Metaphysical Problems of Physics
Lecture 1 (13. Apr 2021)
Maxwell's *Matter and Motion* I: Kinematics

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(date of notes: 4. May 2021)

Contents

1 Reading History

1. difficulty of getting in the mind-set (words, concepts, principles, presuppositions, cognitive biases, processes of thought, forms of argument, standards of evidential warrant—investigative frameworks) of not only a different time, but a profoundly different way of apprehending and moving through the world, and processing one's interactions with it—think how different the experience of self-consciousness must have been before the practice of reading silently to oneself developed—*cf.* Augustine's profound shock, in *Confessions*, on seeing Ambrose reading silently to himself

2. one must have deep and sustained empathy—*einfühlen*—for the writer, the thinker, and the work

3. getting past what seems obvious to us now: "If others had not been foolish, we should be so." – William Blake, *The Marriage of Heaven and Hell*, plate 9, l. 19

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4. what one will, in the best of cases, gain: the freeing of one's mind—coming to see, for instance, that what one had, perhaps without awareness, held to be conceptual necessities are not in fact so; one may then find after questioning that one does want still to hold on to them, in the best of cases for principled reasons, but one now recognizes *that* one holds them in the face of alternatives

5. Nietzsche's "Versuchen wir's!"—experiment with alternative styles of thought

6. come to understand why we today believe the things we do, do not believe other things, the path we took to get here, which gives us the opportunity to move towards a deeper understanding of our own current epistemic situation

2 Maxwell

What's the go of that? What's the particular go of that?

– James Clerk Maxwell
the start of an inquisition by an incessant small child

The man:

1. born to one of the wealthiest families of Edinburgh, minor aristocracy, he grew up isolated in the countryside and home-schooled until age 10

2. from a very early age, showed almost absurdly intent curiosity about how everything worked, and a keen drive to fix and build and construct with his own hands

3. had his first mathematical paper presented at a meeting of the Royal Society of Edinburgh when he was 14, but he was no great shakes in secondary school, although by all accounts he read on his own voraciously on all subject matters

4. he began at the University of Edinburgh at 16 studying philosophy, and moved to Cambridge at 19 to take the mathematical Tripos

5. by all repute, had an extraordinarily lively imagination (even as geniuses go), a dry and wicked extemporaneous wit, an abundance of charm mingled with social awkwardness, and great generosity of spirit

6. became close to and remained life-long friends with several of the greatest scientists of the age, including William Thomson (*viz.*, Lord Kelvin that was to be), Peter Tait, George Stokes and Arthur Cayley

7. became a Fellow of Trinity College (Cambridge) after the Tripos, took the Chair of Natural Philosophy at Marischal College, Aberdeen, at age 25 (lecturing not only 15 hours a week to the students there, but volunteering to lecture *pro bono* at the local working men's college in the evenings), 1856–1860

8. the Chair of Natural Philosophy at King's College (London) 1860–1865

9. 1865–1871 retired to his country estate, carried on research as a gentleman-scientist

10. 1871–1879, returned to Cambridge as the first Cavendish Professor of Physics, overseeing the building and organization of the Cavendish Laboratory, down to the design of many of its precision instruments

11. "a thorough old Scotch laird in ways and speech" – Donald MacAlister, a Cambridge undergraduate in 1877

12. a first-rate poet, satirical, lyrical and ironic[1]

13. one of the great prose-stylists of the English language

14. had the innate style to carry off a super-fly, unkempt, bushy beard, letting it all hang out—he gave, it is clear, exactly 0 fucks concerning what people thought about it (see picture below)

The scientist:

1. in my own estimation, as well as those of 100 eminent physicists polled in 1999, Maxwell is the 3rd greatest physicist who ever lived

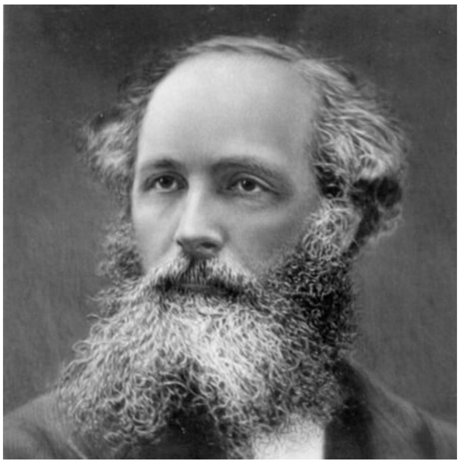
2. made founding contributions to: electromagnetism; electromagnetic theory of light; statistical mechanics; molecular kinetic theory; control theory; dimensional analysis; electrical engineering

