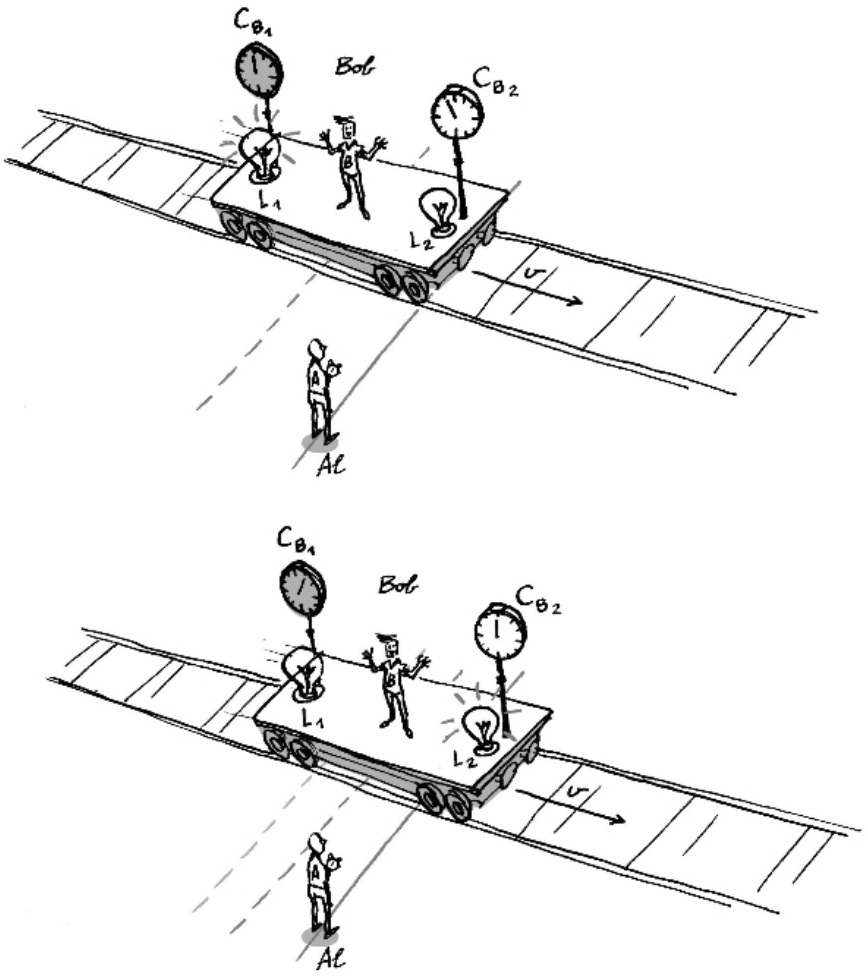
Signaling and Simultaneity 2



Bob's Description

- Light moves at velocity c with respect to me from each light
- Distance traveled by pulse from L_1 = distance from L_2
- Therefore, lights flashed simultaneously

Signaling and Simultaneity 3



Al's Description

- Light moves at velocity c with respect to me from each light
- Bob moves *away* from L_1 and *towards* L_2
- Distance traveled by pulse from $L_1 >$ distance from L_2
- Pulses reach Bob at same time
- Therefore, L_1 occurred *before* L_2

What's Relative?

Bob's Description

- Speed of light $= c$
- Light pulses reach Bob simultaneously
- Distance traveled by pulse from L_1 $=$ Distance traveled by pulse from L_2
- Flash L_1 *simultaneous with* L_2

Al's Description

- Speed of light $= c$
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Relative Quantities

Time elapsed and spatial distance between events *relative* to inertial observers

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Einstein's Insight

Einstein's Two Postulates (1905):

1. *Principle of Relativity*: "...the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good."
2. *Light Postulate*: "... light is always propagated in empty space with a definite velocity independent of the state of motion of the emitting body."

How can these two postulates be reconciled?

