“Singularities, Black Holes, Thermodynamics in Relativistic Spacetimes”: Problem Set 2
(vector fields, tensors, derivations, curvature)

How can we lose when we’re so sincere?

Charlie Brown

1. Malament (2012): problems 1.2.1, 1.3.1–1.3.4, 1.6.1, 1.6.3–1.6.6, 1.7.1–1.7.2, 1.8.2–1.8.4
2. Wald (1984): ch. 2, problems 3, 6; ch. 3, problems 1 (parts a and b), 5
3. For an arbitrary tensor ${\alpha^a}_{bcd}$, show that ${\alpha^a}_{bcd} = 0$ if ${\alpha^a}_{ncd}\xi^n = 0$ for all tangent vectors ξ^a or if ${\alpha^n}_{bcd}\eta_n = 0$ for all cotangent vectors η_a. Assume ${\alpha^a}_{mnr}\xi^m\lambda^n\xi^r = 0$ for all tangent vectors ξ^a and λ^a; can we infer that ${\alpha^a}_{bcd} = 0$? Why or why not?
4. Represent the explicit act of contraction, using ${\delta^a}_b$, in ordinary matrix notation.
5. Does it make sense to (anti-)symmetrize on already contracted indices? Why or why not?

References

Malament, D. (2012). *Topics in the Foundations of General Relativity and Newtonian Gravitational Theory*. Chicago: University of Chicago Press. Uncorrected final proofs for the book are available for download at `http://strangebeautiful.com/other-texts/malament-founds-gr-ngt.pdf`.

Wald, R. (1984). *General Relativity*. Chicago: University of Chicago Press.