3. made fundamental contributions to: thermodynamics; geometrical optics; chaos theory; viscoëlasticity; civil engineering; celestial mechanics; instrument design; neurophysiology of color vision

1. "Rigid Body (sings) – In Memory of Edward Wilson, Who Repented of What Was in His Mind to Write after Section"

Gin a body meet a body
Flyin' through the air,
Gin a body hit a body,
Will it fly? and where?
Ilka impact has its measure,
Ne'er a ane hae I,
Yet a' the lads they measure me,
Or, at least, they try.

Gin a body meet a body
Altogether free,
How they travel afterwards
We do not always see.
Ilka problem has its method
By analytics high;
For me, I ken na ane o' them,
But what the waur am I?



Maxwell, giving 0 fucks

4. the Young-Maxwell-Helmholtz theory is still the basis of the modern trichromatic theory of color vision

5. his first profound contribution to physics, the first solution to the problem of the stability of Saturn's rings (in which he concluded that they must be composed of numerous small particles, rather than being solid or liquid), winning the Adams Prize in 1859, was considered the final word until the Voyager space-crafts confirmed his model by direct inspection in the 1980s

6. he produced the first ever color photograph

7. Einstein credited "Maxwell's influence on the evolution of the idea of physical reality" (primarily, the modern concept of "field") as the most important foundation of his own work

The philosopher:

1. in my estimation, the most philosophically deep and sophisticated physicist who ever lived (or else tied with Newton—it's difficult to judge)

2. certainly the greatest metaphysician and philosopher of science of the 19th Century

3. his development of the concept of a 'field' as a self-subsistent seat of strictly local influence is the metaphysical foundation for our 2 deepest and best current theories of physics, general relativity and quantum field theory

4. his ideas on the role of physical analogy in scientific reasoning and epistemology are still influential and widely studied in philosophy today, and emulated by great scientists

3 *Matter and Motion* Chs. I–II: Kinematics

We treat in this part of the lecture Maxwell (1877), "Preface (1877)" and ch. I–II (§§1–35). Why do we start a course on metaphysical problems of physics by reading a book written as an introduction to a bachelor's course in basic physics in the 19th Century? Because it will teach us not only a bit of physics that will be useful as a touchstone for the testing of the philosophical theses, principles and arguments we consider in the course, but also because it provides the opportunity to see in action how a philosophically sophisticated physicist thinks about the metaphysical foundations and the epistemological requirements of his discipline.

Caveat: Maxwell is far more careful a writer than most of us are readers, and often compresses sophisticated and rich thoughts into concise, seemingly simple remarks; only by reading with care, diligence and reflection can they be uncovered. Do not be gentled into a pleasant stupor by the clarity and elegance of his pithy prose. For this first lecture, therefore, I enter into detailed—some might say neurotically so—exegesis of the opening of the book. We will take a more synoptic approach hereafter (if I can force myself to forgo the pleasures of close reading).

Maxwell's Methodology

> Treat matters as epistemological so far as possible, moving into metaphysics only so far as necessary, and then always grounding it in and constraining it by the knowledge and understanding one has acquired in conformity to the epistemological principles one works with.

Remarks on the methodology:

1. The metaphysics will be developed dialectically, starting from the schematic and general, and proceeding in stages to make parts and aspects more specific, to draw out consequences and to elaborate details in response to advances in our knowledge and understanding; it must never outstrip what we have epistemic warrant for.[2]

2. 'Understanding' here is multi-faceted: it includes the clarification of concepts in response to and grounded in advances in knowledge, and also the capacity to move successfully forward in all aspects of the scientific enterprise.

3. The advancement of knowledge in turn is grounded in, made possible by and responsive to elaborations of the metaphysics and the clarification of concepts—hence the dialectical aspect of the enterprise.

