Metaphysical Problems of Physics
Lecture 4
"Quantities"

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1 *Précis*

I could agree with and endorse much, perhaps most, of Wolff (2020) if the phrase 'non-reductive, restrictive realism' were everywhere replaced by 'non-reductive, restrictive pragmatism à la Peirce, Carnap and Stein', and (concomitantly) quantities were not treated in isolation from the idiosyncrasies of particular theories and from how they, in their pecularity in each theory, relate to each other intra- and inter-theoretically.

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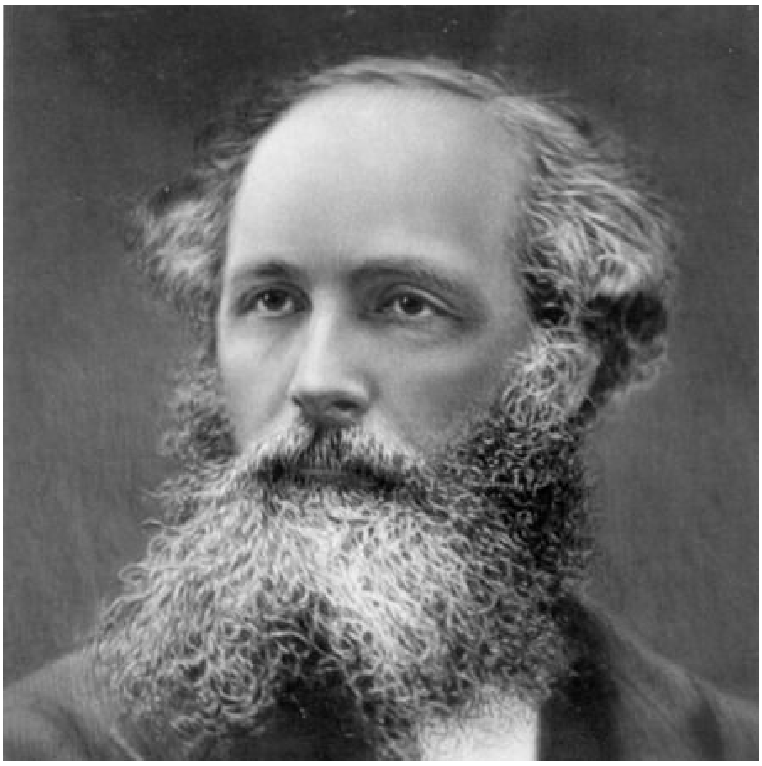
2 Maxwell on the Mathematical Classification of Quantities

You thought you were done with Maxwell! Never! Hahahahahaha...



Maxwell, giving 0 fucks

1. Maxwell (1869, p. 260):

    > The division of the energy into vector factors affords results always capable of satisfactory interpretation. Of the two factors, one is conceived as a tendency towards a certain change, and the other as that change itself.
    >
    > Thus the elementary definition of Work regards it as the product of a force into the distance through which its point of application moves, resolved in the direction of the force. . . .
    >
    > Thus, instead of dividing kinetic energy into the factors "mass" and "square of velocity," the latter of which has no meaning, we may divide it into "momentum" and "velocity". . .

2. Galison (2021), talking about Maxwell (1870), which itself was written immediately after Maxwell (1869):

    > For Maxwell, this congruence, under a common sheltering mathematical structure, was what led to a deeper understanding than any particular physical realization could provide. This class relation, this relation of different realizations, is what I have in mind with the idea of a *lateral coordination* (rather than any form [of] vertical explanation by way of unification or reduction). ¶ Or put another way: Having different realizations (algebraic, geometric, physical) with a mathematical commonality deepens understanding.

    I think this is partly right, but I don't entirely agree (at least, not with the conversational implicature of it). Galison had just quoted Maxwell (1870) on physical quantities that "belong to the same mathematical class." Since Maxwell had written (or, at least published) a paper (or, more precisely, the record of a talk he had given at the London Mathematical Society) the year before (1869) with the title "Remarks on the Mathematical Classification of Physical Quantities", I think we can assume that he had in mind the analysis he gave in that paper when talking in the "Address" of the mathematical class of a physical quantity. Now, the 1869 paper is simply written but deceptively rich, so I can't give a thorough discussion of it, but I think that to understand Maxwell's thought here it is crucial to note that it is not only commonality of mathematical structure that classifies two physical quantities as relevantly the same but, even more importantly, the fact that that structure admits of a *physical* interpretation based on the internal relations definitive of the structure, which interpretation is shared among all the instantiations of quantities of that class. One of the things that makes all the different forms of energy as Maxwell understood it as the time "the same", for example, no matter whether potential or kinetic, or whether thermal or electromagnetic or elastic, or what have you, is that, as a scalar, it can always be factored into the product of two vectors, such that (p. 260) "one is conceived as a tendency towards a certain change, and the other as that change itself."