Possibilities:

1. Revise electrodynamics
2. Revise concepts of space and time!

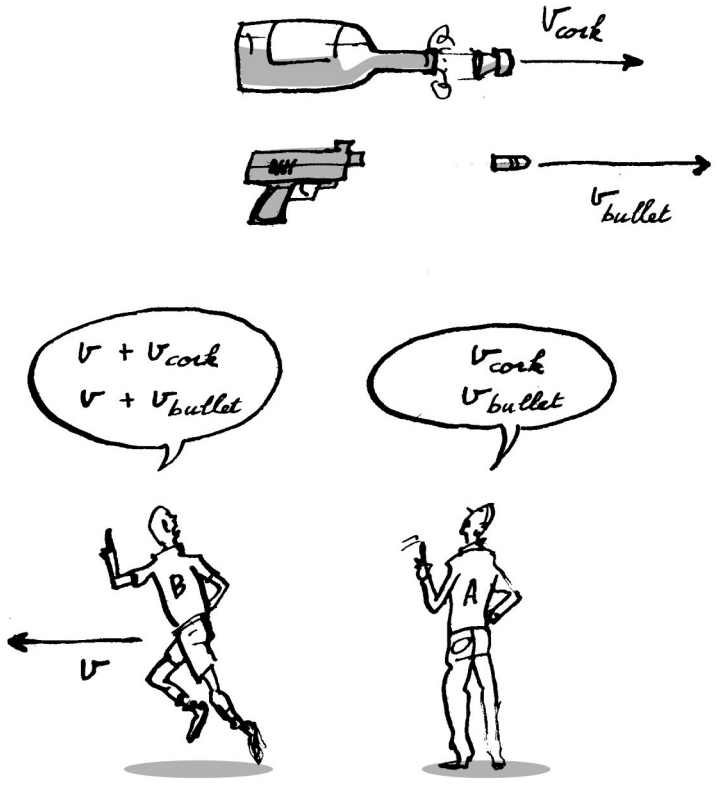
(critical analysis of earlier assumptions regarding space and time)
(agreement about speed of light, *disagreement* about spatial and temporal intervals)

[*** how it follows that speed of light c is an upper bound, which no physical system (observer), other than electromagnetic phenomena, can achieve: if so, an observer would be able to determine his or her absolute velocity (when the electromagnetic field is stationary, a standing wave, she i moving at c, absolutely), violating Galilean relativity, and Einstein's extension of it to cover electromagnetic phenomena; this is how to make the two principles consistent ***]

[*** Question: why is all this not just about meaurements and observations? it has nothing to do *prima facie* with underlying *real* "metaphysical" structure of space and time.

Answer: if we take seriously the regulative idea that lack of conformity to best physical theory can be used to disconfirm metaphysical speculation and conclusions, then special relativity shows us that we should rule out classical conceptions of space and time as live possibilities. If metaphysical views in principle have no empirical, observational or measurable consequences, and they do not conform with our best physical theories, *e.g.*, in the sense that they cannot be used as the ground of a formulation of those theories (in this case, Maxwell theory in conjunction with Galilean relativity extended to cover electromagnetic phenomena), then we should not hold them as live candidates for accurate pictures of real structure of space and time. There is no logical contradiction in holding a metaphysical view of space and time that does not conform with best physics, but that would seem to leave one in the position of having no shared grounds for principled argumentation, agreement and disagreement about those views, and so no possibility for real progress in philosophy. ***]

Velocity Addition



Images by Laurent Taudin, from Janssen (forthcoming)

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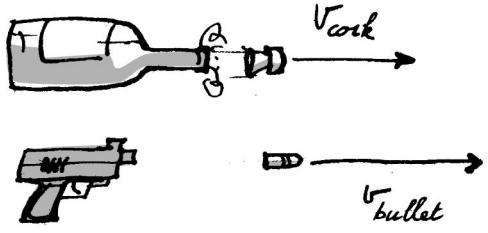

