“Singularities, Black Holes, Thermodynamics in Relativistic Spacetimes”: Problem Set 4 (causal structure)

1. Wald (1984): ch. 8, problems 1–2
2. Prove or give a counter-example: there is a spacetime with two points simultaneously connectable by a timelike geodesic, a null geodesic and a spacelike geodesic.
3. Prove or give a counter-example: if two points are causally related, there is a spacelike curve connecting them.
4. Prove or give a counter-example: any two points of a spacetime can be joined by a spacelike curve. (Hint: there are three cases to consider, first that the temporal futures of the two points intesect, second that their temporal pasts do, and third that neither their temporal futures nor their temporal pasts intersect; recall that the manifold must be connected.)
5. Prove or give a counter-example: every flat Lorentzian metric is temporally orientable.
6. Prove or give a counter-example: if the temporal futures of two points do not intersect, then neither do their temporal pasts.
7. Prove or give a counter-example: for a point p in a spacetime, the temporal past of the temporal future of the temporal past of the temporal future... of p is the entire spacetime. (Hint: if two points can be joined by a spacelike curve, then they can be joined by a curve consisting of some finite number of null geodesic segments glued together; recall that the manifold is connected.)
8. Prove or give a counter-example: if the temporal past of p contains the entire temporal future of q, then there is a closed timelike curve through p.
9. Prove or give a counter-example: if $q \notin I^-(p)$, then there exists a maximally extended past-directed causal curve from q that never enters $I^-(p)$.
10. Prove or give a counter-example: if q is simultaneously on the boundary of $I^+(p)$ and $I^-(p)$, then $q = p$.

References

Wald, R. (1984). *General Relativity*. Chicago: University of Chicago Press.