Caveat: do not come with preconceived notions of what metaphysics "must mean"; Maxwell has his own conception of it, and it is one of our tasks as careful and just readers to figure out

2. Maxwell adhered to this methodology in his practice of physics as well (Maxwell 1879, p. 215):

> In cases like that of the planets, when the motions we have to account for can be actually observed, the equations of Maclaurin, which are simply a translation of Newton's laws into the Cartesian system of co-ordinates, are amply sufficient for our purpose. But when we have reason to believe that the phenomena which fall under our observation form but a very small part of what is really going on in the system, the question is not—what phenomena will result from the hypothesis that the system is of a certain specified kind? but—what is the most general specification of a material system consistent with the condition that the motions of those parts of the system which we can observe are what we find them to be?
>
> It is to Lagrange, in the first place, that we owe the method which enables us to answer this question without asserting either more or less than all that can be legitimately deduced from the observed facts.

what that is by how he uses the term and how the concept comes into play implicitly otherwise. An excellent clue is provided by §15, where he explicitly introduces the term, and raises the issue of its relationship with physics.

3.1 Object Lesson for Intellectual Work

You may find it odd that I have written and rewritten these notes several times now, adding and emending parts, elaborating on and correcting previous claims, introducing entirely new ideas. It is because, even though I have read *Matter and Motion* many times, it is such a rich and deep book that I always find something new when I return to it, and when I do return to it and find myself immersed in it, I find it difficult to tear myself away from it, and so a new idea, a new connection will occur to me, and I will go back to read earlier parts of the book again and again until I am satisfied I have a better grasp on how it all fits together. That is how one ought to read, and learn from what one reads, when the work one is reading is worth it.

3.2 *Précis* of Themes

Epistemology §3.3 below:

1. knowledge consists not only of what can be expressed by a statement of determinate fact, but also precise rules governing the general properties and behavior of relevantly related individuals, and loose characterizations of general features of kinds of systems
2. knowledge claims are to be justified in large part by their fruitfulness in allowing the successful continuance of the enterprise of science

Natural Kinds §3.3 below: dictated by the needs of the framework within which systems are to be modeled

Ontology §3.3 below: again, dictated by the needs of the framework within which systems are to be modeled, never outrunning the epistemology

Concepts §3.4 below: views on formation and clarification given by way of exemplification:

1. simple by definition; complex by synthesis
2. criteria of goodness is fruitfulness in application

Space, Time, Motion §3.5 below: space and time are complex concepts, built up slowly from the simpler, more basic ideas of motion and change

Representation §3.6 below: math does not represent the world in any interesting sense; rather the way our concepts represent (latch on to, make substantive contact with) the world, the way they acquire empirical content, is mediate by math

3.3 Chapter I

§1 "Physical Science is that department of knowledge which relates to the order of nature, or, in other words, to the regular succession of events."

1. "relates to": a cautious and useful ambiguity, not restricting it to "prediction" or "control"
2. the "order of nature" ≈ "regular succession of events": metaphysics enters, unavoidably; a choice must be made for an organizational principle to regiment empirical knowledge, and Maxwell posits *ordered behavior* and *structured change* of *events* as the fundamental concepts (rather than, *e.g.*, 'substances' and their 'essential properties', as Aristotelian physics had done, and of more relevance, in contradistion to a metaphysics based on the concept of "matter" and its "properties", as Newton 1726 assumed, as given in the "Definitions" of that work)
3. note that the discussion of "material system" (§2) comes only after the introduction of the basic object of study, the "change in the arrangement of certain bodies", *i.e.*, an *event*
4. what it means, in part, that 'event' is the fundamental concept of the metaphysics: they are to serve as the "individuals", *i.e.*, the entities forming the extensions of 'first-order' concepts; and as such, they serve as the targets of simple and basic knowledge claims
5. not beholden to the constricted empiricism of Mill (much less the primitive one of Locke) nor the abusive positivism of Mach:

   > [W]hen water freezes we know that the molecules or smallest parts of the substance must be arranged differently in ice and in water. We also know that this arrangement in ice must have a certain kind of symmetry, because the ice is in the form of symmetrical crystals, but we have as yet no precise knowledge of the actual arrangement of the molecules in ice.

6. thus: knowledge in physics consists not only of specific predictions and descriptions, and statements of exact laws, but also characterizations of general features and properties of kinds of systems—a richer picture, I'm sorry to say, than many contemporary philosophers seem able to achieve
7. nonetheless:

   > [W]henever we can completely describe the change of arrangement we have a knowledge, perfect so far as it extends, of what has taken place, though we may still have to learn the necessary conditions under which a similar event will always take place.

   we do aim for the statement of exact laws of regularity, which may take the form of necessary and sufficient conditions for the occurrence of a kind of event
8. QUESTION: what is the relevant sense of 'similar' here? how, that is, are events (*NOT*, *n.b.*, physical systems) to be differentiated into 'natural kinds'? by the fact that the description of their instantaneous state and of the pattern of their behavior (eventuation, *i.e.*, transformation into another event) admits of representation in such a way that we can judge whether a given set of necessary and sufficient conditions defining a law of regularity is satisfied?
9. we see the method: epistemology first (a statement of knowledge conditions), followed by the intimation of the need for metaphysics (differentiation of events into kinds, form and character of laws governing their regular succession), which we will not yet essay, for we do not yet know enough to do so with confidence; this clearly has an eye towards the discussion of the "statement of the general maxim of physical science" in §19.