    I thus think that Maxwell's class relation is based on elements of *both* Galison's idea of lateral coordination *and* some kind of unification. I do agree, however, that the unification is not by way of anything like a "vertical explanation". It is rather a unity of abstract physical significance, as it were.

> In the same vein, I think that the different kinds of realization Galison mentions of quantities that share a relevant mathematical commonality—"algebraic, geometric, physical"—are not exhaustive of the kinds of realization relevant to what Maxwell is getting at. I agree, he certainly does have at least those kinds of realizations in mind, however, as is shown in Maxwell (1869) in his contrast of the "arithmetic" understanding of "position and form" afforded by Descartes' coordinates with the "spatial conception" given by Hamilton's quaternions.

3 Wolff on the Metaphysics of Quantities

In her own words:

> This book has two main aims. On the one hand, the book offers a defence of a novel position in the metaphysics of quantities: substantival structuralism. According to this view, quantitativeness is an irreducible feature of attributes, and quantitative attributes are best understood as relationally structured spaces. On the other hand, the book provides a thorough examination of the metaphysical questions posed by quantities as well as a careful assessment of possible answers to these questions. It thereby provides a resource for metaphysicians and philosophers of science interested in the topic of quantities, and connects quantities to a number of debates in metaphysics and philosophy of science. . . . ¶
>
> My topic is not the status of this or that quantity in a particular scientific theory, for example mass in Newtonian mechanics or temperature in thermodynamics, or even the status of particular quantities across different theories. Instead, I'm interested in the question of what it means for an attribute to be quantitative, and what metaphysical implications a commitment to quantitative attributes has.
>
> Jo Wolff
> *The Metaphysics of Quantities*

Let's review her arguments and views in some detail.

3.1 Architectonic Disputes and Synopsis

ch. 1, §2:

reductionist versus non-reductionist "whether quantitativeness is a distinct, sui generis feature [of attributes], or whether it can be reduced to qualitative features"

operationalism versus realism "Realists. . . hold that the target of measurement operations is attributes and that quantitativeness is a feature of attributes independent of our ability to measure them. [O]perationalism. . . holds that quantitativeness is a feature of concepts, not attributes, and that a concept's quantitativeness or lack thereof is a matter of our ability to construct a suitable measurement procedure."

permissivism versus restrictivism "whether quantitativeness is restricted to a certain class of attributes, or whether any attribute is (potentially) quantitative"

Wolff describes her own position as a *non-reductive, restrictive realism*, which she calls *substantival structuralism.*

1. she argues in favor of non-reductivism and realism in chs. 2–4, primarily by pointing out problems with the alternatives, not so much by spelling out how what exactly the positive views consist of and how they apply in practice, how, that is, we are to understood them as doing the job required of them (indeed, it is not entirely clear to me what Wolff thinks the job of a metaphysics of quantities to be—at a minimum, she thinks it is required to underwrite and explain the epistemic value of measurements, but is that all?)
2. she introduces in ch. 5 the representational theory of measurement (RTM) as the framework within which to construct her metaphysical position
3. in ch. 6, she argues against common interpretations of RTM (it is not necessarily operationalist and permissive), and uses it to articulate and defend her own restrictive criterion for quantitativeness ("quantitative attributes are special because they can be given particularly informative representations")
4. ch. 7 argues for the need of a substantival ontology of quantities
5. ch. 8 argues that standard absolutist and comparativist views of quantity are inadequate, and that a substantival view is superior
6. ch. 9 considers recent debates about hyperintensional metaphysics
7. ch. 10 draws all the strands together to conclude that her view is a form of structuralism

3.2 Clearing the Ground, Planting the Crops (Chs. 2–4)

ch. 2: clearing the ground

1. She considers and rejects a traditional and standard account (Johnson 1921; Prior, no date), according to which quantities are to be conceived on the model of *determinables* and their *determinates.*
2. This gives her the opportunity to propose an ingenious and compelling (albeit logically flawed) answer to the puzzle of "the single-value principle", that a quantity, when it has any value in a given circumstance, has exactly one value—she observes that this is entailed by the the fact that quantities allow one to place objects bearing them in a weak linear ordering, but that is possible if and only if each object that is assigned any value of a quantity is assigned exactly one value. The logical flaw (the 'only if' does not in fact hold—exercise for the reader) does not detract from the value of the insight.