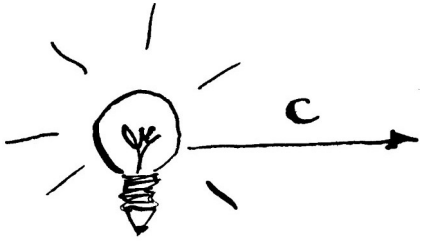

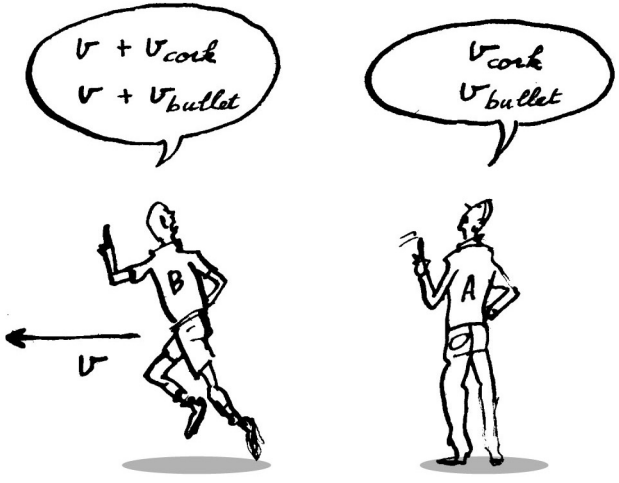

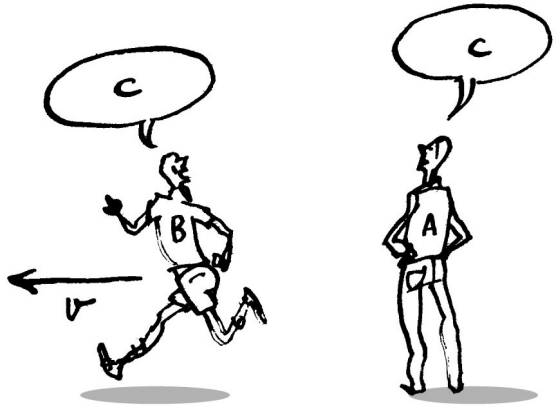

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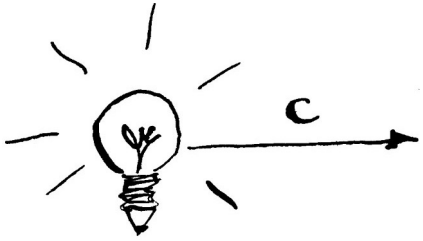
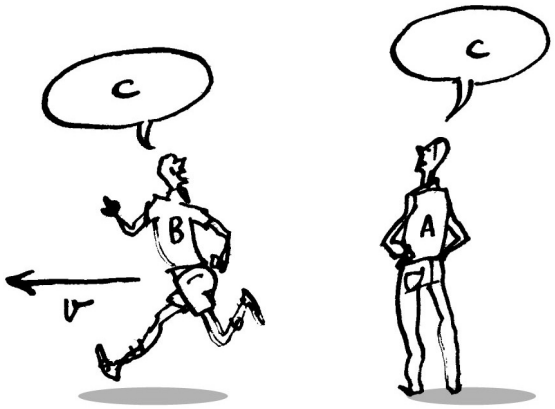
Revising Velocity Addition, Option 1

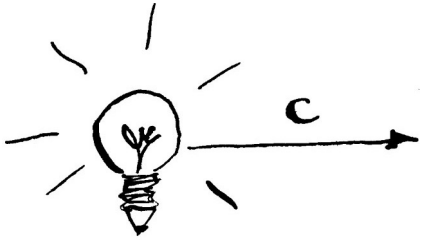
- Postulate 1: Principle of Relativity
- Light moves at c *with respect to emitter*
- . . . modify theory, compatible with null results

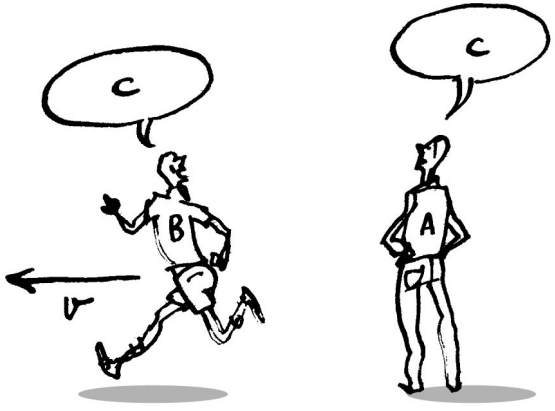




Revising Velocity Addition, Option 2

- Both postulates $\rightarrow$ all inertial observers measure *same* speed of light
- . . . but velocity addition follows from usual ideas about space and time
- . . . so these need to be revised!