§2 Maxwell does not here give a definition of the general, abstract concept of 'material system', but rather states the need to define the specific kind of system one will study in a given scientific endeavor

1. again, the details of the metaphysics (a fixing of the concept of a general or abstract material system) must await further knowledge and understanding
2. "In all scientific procedure we begin by marking out a certain region or subject as the field of our investigations": note, again, the useful ambiguity of "marking out" and "region or subject"—Maxwell is aware that we do not always have available clear concepts to use for unambigous and precise definitions when we start a scientific investigation, that the clarification of concepts often comes only after knowledge has been acquired (contrary to the stories told by many philosophers; see, *e.g.*, Stein 2004)
3. "To this we must confine our attention, leaving the rest of the universe out of account till we have completed the investigation in which we are engaged.":
   a. seems to be a strong presupposition of the possibility of isolating physical systems from all external effects
   b. but is it?
   c. even if we construe the injunction in this way, we must allow that any external effect that is negligible (with respect to the properties and behavior of the subject under investigation), or that can be accommodated and controlled for when non-negligible and otherwise unavoidable, will pose no problem—Maxwell knew, *e.g.*, that one cannot shield against gravity, but one can control for it
   d. this, indeed, is shown by §3—but is that all there is to this injunction?
   e. I think not: an assumption of causal isolability at this point in the exposition would be a violation of Maxwell's methodology, a *very* strong assumption and detailed about the metaphysics of physical systems unwarranted by any detailed knowledge we can at this point assume to have about them
   f. it seems better, thus, in line with the methodology, to interpret the injunction as epistemological: we confine (as he explicitly says) our *attention* to the demarcated region or subject; we do not necessarily confine (isolate) the region or subject itself as a physical entity
   g. this is substantiated and clarified immediately in §3

§3 "[External relations or actions] we study only so far as they affect the system itself, leaving their effect on external bodies out of consideration."

1. so there is a metaphysical assumption being made implicit in the injunction of §2, and it is a deep and subtle one: it is always possible to make a clean separation between the degrees of freedom of the system under study on the one hand and the degrees of freedom of the 'environment' (*i.e.*, that part of the external world whose effects one must account for in studying the system, that part the system non-negligibly interacts with) on the other
2. this means, in particular, that one can represent any external action as an abstractly characterized force (intensity and direction) without a specification of the kind of system (the particularities of its degrees of freedom and dynamics) that gives rise to the force

3. a mark of the depth and subtlety of this assumption—which Maxwell may not even have been aware of, so deeply ingrained was it, lying so near the bottom bedrock of the conceptual framework of the classical world-view in physics—is that it does not hold in quantum mechanics: one must reject it in order to make sense of entanglement

§§4–14 see §3.6 ("Representation") and §3.4 ("Concepts") below

§§15–17 besides what I discuss here, see also §§3.4–3.5 ("Concepts", "Representation" and "Space, Time, Motion") below

§15 the most explicit expression and use of the Methodology yet:

1. a beautiful epistemic progression leads us to the metaphysical concept of Space; note that the elaboration of the concept of space is grounded on the same simple idea (or operation or idea of an operation) as that on which the concept of "vector" is based, *viz.*, motion (see §3.4 "Concepts" below for further discussion)
2. it is of utmost importance in understanding Maxwell's epistemology, and in turn his metaphysics, to note that the progression does *not* proceed by induction
3. when direct inspection and the reports of others are exhausted, we ascertain *by calculation* that every place has a definite position: an *abduction* supported by a mathematical representation

Thus, we arrive (implicitly) at the idea of Space: every place has a definite position with respect to every other place; and these relative positions are independent of the material content of any systems occupying the places

