"Singularities, Black Holes, Thermodynamics in Relativistic Spacetimes": Problem Set 3
(metrics, relativistic spacetimes, causal structure)

1. Malament (2012): problems 1.9.1–1.9.7, 1.10.1–1.10.4, 1.11.1–1.11.2, 2.1.1–2.1.3, 2.2.1–2.2.3, 2.5.1–2.5.3, 2.7.2–2.7.3, 2.8.1–2.8.3, 2.9.1
2. Wald (1984): ch. 3, problems 3, 5–8; ch. 4, problem 6
3. What is the geometrical significance of each index of the Riemann tensor?
4. Prove or give a counter-example: a curve is a geodesic of a Riemannian metric if and only if its tangent vector has constant length along it.
5. Show that $\mathbb{S}^2$ does not admit a Lorentz metric. (Hint: $\mathbb{S}^2$ has no everywhere non-zero vector field, and a manifold admits a Lorentz metric if and only if its universal covering space does.)
6. Show that the definition of proper time for a given timelike curve is invariant under monotonic (but otherwise arbitrary) reparametrizations.
7. Can a manifold with no everywhere non-zero vector field (*e.g.*, $\mathbb{S}^2$) be parallelizable? Explain.
8. Can a manifold be not orientable but parallelizable? Is the Möbius strip parallelizable? Prove not or give an explicit representation of the parallelization.
9. Does $6g_{ab}$ yield the same spacetime physically? $6\nabla_a$? $6R^a{}_{bcd}$? Why or why not?
10. Show that $\nabla^n T_{na} = 0$, for dust and for perfect fluids respectively, implies that the worldlines of dust and the flowlines of perfect fluids are geodesics.

References

Malament, D. (2012). *Topics in the Foundations of General Relativity and Newtonian Gravitational Theory*. Chicago: University of Chicago Press. Uncorrected final proofs for the book are available for download at http://strangebeautiful.com/other-texts/malament-founds-gr-ngt.pdf.

Wald, R. (1984). *General Relativity*. Chicago: University of Chicago Press.