Signaling and Simultaneity

- Meaning of time for distant events
  - Newtonian assumption: *instantaneous signal*, global synchronization of clocks and absolute time
  - . . . But in fact use light signals. (1) Finite velocity; (2) Does not obey usual velocity addition rule
- Consequences of using light signals
  - Observers in relative motion *disagree* about spatial distances, temporal intervals between two events

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Contrast

Newtonian View

- Moving Rods: same length
- Moving Clocks: tick at same rate
- Velocities: simple addition $v'_b = v_b + v$

Relativistic View

- Moving Rods: contract
- Moving Clocks: tick more slowly
- Velocities: addition with "correction factor," such that $v'_b < c$

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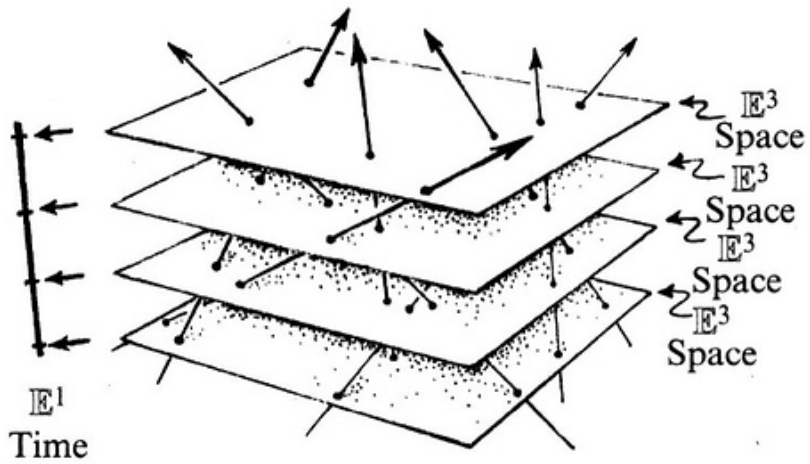
Measuring Absolute Time



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Measuring Time

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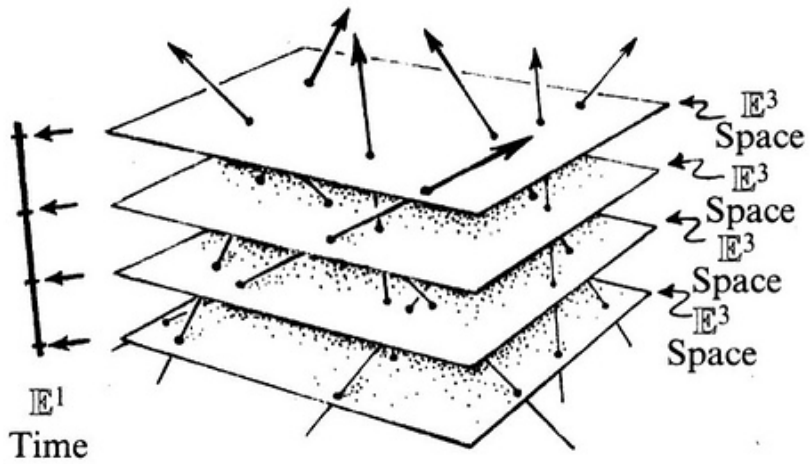

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Einstein's Analysis

Newtonian View

- Based on implausible assumptions: *infinite* signal velocity, global time
- Incompatible with theory of electromagnetism