1. note that this does not constitute a metaphysical account of Space that would satisfy a contemporary analytic metaphysician—it is too skeletal, too schematic, does not tell you what the *essence* of space is, whether space is "really" a substance or is "really" relational in itself, and so no—but we have no epistemic warrant for any of that yet, indeed, not even warrant to assume that those are the correct questions to ask, the correct concepts to attempt to employ
2. in any event, it *does* tell you something about the *modal* properties of space, that are in fact largely in line with Newton's account of "absolute, true and mathematical space" as he gives it in the Scholium to the Definitions in *Principia*, in tandem with some of the characteristics of Newton's idea of "relative, apparent and common space", without endorsing the entirety of Newton's position—Maxwell admits the bare minimum supported by the epistemology

§16 we know his heart is in the right place: he beats up on Descartes; it warms the cockles of my heart to see; but at least as important is the further insight given into his metaphysics: the Laws of Motion will not only inform, shape and underpin the metaphysics, but they will themselves form part of what Maxwell means by the metaphysics of this system of physics

§19 The General Maxim of Physical Science

1. Maxwell is careful not to assume that slippery words such as 'cause' and 'effect', which have meant a multitude of different things to a multitude of different people at different times and

in different contexts, have unambiguous and perspicuous meaning—certainly not enough for the purposes of physical science, which requires appropriate precision and clarity in thought, expression and representation

2. note that, in conformity with the schema of the metaphysics begun in §19, the fundemental unit of physics, its basic genus of natural kinds, is the *event*
3. why is it desirable—or perhaps necessary?—for the statement of the maxim to be "more explicitly connected with the ideas of space and time"? I suspect: because these are required to serve as the basis for the framework within which mathematical representations of material systems can be constructed
4. "the nature, configuration, and motion of bodies" are the differentiæ that classify events into natural kinds relevant to the construction of equations of motion for their regular succession to conform to

3.4 Concepts

1. simple (or basic) concepts: explicit definition using basic terms (*e.g.*, "external action" and "internal action", §3)
2. complex concepts: built up in steps by developing simple concepts and elaborating physical operations and procedures, integrating into a synthesis (*e.g.*, "space" starting with "configuration" in §4 and culminating in §15; "material system" in §2 and clarified and elaborated in all subsequent chapters, *e.g.*, in §82)
3. there is often a virtuous circular feedback loop, simple concepts being initially articulated, used to build up complex concepts; the knowledge we gain from those complex concepts is then used in turn to elaborate and clarify the simple characterization of the initial concept into something richer and clearer, with cleaner articulation and wider and deeper scope for application—this is explained with great clarity in §102.
4. criterion of goodness is fruitfulness in application

Consider an example, to get an idea of the conceit and the method:

1. His definition of "vector" (§8) is based on the simple idea of motion, more particularly, the *operation* of "carrying a tracing point" along a line, *i.e.*, something that someone does. This is the same basis he uses to begin to build up the idea of space in §15. This, I think, is why vectors are peculiarly suited to represent the relative configuration of points in space. It is also why (§5) a "diagram is supposed to resemble the material system only in form, not necessarily in any other respect," *i.e.*, it "completely represents the configuration, but . . . has none of the other properties of the material system," and why "diagrams of velocity, of stress, &c, . . . do not represent the form of the system."
2. There is no vicious circularity here ("but motion is just change of position in space!"). We use the simple idea of the *operation* of an agent—something she *does*, for the understanding of which she needs no physical theory—to ground the idea of motion to get the idea of space

off the ground. There is a virtuous, dialectical circularity: as the idea of motion is refined, so in turn is the idea of space, and back again, each being clarified by deeper understanding of the other (see, again, §102). Thus, in §15, he says,

> We have described the method of combining several configurations into one system which includes them all. In this way we add to the small region which we can explore by stretching our limbs the more distant regions which we can reach by walking or by being carried.

The method he refers to is that of §13 (requiring the construction of a master affine module with congruence): the refinements in the concept of relative position up to this point, based on the idea of a vector itself grounded in the operation of a simple motion, allow us to construct the full concept of space needed, in turn, in ch. II, to elaborate and clarify the full concept of motion, giving it a wider and deeper scope of application. In particular, by this clarification and deepening, the concept of vector will have its scope expanded so as to cover velocity and acceleration, not just spatial configuration—thus further clarifying the concept of motion.