ch. 3: initial ploughing, first seeds sown: against permissivism

1. She analyzes the treatment of Carnap (1966), a species of empirical restrictivism, because (p. 23) "it captures several features of empiricism about quantities: quantitativeness is a feature of concepts, not attributes, only some concepts are quantitative, and concepts are quantitative in virtue of satisfying operational axioms."
2. She ultimately rejects Carnap's position because (p. 27) "it overestimates the extent to which orderings and numerical relations can be based on purely observational axioms. . . . [P]erceptual judgements alone are not sufficient to justify the claim that an attribute satisfies the stringent axioms governing quantities." (This is unjust for several reasons, but now is not the time to go into it.)
3. She considers and rejects a silly restrictive realism and a silly permissive empiricism.
4. "Representationalism" is introduced: numbers are tools for representing measurements. The fundamental question of a theory of measurement is then: is a numerical representation of a type of measurement possible, and, if so, "how unique" is it?
5. She claims (to be argued for in ch. 6), *contra* orthodoxy, that representationalism is not the sole tool of the empiricist or operationalist, and does not require dropping any distinction between qualities and quantities.
6. Nonetheless, numerical representability is neither necessary nor sufficient for quantitativeness.
7. But then why draw a distinction at all between qualitative and quantitative? Because quantities admit numerical representation of certain kinds, *viz.*, they are highly informative. And the realist claims that this holds in virtue of the character of the attribute itself the representation refers to, not that of any operationalization of it.

ch. 4: last acre ploughed, final seeds sown: against operationalism, for (a mild) realism

1. Of several different possible features one may think characteristic of or necessary for realism about quantities, she holds that, "for a metaphysics of quantities," one needs only this (pp. 41–42, her emphases):

   > A metaphysics of quantities presupposes that quantities are special kinds of attributes that are quantitative independently of our measurement procedures and conventions. Otherwise a metaphysics of quantities would seem to lack a subject matter and should instead be replaced by an epistemology and semantics of measurement. More specifically, what is at stake is whether *quantitativeness* is a feature of attributes, which we might discover through the construction of measurement procedures, but which is not brought about by measurement procedures. As we shall see, operationalists deny this, and replace the question of the quantitativeness of particular attributes with the question of whether certain *concepts are measurable*, where measurability is a feature a concept only acquires through

> the construction of a measurement procedure. The question of the independence of quantitativeness from measurement procedures is often intermingled with the question of the independence of the *attribute* from the measurement procedure. It is vital for the project of providing a metaphysics of quantities that measurability of concepts depends at least in part on whether the attribute to which the concept purports to refer is quantitative.

All other standard claims with regard to realism about quantities (*e.g.*, "individual quantities are actually related by numerical ratios") are "particular positions within the metaphysics of quantities."

2. Problem: it is still not clear to me what a "metaphysics of quantities" is supposed to be.

3. She considers standard operationalist, conventionalist and coherentist attacks on this reduced form of realism, all of which boil down to the idea that we cannot find out *empirically* whether an attribute is quantitative. . .

4. and rejects them: realism explains why some attributes (*e.g.*, temperature) admit scales of measurement and others (*e.g.*, hardness) do not; other positions don't explain this (p. 61):

> The key to defending the claim that quantities are independent of measurement procedures is to distinguish quantitativeness from measurability. An attribute can be quantitative without its concept being measurable; conversely we may believe a concept to be measurable without there being a quantitative attribute to which it refers. The realist idea is that for measurement to be successful, two things have to come together: we need to be able to construct a measurement procedure that yields empirical data that can be interpreted as satisfying the axioms for numerical representability and we need to have a concept whose (purported) referent has the right sort of structure.