Relativistic View

- Theory of electromagnetism: light postulate
- Revise space and time concepts to insure compatibility with Relativity Principle
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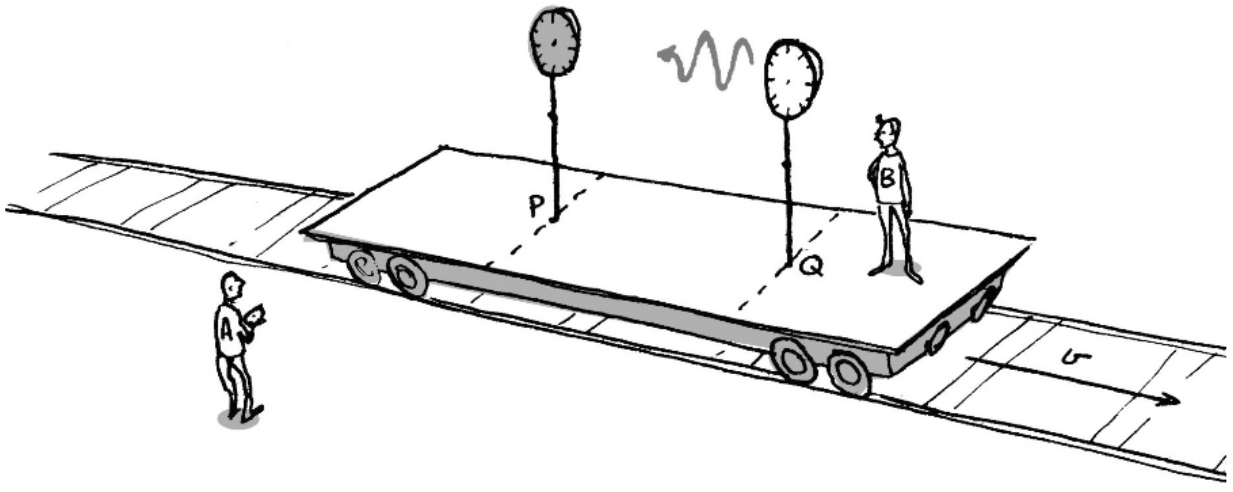
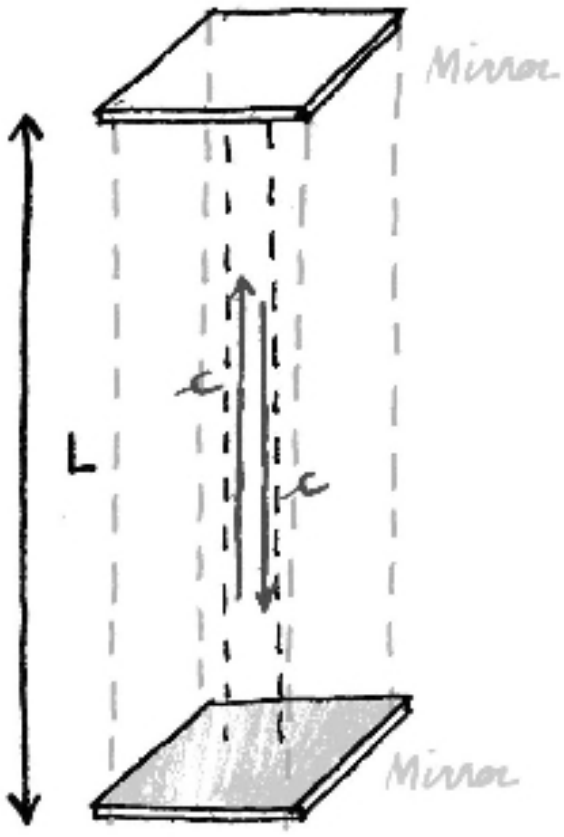
Illustration: Al and Bob



- Distance from P to $Q = d$
- Send a light signal from Q to P at t_Q
- Clock at P synchronized if it reads $t_Q + d/c$
- . . . but Al will not agree

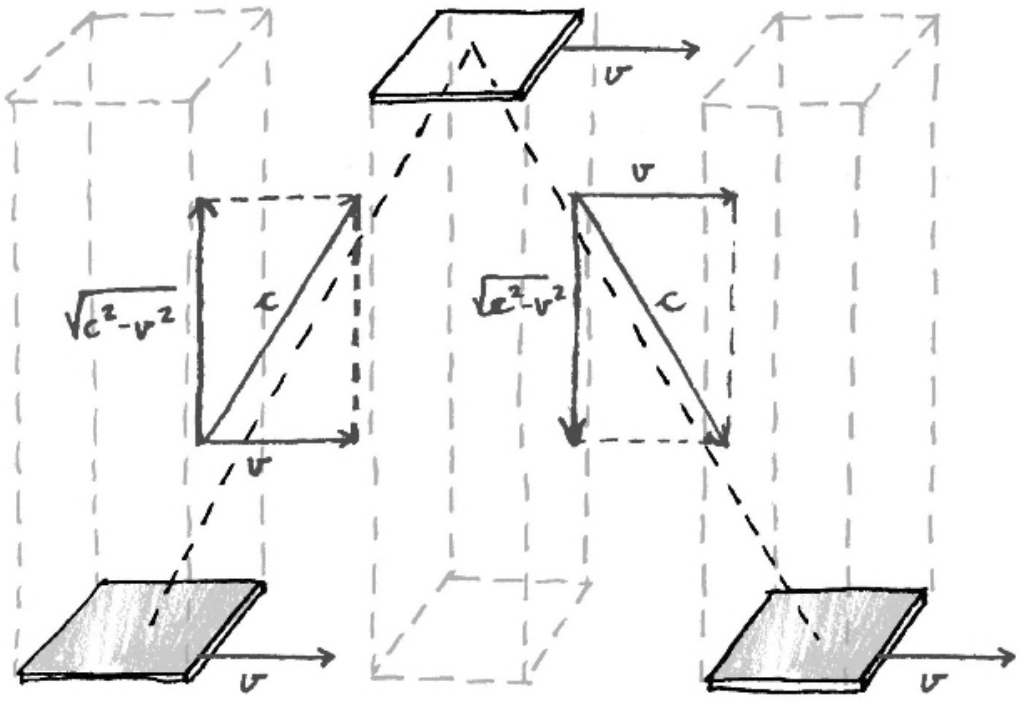
Illustration: Light Clock



Simple clock:

- Two mirrors, spaced a distance L apart
- "Tick": light beam reflecting off base mirror
- Ticks $\frac{2L}{c}$ times per second

Moving Light Clock



- *Vertical* velocity is much lower (by factor $\sqrt{1-v^2/c^2}$)
- Result: round trip takes *longer*, clock ticks *slower* ($\frac{2L}{c}\sqrt{1-v^2/c^2}$ per second)

Time Dilation

- Direct consequence of Einstein's postulates: *moving clock runs more slowly*
- Analysis for simple case (light clock), but also applies to other clocks
- . . . as a consequence of the principle of relativity (Exercise: why?)

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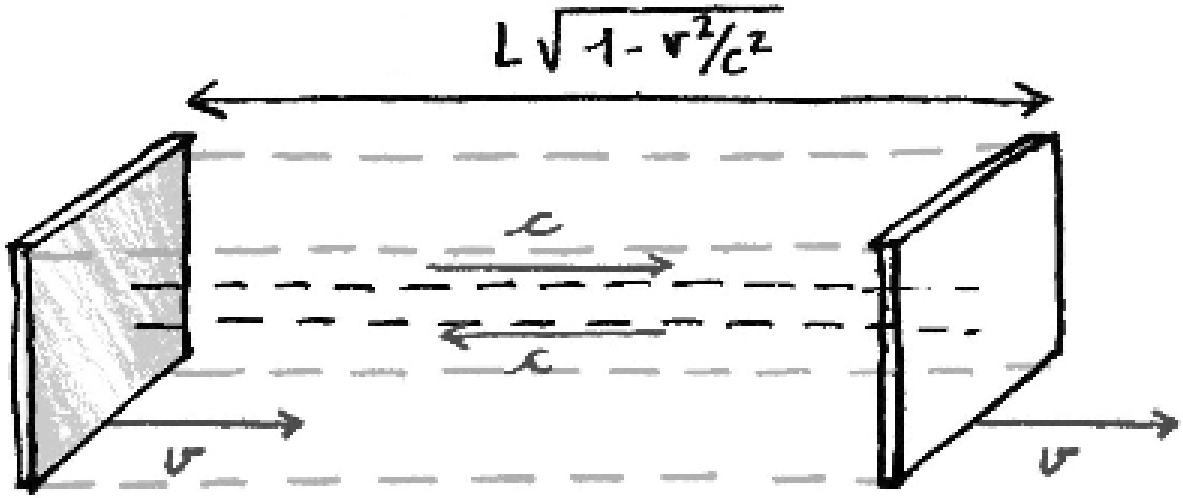
Length Contraction



- Length for moving light clock: $\left(\sqrt{1-v^2/c^2}\right) L$ (shorter, $0 < \sqrt{1-v^2/c^2} < 1$)
- Derive by requiring equal travel times for two perpendicular light clocks (see Janssen (2013) for the details)

Common Mistake

- "Appearance Simultaneity"
  - Corrections for different signal speeds (sound vs. light)
  - Signals arriving simultaneously from events at different distances
- Contrast: relativity of simultaneity holds after these have been taken into account...

Relativity of Simultaneity

- Einstein's Definition of Time
  - Compatible with his basic postulates
  - Acknowledges importance of light signals in synchronizing distant clocks
- Time elapsed between two events *relative to observer*
  - Common sense, pre-1905 physics $\rightarrow$ approximation, reflects experience with low velocities
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Relativistic Effects: FAQs

Question 1

Do length contraction and time dilation only effect *special* clocks and measuring devices?

Question 2

Are length contraction and time dilation *real* effects? (Does Lorentz contraction hurt?)

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Perspectival does not imply "imaginary" or "unreal"

- "Relative Quantities", values depend on state of motion of observer who measures them ("perspectival"): space, time, energy, momentum, electric field, magnetic field, ...
- but underlying *absolute quantities* all observers agree on: spatiotemporal interval, energy-momentum, electromagnetic field, ...

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