3. Maxwell is able to identify and correct Descartes' error (§16) because he has a richer and more fruitful conception of the idea of a "concept", and of concept formation, development, elaboration and clarification than Descartes had. Descartes' conception: start with "clear and distinct idea" and analyze by stripping away all features and parts seen as non-essential. Maxwell's conception is: start with simple, basic ideas and operations ans use these to build up complex concepts by construction and clarification, *i.e.*, by *synthesis*.

4. The former takes away what seems not to be needed, but it is never verified that what remains is what is actually needed. The latter from the start focuses only on what is needed for the concept to be able to support the tasks at hand. This is shown most piquantly by the fact that Descartes *nowhere uses this analysis of space nor his official definition of matter to support any substantive reasoning in his physics.* Maxwell, to the contrary, uses his own left and right. (Note also that Maxwell has shown by his construction that what Descartes had taken to be a "conceptual necessity" is in fact not so.)

5. This leads (§16) to the true criterion for the cogency, clarity—"soundness" ("I have referred to this opinion of Descartes in order to show the importance of sound views in elementary dynamics")—of concepts in physical science: *fruitfulness of application*:

> We shall find it more conducive to scientific progress to recognise, with Newton, the ideas of time and space as distinct, at least in thought, from that of the material system whose relations these ideas serve to coordinate.

3.5 Space, Time, Motion

How are they inter-related? We deal with space and motion first.

space

1. The idea of motion seems simpler, more basic, "prior" in some sense to that of space, at least: as remarked on already in these notes, it is used to build up the concept of space, and to define the concept of the geometrical object ("vector") that represents relative spatial positions. But in what sense, then, "prior"? As a matter of conceptual necessity? conceptual convenience? logical or causal or metaphysical necessity or convenience?
2. I suspect: conceptual convenience. The idea of space must be suited to the requirements of the kinematics and the dynamics; it thus makes sense to build it up from the start out of kinematical ideas and operations.
3. This leads us to the question: what geometrical structures is space (and its required to have in order to support the formulation of the more complex kinematical and dynamical concepts, relations, rules, principles, *et al.*? As always, Maxwell builds this up slowly, never assuming or positing more than is required up to that point in the investigation. In order:
   a. §7: he assumes the idea of straightness, so we need a projective structure; he speaks of directions, so we need an orientation on the projective structure
   b. §8: "parallel" requires an affine structure, compatible with the oriented projective structure
   c. §10: addition of vectors and ascertaining equality of line-segments requires the structure of a monoid and a congruence function compatible with the binary operation of the monoid (not yet a length or distance function)
   d. §11: subtraction of vectors requires the structure of a an algebraic module in order for subtraction to be compatible with the binary operation of the monoid and to retain compatibility with the congruence function (not yet a vector space)
   e. §12: freedom in choosing the origin requires the structure of an affine module compatible with the congruence function
   f. §13: fixing the relative positions of two independent systems requires the construction of a "master" affine module with congruence such that that the affine models with congruence associated with the two systems can each be mapped isomorphically into it—this is an *operation* as well
   g. §14: the practical application of the idea of fixing the relative positions of two independent systems requires a conformal structure (measurement of angles); but this is not yet required by the abstract formal treatment of the kinematics

time

1. the procedure is similar to that for space: we build up the concept of time, construct it by physical operations, making the idea ever more precise and clear by putting ever clearer ideas of physical in coordination with each other
2. we start with the idea of an ordered succession, based on a primitive notion of change derived from our own mental activity: thus we require the structure of an ordered set
3. all events find their place in the order, so it is a total order, and thence defines a 1-dimensional topology

4. we require the capacity to compare lengths of different temporal intervals, which, on a 1-dimensional topological space, is equivalent to an affine structure

3.6 Representation

1. Maxwell's views on mathematical representation of physical phenomena, how they acquire empirical content, and how representation more generally works in scientific reasoning, is implicit in early remarks and the kinds of diagrams he describes and explains:

§2 we must demarcate the regime to include in our representations

§3 we include external systems in our representations only to th degree they influence those in our fixed regime

§5 What is it for a digram to *resemble* a material system "in form"?

a. "resemble" ≈ analogical? implicational? depictive?
b. Note that "form" here refers to the *configuration* (relative positions) of a system's constituent parts.
c. In what sense can a 3-d model "*completely*" represent a 3-dimensional system?
d. Is it any more than definitonal that velocity diagrams do not represent (resemble) the "form" of a system?
e. I think I have in hand one example where the idea of resemblance is clear: how vectors represent relative configuration in space (see §3.4 "Concepts" below)