3.3 The Positive View

The Representational Theory of Measurement (RTM), Chs. 5–6

4 Questions

1. what is a formal framework?
   a. at a minimum, it should include, at least implicitly, the fixing of a logic
   b. a logic brings along in its turn restrictions on the possibility of an ontology, an epistemology, a philosophy of language, a philosophy of science, *et al.* (think of how early analytic philosophy developed novel, even revolutionary, philosophical positions in the early 20th Century after the development of formal predicate and quantificational logic at the hands of Peirce and Frege, such as Russell's "On Denoting")

2. how to distinguish quantities with isomorphic mathematical structures as determined by use in theoretical and experimental practice, but which are such that the same operations have different physical significances—both as used for the same quantity, and as used for the different quantities?

   a. think of addition as used for spatial position in Newtonian mechanics and as used for linear momentum in Newtonian mechanics

   b. relationalism is too coarse a tool to say what the relevant operations are

   c. this cannot be relegated to the domains of the special sciences, because of the special sciences do not, in general, provide univocal, unambiguous answers

   d. and there is no way to choose a single "natural, canonical" structure for any given attribute

3. put aside quantum theory for the moment—what about relativity theory? the account cannot handle the spacetime metric. How is this supposed to be realist if it cannot deal with *either* of our two fundamental theories?

5 My Assessment of Wolff's Views

I think Wolff does as good a job as can be done approaching questions about the metaphysical nature of physical quantities in a formal way without attending to the Sturm and Drang of the idiosyncracies of particular theories and the pecularities of the quantities one finds therein, her discussion rich with interesting insights and ideas.[1] Nonetheless, as I shall argue, it is not good enough, and that by the very nature of the approach itself—without concern for the idiosyncrasies and peculiarities, how can one even know whether what one says is meaningful, much more relevant? In particular, the discussion of "energy" above in §2 of these notes and Curiel (2021) suggests that it is too quick to attempt a "metaphysics of quantities" without first accounting for, not only the different operationalizations of a quantity, but also the dirempt variegation one finds in theory of "the same" quantity, the different structures it can manifest, and how physical interpretation of those structures is required to substantiate the idea that they are all "the same quantity". Indeed, more generally speaking, I claim that physical interpretation of a very rich kind indeed is required for the formal structures Wolff invokes and relies on to be able to do the work she requires of them.

Thus, I find Wolff's attempts at minimal commitment unsuccessful, *e.g.* (p. 63):

> The metaphysics of quantities is concerned not with the question of the ontological commitments of a particular theoretical concept, like temperature, but with the on-

1. I like her description of her research profile, on her university webpage (`https://www.ed.ac.uk/profile/jo-wolff`):

> I specialise in the philosophy of science and in metaphysics. In addition to particular topics in either area, I'm interested in their overlap (if any), and the boundary between them.

The "(if any)" is marvelous.

> tological commitments required for quantitativeness [which depend on the “structure” inherent in the relevant attribute].

Nothing in the role energy plays in physics suggests there is in any straightforward way a single entity in the world promiscuously playing the role of referent to all that theoretical talk. The conservation principle, however, suggests that talk is all related in *some* intimate way—just not one readily capturable by a flat-footed notion of a “real quantity”. “Quantities do not pose particular problems for the scientific realist in virtue of being theory dependent” (p. 64)—I think they do. In particular, showing that the “formal structure” of a property (or: a concept representing a property) satisfies a particular set of representational axioms cannot suffice for showing anything of physical significance about the property until one know which bits of the formal structure have physical significance and how, and which do not and why.

Spatial position in Newtonian mechanics gives a particularly simple example of the issues here. It is represented by elements of a 3-dimensional vector space, structurally identical to that used to represent linear momentum. Nonetheless the same “parts” of the structure have radically different meaning in the two cases. It makes physical sense to add two vectors representing the respective linear momenta of two particles as itself a representation of a physical interaction (collision, *e.g.*). It makes no physical sense to add two vector representing spatial positions as itself a representation of anything with intrinsic physical significance. (It does make sense to add them as part of the calculation of a factitious quantity such as “center of mass” of a system.) Linearity in the sense of supporting an additive operation, however, is one of the crucial structures often used to argue that a property is quantitative. Indeed, spatial *length* can be represented by the *self-same* vector space as is used for spatial position, and in that representational role addition is physically meaningful, *viz.*, the concatenation of two lengths. Thus: structure in isolation from physical interpretation cannot determine quantitativeness.

6 Invitation to a Short Essay

I invite you to write me a short discussion (no more than 2 pages, *i.e.*, no more than 1000 words) on any issue discussed in any of this week’s three readings. You can raise further questions, propose answers or interpretations, or whatever seems of most interest to you. If you get it to me by the start of next lecture (18. May), then I will return it to you with my comments the following week.

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