§6 "*for the purposes of our investigation*": the context-dependence, pragmatic character of representation

§7 operationalization ("start from A and travel in the direction indicated by the line $\overline{AB}$...") may not be necessary but it is sufficient, when carefully deployed with the right conceptual underpinning already in place (which Maxwll has provided in the initial sections) for empirical content to accrue to mathematical representations; and note how careful indeed Maxwell is in spelling out all the operations

§8 the role of operationalization comes out even more clearly now:

> The expression $\overline{AB}$, in geometry, is merely the name of a line. Here it indicates the operation by which the line is drawn, that of carrying a tracing point in a certain direction for a certain distance. As indicating an operation, $\overline{AB}$ is called a Vector...

the representation of mechanical systems is more than applied geometry; the concepts themselves are transformed in meaning in the translation into the context of physics

§§12–13 the arbitrariness, the freedom in our constructions indicates "slippage" between our mathematical models and the world, a *necessary* one (in the sense that it is unavoidable), but this is a *virtue* not a demerit: think how constrained and difficult physics would be without this freedom to arbitrarily choose features of our representations to best suit the needs of the tasks at hand

§§16–17 "absolute" time and space: both are discussed only in so far as Newton or a generic investigator *conceives* of them, and the use to which they are put in the system of physics; Maxwell does not say we are licensed to assert their metaphysical existence, much less attempt to do so (thus: ignore the editorial footnote inserted by Larmor in §18): thus, the math we use to codify and regiment our concepts of space and time do not themselves represent metaphysically existing entities; rather they mediate the way that our concepts represent the world

2. he hews to these views, thereby not only exemplifying but also illuminating them, in his elaborately detailed instructions for producing diagrams of different kinds—especially helps to make sense of why he spend so much time on diagrams that seem essentially isomorphic to each other

3. the fundamental point: math does not represent the world in any interesting sense—it rather mediates the way that our concepts represent the world; for brevity, however, I shall continue to use 'represent' and 'representation', *etc.*, in connection with mathematical structures

4. this, by the way, gives insight into why Maxwell relies so heavily on the method of physical analogy in his own research, and what his justification for it can be—but that is a story for another time

Maxwell (1879):

1. on the idea that "matter" must be defined in order to discuss the idea of mass in abstract dynamics (p. 214):

   > Why should we find it more difficult to endow moving figures with mass than to endow stationary figures with motion? The bodies we deal with in abstract dynamics are just as completely known to us as the figures in Euclid. They have no properties whatever except those which we explicitly assign to them.

   The mathematics represents nothing; we put it to use to help us bring our concepts and ideas into contact with the world. Math does to represent the world; it mediates the way our concepts grasp the world (when they do), and it does so by the way that we put the math to use.

2. representation, and concomitantly our knowledge of the "inner, hidden metaphysics" of the world, is limited by the concepts we have available to apply to the mathematics we use in modeling physical systems (pp. 214–215):

   > The phenomena of real bodies are found to correspond so exactly with the necessary laws of dynamical systems, that we cannot help applying the language of dynamics to real bodies, and speaking of the masses in dynamics as if they were real bodies or portions of matter.
   >
   > We must be careful, however, to remember that what we sometimes, even in abstract dynamics, call matter, is not that unknown substratum of real bodies, against

> which Berkeley directed his arguments, but something as perfectly intelligible as a straight line or a sphere.
>
> Real bodies may or may not have such a substratum, just as they may or may not have sensations, or be capable of happiness or misery, knowledge or ignorance, and the dynamical transactions between them may or may not be accompanied with the conscious effort which the word force suggests to us when we imagine one of the bodies to be our own, but so long as their motions are related to each other according to the conditions laid down in dynamics, we call them, in a perfectly intelligible sense, dynamical or material systems.
>
> . . . . ¶
>
> Whatever may be our opinion about the relation of mass, as defined in dynamics, to the matter which constitutes real bodies, the practical interest of the science arises from the fact that real bodies do behave in a manner strikingly analogous to that in which we have proved that the mass-systems of abstract dynamics must behave.

4 Invitation to a Short Essay

I invite you to write me a short discussion (no more than 2 pages, *i.e.*, no more than 1000 words) on any issue discussed in this lecture or any of this week's readings, required or suggested. You can raise further questions, propose answers or interpretations, or whatever seems of most interest to you. If you get it to me by the start of next lecture (4. May), then I will return it to you with my comments the following week.

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